Turbulence Homework 1

Stochastic Tools in Turbulence, AEM-ADV18
Instructor: William K. George, 2013-01-22

References: Chapters 2 and Appendix B of WKG's 'Lectures in Turbulence for the 21st Century

Do the following experiment: Carry out 100 coin flips. (You can share data with each other.) Assign the value of 1 to each head, and 0 to each tail.

1. Use a simple arithmetic estimator for the mean value, say X_N, and plot how its value varies with N.

2. On the same plot show lines that show +/- the square root of the variance of X_N (i.e.,, $\sqrt{var\{X_N\}}$) versus N using the theoretical value determined in class (and in the notes).

3. Assuming the probability density function of $X_N - X$ to be Gaussian (at least for large enough N), what is the probability that that your value of X_N is more than one standard deviation larger (or smaller) than than your estimate.

4. Now use your data to compute an estimate of the variance of your coin flip values and show plot how it varies with N. Try it two ways, one using the theoretical value of 1/2 for the mean, and the other using your measured values of X_N. Which do you think is more accurate?

5. Now on your seco nd plot, show +/- the square root of the variance of your variance estimator (i.e., $\sqrt{(var\{var_N\})}$

6. Use the fact that the probability density function of your coin flips is bimodal (1 or 0) to compute the theoretical standard deviation and variance for a perfect coin.

7. How many coin flips would you need to determine the variability of your estimator for the mean (i.e. $\sqrt{var\{X_N\}}/X$) to within 0.1 %?

8. How many coin flips would you need to determine the variability of the estimator for the variance (i.e., $\sqrt{(var\{var_N\})}/\sqrt{var}$) to within 0.1 %?

9. Now use your data to make a histogram to estimate the probability of occurence of each value of the original realizations. Try it first using only 10 flips, then 20, etc. See if you can figure out what the variability of your histogram column is and how it varies with N and the probability of observing a given value. Do you see any relation to our results for estimators of the mean and variance?