Pressure Fields and Cavitation in Turbulent Shear Flows

Roger E. A. Arndt
University of Minnesota
Minneapolis, Minnesota

William K. George
State University of New York at Buffalo
Buffalo, New York

ABSTRACT

Cavitation in turbulent shear flows is the result of a complex interaction between an unsteady pressure field and a distribution of free stream nuclei. Experimental evidence indicates that cavitation is incited by negative peaks in pressure that are as high as ten times the rms level. This paper reviews the current state of knowledge of turbulent pressure fields and presents new theory on spectra in a Lagrangian frame of reference. Cavitation data are analyzed in terms of the available theory on the unsteady pressure field. It is postulated that one heretofore unconsidered factor in cavitation scaling is the highly intermittent pressure fluctuations which contribute to the high frequency end of the pressure spectrum. Because of limitations on the response time of cavitation nuclei, these pressure fluctuations play no role in the inception process in laboratory experiments. However, in large scale prototype flows, cavitation nuclei are relatively more responsive to a wider range of the pressure spectrum and this can lead to substantially higher values of the critical cavitation index. Unfortunately, this issue is clouded by the fact that higher cavitation indices can be found in prototype flows because of gas content effects. Some cavitation noise data are also examined within the context of available theory. The spectrum of cavitation noise in free shear flows has some similarity to the noise data found by Blake et al. (1977) with the exception that there appears to be a greater uncertainty in the scaling of the rate of cavitation events which leads to a substantial spread in the available data.

1. INTRODUCTION

The physical processes involved in cavitation inception have been studied for many years. Much of this research has been directed toward an understanding of the dynamics of bubble growth and the determination of the sources of cavitation nuclei and their size and number in a given flow situation. This research has led to a general understanding of some of the environmental factors involved in scaling experimental results from model to prototype. More recently, considerable attention has been paid to the details of the boundary layer flow over streamlined bodies and the role of viscous effects in the cavitation process. This research has shown that viscous effects such as laminar separation and transition to turbulence can have a major impact on the inception process and that there can be considerable variation between model and prototype in the critical conditions for cavitation.

In the absence of viscous effects, the scaling problem reduces to an understanding of the size distribution of nuclei and the temporal response of these nuclei to pressure variations as viewed in a Lagrangian frame of reference. This was first treated in detail by Plesset (1949). As already mentioned, consideration of viscous effects shows that the cavitation inception process can be considerably altered by either laminar separation or transition to turbulent flow. Obviously these phenomena are interrelated and are strongly Reynolds number dependent. The recognition of the importance of these factors has had considerable impact on the direction of cavitation research in recent years. Several papers in this symposium deal directly with this aspect of the cavitation scaling problem.

It is reasonably well understood that intense pressure fluctuations, either at the trailing edge of a laminar separation bubble or in the transition region, can have a major effect on the inception process on streamlined bodies. However, these phenomena will be excluded from this review. The focus of this paper will be on the relationship between the temporal pressure field and cavitation inception in free turbulent shear flows and fully developed boundary layer flows. Scant attention has been given to this problem, even though the topic is of practical significance. Turbulent shear flows are very common in practice and what cavitation data are available for these flows indicate that there can be significant scale effects. For example, Lienhard and Goss (1971) present a collection of cavitation data for submerged jets. It is observed that the critical value of the cavitation index increases with an increase in jet diameter, with no upper bound on the cavitation index being defined by the available data. The cavitation index is observed to vary from 0.15 to 3.0 over a size range of 0.1 cm to 13 cm. Arndt (1978) reviews the available data for cavitation in the wake of a sharp edged disk. These data increase monotonically with Reynolds number and again no upper limit on the critical cavitation index can be determined from the available data. At present, it can be said that laboratory experiments do not provide a reasonable estimate of the conditions that can be encountered under prototype conditions. From a practical point of view the situation is much more critical than the scaling problems associated with streamlined bodies since at present there is no definable upper limit on the cavitation index for these free shear flows.

There are a myriad of factors that enter into the inception process in turbulent shear flows. As a minimum, we need information on the turbulent pressure field, such as spectra and probability density. We require an understanding of the diffusion of nuclei within the flow, and we need to know how these nuclei respond to temporal fluctuations in pressure. In taking into account the bubble dynamics inherent in the problem, consideration must also be given to gas in solution which can have an influence on both bubble growth and collapse.

The theory of bubble dynamics is well founded and reasonable estimates of critical pressure can be determined under flow conditions that are well defined. Needless to say, the flow conditions in a turbulent shear flow cannot be defined in sufficient detail. However, the problem of flow noise has led to a more comprehensive understanding of turbulence; in particular, recent aeroacoustic research has provided a wealth of data on turbulent pressure fluctuations. These data are a by-product of the need for understanding turbulence as a source of sound. At this point in time, it seems only logical to review the inception problem in terms of both classical bubble dynamics and the more recent results of the field of aeroacoustics.

2. THEORETICAL CONSIDERATIONS FOR CAVITATION

Cavitation Index

The most fundamental parameter for cavitating flows is the cavitation index

$$\sigma = \frac{p_o - p_v}{\frac{1}{2}\rho U_o^2}$$

wherein p_o is a reference pressure, p_v the vapor pressure, U_o a reference velocity, and ρ the density of the liquid. The flow state of primary interest in this paper is characterized by a limited amount of cavitation in an otherwise single phase flow. There is a specific value of σ associated with this flow condition, which for convenience will be defined as the critical index:

$$\sigma_c = \frac{p_{oc} - p_v}{\frac{1}{2}\rho U_o^2}$$

If it is necessary to have completely cavitation free conditions, one design objective for various hydronautical vehicles is the minimization of σ_c.

Cavity flows are assumed identical in model and prototype for geometrically similar bodies when σ is constant, irrespective of variations in physical size, velocity, temperature, type of fluid etc. In practice σ_c is found to vary over wide limits. Simply stated, these so-called scale effects are due to deviations in two basic assumptions inherent in the cavitation scaling law; namely that the pressure scales with velocity squared and the critical pressure for inception is the vapor pressure, p_v. As will be shown, the two factors can be interrelated, since in principle the critical pressure is a function of the time scale of the pressure field.

In order to provide a foundation for the ensuing discussion, consider a steady uniform flow over a streamlined body devoid of any viscous effects. The following identity can be written:

$$\sigma = \frac{p - p_v}{\frac{1}{2}\rho U_o^2} - C_p$$

wherein C_p is a pressure coefficent defined in the usual manner. Generally speaking, C_p is defined by the pressure on the surface of a given body. It is generally assumed that cavitation first occurs when the minimum pressure, p_m, is equal to the vapor pressure, p_v. This results in the well-known scaling law

$$\sigma_c = -C_{p_m}$$

Consider next the case where the pressure in the cavitation zone is less than the minimum pressure measured on the surface of the body, then

$$\sigma = \frac{p_{ml} - p_v}{\frac{1}{2}\rho U_o^2} - C_p + \frac{p - p_{ml}}{\frac{1}{2}\rho U_o^2}$$

Here we have to distinguish between the pressure at the surface of the body p, and the pressure sensed by cavitating nuclei, p_{ml}. Assuming cavitation occurs when $p_{ml} = p_v$ we have

$$\sigma_c = -C_p + \frac{p - p_{ml}}{\frac{1}{2}\rho U_o^2} \qquad (1)$$

Equation (1) is one version of the superposition equation that is commonly referred to in the literature.

Bubble Dynamics

It is generally accepted that the process of cavitation inception is a consequence of the rapid or explosive growth of small bubbles or nuclei which become unstable due to a change in ambient pressure. These nuclei can be either imbedded in the flow or find their origins in small cracks and crevices at the surfaces bounding a given flow. The details of how these nuclei can exist have been considered by many investigators. A summary of this work is offered by Holl (1969, 1970).

Theoretically, liquids are capable of sustaining large values of tension. However, the nuclei in the flow act as sites for cavitation inception and prevent the existence of significant tensions. The mechanics of the inception process are adequately described by the Rayleigh-Plesset equation, which considers the dynamic equilibrium of a spherical bubble containing vapor and non-condensable gas and subject to an external pressure $p_{ml}(t)$:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left[p_v + p_g - p_{ml}(t) - \frac{2S}{R} - 4\mu\frac{\dot{R}}{R}\right] \quad (2)$$

wherein R is the bubble radius and dots denote differentiation with respect to time. It should be emphasized here that even for the case of steady flow over a streamlined body, $p_{ml}(t)$ is a function of time since we are concerned with the pressure history sensed by a moving bubble. If the problem is simplified to consider the static equilibrium of a bubble, we find that there is a critical value of $p_v - p_{ml}$ below which static equilibrium is not possible. This is found to be

$$(p_v - p_{ml})_c = 4S/3R^* \quad (3)$$

wherein R^* is defined as the critical bubble radius. Substitution of Eq. (3) into Eq. (2) with dynamical terms identically zero will indicate that R^* is a function of the partial pressure of noncondensable gas within the bubble. If $p_{ml}(t)$ varies rapidly in comparison to the response time of the nuclei, then even greater values of tension are possible. Thus in general we can write

$$\frac{p_v - p_{ml}}{4S/3R^*} = \phi\left[t_c\sqrt{\frac{S}{\rho R_*^3}}\right]$$

where

$$\phi(o) = \infty, \quad \phi(\infty) = 1$$

The function ϕ depends on the flow field. The argument of ϕ contains a characteristic time scale of the pressure field (t_c) and a characteristic response time of the nuclei, $(\rho R_*^3/S)^{1/2}$. In the case of a streamlined body in the absence of viscous effects, t_c would be proportional to the quotient of body diameter and velocity. In the case of cavitation induced by turbulence, the characteristic time scale could be any of the turbulence time scales. For example,

$$L_{11}/\sqrt{\overline{u_1'^2}}$$

is often appropriate. The factor $(\rho R_*^3/S)^{1/2}$ is derived from the asymptotic solution to Eq. (2) for the case of negligible gas diffusion. Under these conditions

$$p_g \sim \frac{1}{R^3}$$

and the growth rate stabilizes at a value given by

$$\dot{R} = \sqrt{\frac{2}{3}\frac{p_v - p_{ml}}{\rho}} \quad (4)$$

Assuming a characteristic bubble response time given by $R^*/\dot{R}$, with $p_v - p_{ml} = 4S/3R^*$, we obtain

$$T_B \equiv \frac{R^*}{\dot{R}} = 0.87\sqrt{\frac{\rho R_*^3}{S}} \quad (5)$$

A typical variation of ϕ based on the theoretical computations of Keller (1974) is given in Arndt (1974).

The Influence of Dissolved and Free Gas

The discussion in the previous section is based on the assumption of a healthy supply of free nuclei which is generally the case in recirculating water tunnels and in the field. Generally speaking, a reduction in σ_c due to bubble dynamic effects usually only occurs on model scale. To some extent the level of dissolved gas and the number and size of free nuclei are interrelated. Some recent experimental results are documented in Arndt and Keller (1976). The level of dissolved gas can play an important direct role when the time of exposure to reduced pressure is relatively long. Under these circumstances Holl (1960) has shown that gaseous cavitation can occur at values of σ much greater than those for vaporous cavitation. Using an equilibrium theory, Holl (1960) deduced an upper limit on σ_c given by

$$\sigma_c = -C_{pm} + \frac{\alpha\beta}{\frac{1}{2}\rho U_o^2} \quad (6)$$

wherein α is the concentration of dissolved gas and β is Henry's constant.

In summary, an overview of the effects of bubble dynamics and free and dissolved gas indicates that short exposure times such as are the case in a model implies that cavitation will occur at pressur lower than vapor pressure and σ_c is less than expected. Long exposure time, such as can occur in vortical motion of all types, including large scale turbulence, implies the possibility of gaseou cavitation with σ_c being greater than expected.

3. PRESSURE FLUCTUATIONS IN TURBULENT SHEAR FLOWS

Background

Considerable progress has been made over the last five years in the understanding turbulent pressure fluctuations in free shear flows in an Eulerian frame of reference. Of particular importance is the development of pressure sensing techniques which under certain circumstances can lead to reliable measurements of pressure fluctuations.

The first theoretical arguments on the pressure fluctuations associated with turbulent flow appear to be due to Obukov and Heisenberg [Batchelor (1953)]. Heisenberg argued that Kolmogorov scaling should be possible for small scale pressure fluctuations. Batchelor (1951) was able to calculate the mean square intensity of the pressure fluctuations as well as the mean square fluctuating pressure gradient in a homogeneous, isotropic turbulent flow. This work was extended by Kraichnan (1956) to the physically impossible but conceptually useful case of a shear flow having a constant mean velocity gradient and homogeneous and isotropic turbulence.

Apparently there were no attempts made to extend this theoretical work until the 1970's when George (1974a), Beuther, George, and Arndt (1977a, b, c) and George and Beuther (1977) applied the concepts developed by Batchelor and Kraichnan to the calculation of the turbulent pressure spectrum in honogeneous, isotropic turbulent flows with and without shear. When compared with experimental evidence gathered in turbulent mixing layers, the theory is found to be remarkably accurate. The predicted spectrum (with no adjustable constants) agrees with pressure measurements in turbulent jet mixing layers from several sources, including those of Fuchs (1972a), Jones and his co-workers (1977), and the authors themselves. As shown in Figure 1, the experimental data and the theory are remarkably consistent, especially in light of the fact that several different experimental techniques and different flow facilities are involved.

The current state of knowledge of turbulent pressure fluctuations can be summarized as follows:

1) Pressure fluctuations in a shear flow can arise from three sources. The first two involve interaction of the turbulence with the mean shear. These are second order and third order interactions, of which only the second order interactions are important at small scales. The last involves only interactions of the turbulence with itself.

2) Kolmogorov similarity arguments can be applied to each of the spectra arising from these terms. These arguments are valid for the small scale fluctuations.

3) If the turbulent Reynolds number is high enough, there exists an inertial subrange in each of the three spectra in which

$$\pi_{MS1}(k) = \alpha_{s1}\rho^2 \varepsilon^{2/3} Kk^{-11/3}$$

$$\pi_{MS2}(k) = \alpha_{s2}\rho^2 \varepsilon Kk^{-9/3}$$

$$\pi_T(k) = \alpha_T\rho^2 \varepsilon^{4/3} k^{-7/3}$$

wherein $\alpha_{s1} \cong 2$, $\alpha_{s2} \cong 0$, $\alpha_T \cong 1.3$, ε is the rate of dissipation of turbulent energy per unit volume, K is the mean shear, and k is the disturbance wave number.

4) There is considerable evidence that coherent structures play an important role in determination of at least the large scale pressure fluctuations [Fuchs and Michalke (1975), Fuchs (1972a, b), Chan (1974a, b), and Chan (1976)].

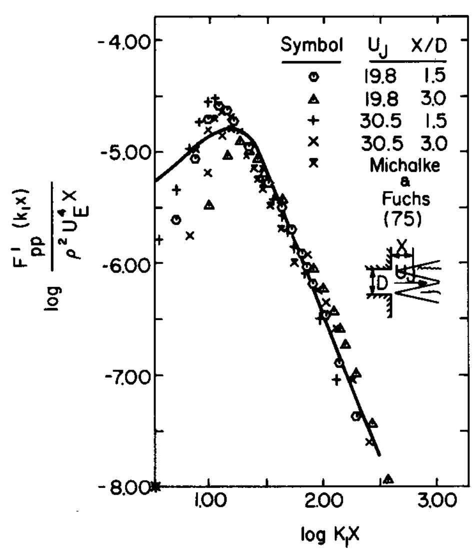

FIGURE 1. Experimental confirmation of the theoretical pressure spectrum for a turbulent jet.

Relation to Cavitation

Since the above spectral results are expressed in Eulerian frames, they cannot be directly applied to the problem of cavitation inception which is a Lagrangian problem. Nonetheless, Kolomogorov scaling has been successful in an Eulerian frame of reference and therefore we can, with some confidence, infer that similar scaling will be valid for Lagrangian time spectra (i.e. the frequency spectra that would be seen by a moving material point). The results of such an exercise are as follows:

1) The Lagrangian turbulent spectrum can be separated into interaction of the turbulence with the mean shear and the interaction of the turbulence with itself.

2) The high frequency (analogous to small scale) will be well described by Kolmogorov scaling such that

$$\frac{1}{\rho^2} \Lambda pps(\omega) = K^2 \nu^{5/2} \varepsilon^{-1/2} f_s\left(\frac{\omega}{\omega_d}\right)$$

$$\frac{1}{\rho^2} \Lambda ppT(\omega) = \nu^{3/2} \varepsilon^{1/2} f_T\left(\frac{\omega}{\omega_d}\right)$$

where

$$\omega_d = \left(\frac{\varepsilon}{\nu}\right)^{1/2}$$

3) In the inertial subrange these reduce to

$$\frac{1}{\rho^2} \Lambda pps(\omega) = K^2 \nu^{5/2} \varepsilon^{-1/2} \left(\frac{\omega}{\omega_d}\right)^{-5}$$

$$\frac{1}{\rho^2} \Lambda ppT(\omega) = \nu^{3/2} \varepsilon^{1/2} \left(\frac{\omega}{\omega_d}\right)^{-3}$$

In summary it appears plausible to assume that the basic picture of pressure fluctuations arising from mean-shear turbulence interactions will be unchanged in a Lagrangian frame of reference, although the actual spectra are different. The postulated relations for Lagrangian spectra should be directly applicable to any Lagrangian phenomenon; in particular the relations should be applicable to the inception of nuclei in a fluctuating pressure field.

In relating the information on the pressure field to the problem at hand, it is evident that two criteria must be satisfied for turbulence induced inception:

1) The pressure must dip to the vapor pressure or lower.

2) The pressure minimum must persist for a time that is long in comparison to the characteristic time scale of the bubble, say T_B (taken to be the time scale for growth at inception).

Both factors lead to scale effects. Consider first the second factor. The preceding arguments for the pressure field in a Lagrangian frame of reference lead to the hypothetical spectrum shown in Figure 2. For convenience we have normalized the spectrum with respect to the mean square pressure and the Lagrangian time scale $\mathcal{T}$. (c.f. Tennekes and Lumley, Chapter 8). Requirement (2) for bubble growth is plotted at the frequency $\omega = 1/T_B$. It is clear that as long as $\omega << 1/T_B$, any pressure flucuation persists for a time longer than the time scale of the bubble. Thus at frequencies less than $\omega = 1/T_B$ cavitation inception can occur with minimal local tension. Moreover, by integrating the spectrum from $\omega = 0$ to $\omega = 1/T_B$, we can determine that fraction of the mean square pressure which can contribute to bubble growth without appreciable tension (assuming a normal distribution of nuclei).

Consider now the effect of maintaining T_B constant while varying the Reynolds number. Taking $\mathcal{T} \sim \ell/u'$ and noting that there are essentially no pressure fluctuations of interest above the Kolmogorov frequency, $\omega = (\varepsilon/\nu)^{1/2}$ we find that after $1/T_B$, exceeds $(\varepsilon/\nu)^{1/2}$, the entire spectrum can potentially contribute to bubble growth. This will occur when the Reynolds number is roughly

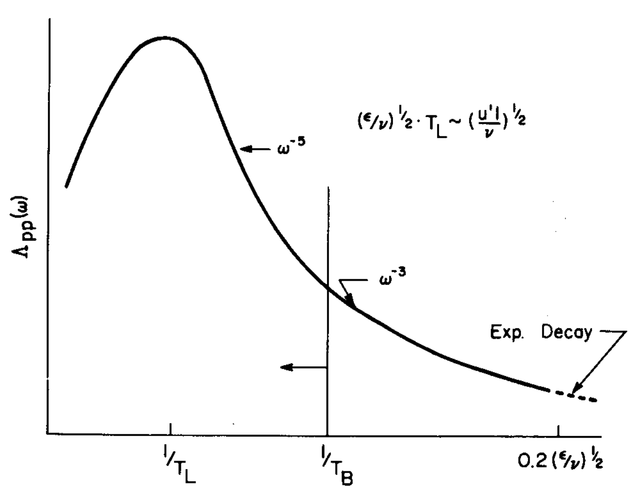

FIGURE 2. Hypothetical pressure spectrum in a Lagrangian frame of reference.

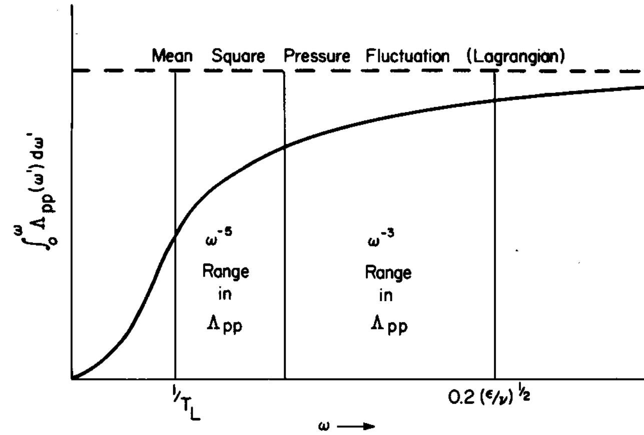

FIGURE 3. Integration of Lagrangian pressure spectrum.

$u\ell/\nu \sim (\ell/uT_B)^2$. By noting the spectral dependence on frequency and performing a running integral, a plot such as shown in Figure 3 can be generated. This graph illustrates how rapidly the asymptotic state is reached. This occurs when $\mathcal{T}/T_B^{-1} > \mathcal{T}(\varepsilon/\nu)^{1/2} \geq (u'\ell/\nu)^{1/2}$ or when $\ell/u'T_B > (u'\ell/\nu)^{1/2}$ as previously stated.

As an example,* cavitation is observed to occur in submerged jets at an axial position, x, that is roughly one diameter from the nozzle. Assuming the dissipation rate to be approximately $0.05U_J^3/x$, where U_J is the jet velocity, results in a criterion that the jet diameter must exceed the following before scale effects are absent: $d > 0.05U_J^3T_B^2/\nu$. Using typical values of $U_J = 10$ m/s and $T_B = 10^{-3}$ sec., we conclude that the asymptote is reached for $d \sim 50$ meters. Thus size effects could be important in many model experiments.

Effect of Intermittency at Small Scale

In 1947, Batchelor and Townsend concluded from observations of the velocity derivatives in turbulent flow that the fine structure of the turbulence (small scales, high frequency) was spatially localized and highly intermittant in high Reynolds number flows. Subsequent work [c.f. Kuo and Corrsin (1971)] has confirmed that there is a decrease in the relative volume occupied by the fine structure as the Reynolds number is increased. Thus the spatial intermittancy increases with Reynolds number. The effect of this phenomenon on filtered hot wire signals is shown in Figure 4. These data are derived from Kuo and Corrsin (1971). It is obvious from these data that the signal is increasingly intermittant as the filter frequency is moved to higher and higher values.

Since the dissipation of turbulent energy takes place at the smallest scales of motion, it is clear from these observations that the rate of dissipation of turbulent energy must vary widely with space and time. It was this consideration that led

*Strictly speaking, these results are only applicable when the Lagrangian turbulent field is stationary. In most flows of interest this is seldom the case. However, the smallest scales of motion can often be considered to be in quasi-equilibrium.

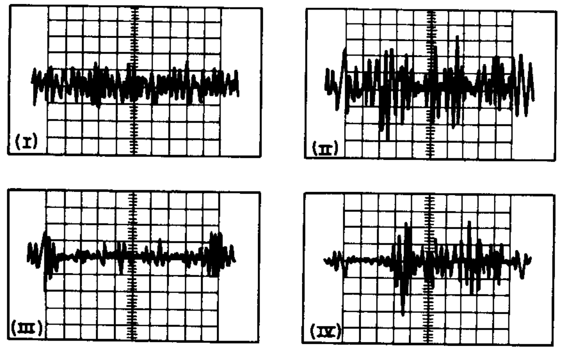

FIGURE 4. Filtered hot-wire signals in grid-generated turbulence [adapted from Kuo and Corrsin (1971)]. (i) f_m = 200 Hz, f/f_m = 0.52, 20 ms/division (horizontal scale); (ii) 1 kHz, 0.52, 4; (iii) 6, 0.52, 1; (iv) high-pass signal, f_c = kHz, 1 ms/division.

Kolmogorov (1962) to reformulate his original similarity hypothesis in terms of the average rate of dissipation of turbulent energy $\langle\varepsilon\rangle$, and to assume that the logarithm of ε was governed by a normal distribution. Later work by Gurvich and Yaglom (1967) showed that any non-negative quantity governed by fine scale components has $a_2$log normal distribution with a variance given by $\Sigma = A + B \ln R_\ell$ where A is a constant depending on the structure of the flow, B is a universal constant and R_ℓ is the turbulence Reynolds number.

These results have implications for the cavitation problem at hand. Beuther, George, and Arndt (1977a, b) have shown that Kolmogorov similarity scaling is applicable to the high wave number turbulent pressure spectrum. As a consequence of this and the observed intermittancy and spatial localization of small scale velocity fluctuations, it is reasonable to expect the same trend in the small scale pressure fluctuations. This could result in an important cavitation scale effect.

To make this point clear, a set of hypothetical band passed pressure signals at high and low Reynolds number are presented in Figure 5. For the sake of argument, assume that the filter is set around a range of frequencies which will result in bubble growth ($\omega T_B \leq 1$). Since the spectra of these two signals will be identified in terms of Kolmogorov variables and since the low Reynolds number signal is less intermittant, there is a greater probability that the high Reynolds number signal will contain more intense deviations from the mean. In particular, with all other factors held equal it is more likely that the local pressure will fall below the critical pressure when the Reynolds number is high, even though the spectra are identical. This is shown in Figure 5. If the log normal arguments were applicable, then it can be expected that this will depend on the Reynolds number.

The effect of intermittancy coupled with effects cited earlier could be of considerable importance to the problem of predicting cavitation inception in the prototype from small scale experiments in the laboratory. The Reynolds number in model and prototype can vary by many orders of magnitude. For example, experimental observations of boundary layer cavitation by Arndt and Ippen (1968) were carried out at Reynolds numbers, $u'\delta/\nu$, of the order 5000. On large ships, Reynolds numbers of 10^6 and greater are not uncommon.

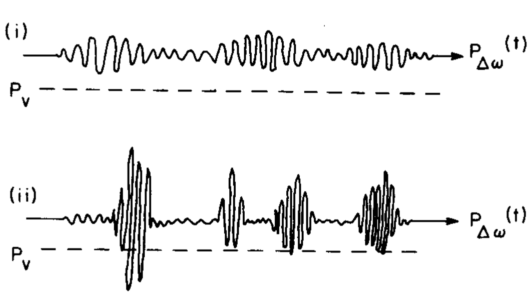

FIGURE 5. Hypothetical band-passed pressure signals: (i) low turbulent Reynolds numbers, (ii) high turbulent Reynolds number.

Coherency of the Pressure Field

An important factor related to cavitation inception in jets is the existence of coherent structure in the flow. Cavitation in highly turbulent jets is observed to occur in ring like bursts, smoke rings if you will. These bursts appear to have a Strouhal frequency fd/U_J of approximately 0.5. This point is underscored by some recent work of Fuchs (1974). Fuchs made 2 and 3 probe pressure correlations as shown in Figure 6. His results are summarized in Table 1. Signals filtered at a Strouhal number of 0.45 were highly coherent. For comparison, velocity correlations are shown in parentheses indicating that the velocity field is much less coherent than the pressure field.

The Turbulent Boundary Layer

Because of the relative ease of measurement, there exists a considerable body of experimental data

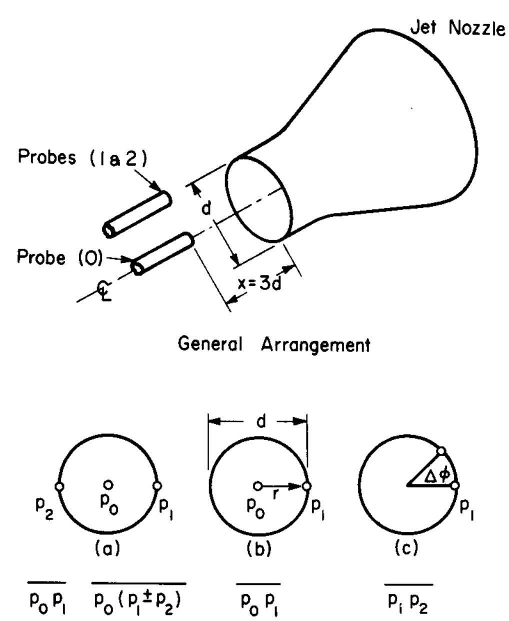

FIGURE 6. Measurement of pressure coherency in a turbulent jet [adapted from Fuchs (1974)].

for wall pressure due to turbulent boundary layer flow. However, in many ways less is known about the turbulent pressure field for boundary layers than for free turbulent shear flows. Not only is the theoretical problem made more difficult (impossible to the present) by the presence of the wall, the experimental problem is considerably complicated by the dynamical significance of the small scales near the wall.

Thus, in spite of over two decades of concentrated attention we cannot say with confidence even what the rms wall pressure level is, although recent evidence points to a value of [Willmarth (1975)]:

$$p' = c \rho u_*^2$$

$$c = 2 \text{ to } 3$$

The basic problem is that the most interesting part of a turbulent boundary layer appears to the region near the wall where intense dynamical activity apparently gives rise to the overall boundary layer activity. While the details of the process are debatable, most investigators concur on the importance of the wall region on overall boundary layer development. Unfortunately, under most experimental conditions, the scales of primary activity are smaller than standard wall pressure probes can resolve [Willmarth (1975)]. Thus we have virtually no information concerning the contribution of the small scales to the pressure field, although we suspect that the small scales are significant or even dominant.

Pressure Spectra in Boundary Layers

Our knowledge of the pressure spectra may be summarized as follows:

1) Pressure fluctuations arising from motions in the main part of the boundary layer ($y/\delta > 0.1$) scale with the outer parameters u_* and δ.

2) Pressure fluctuations arising from the inner part of the boundary layer scale with the inner parameters:

a) hydraulically smooth, u_*, ν

b) hydraulically rough, u_*, h; where h is roughness height

3) Pressure fluctuations arising from the inertial sublayer (logarithmic layer) scale only with u_* and y, the distance from the wall.

4) The wall pressure spectrum is a composite of all these factors and has a distinct region corresponding to each factor.

A composite picture of the wall pressure spectrum is shown in Figures 7a and 7b. The 1/k range is evident in both the inner and outer scalings and arises from the inertial sublayer contribution [Bradshaw (1967)].

The pressure spectrum within the near wall region should closely resemble the wall spectrum (although this has never been confirmed). The spectrum in the main part of the boundary layer, should, however, resemble that obtained for a free shear flow at high Reynolds numbers. Again there is no information available to either prove or disprove this conjecture

The Lagrangian model developed in the preceding section depends in part on the assumption that a material point is in a stationary random field. As long as the Eulerian field is homogeneous, there is no problem. This is approximately true in many shear flows, but is never true in a turbulent boundary layer. Thus our Lagrangian spectral picture must be abandoned entirely (or used with great restraint).

However, a number of features of the Lagrangian model can be applied to this problem. In particular, the "spectral peaks" in the outer flow can be identified with the Lagrangian integral scale, $\mathcal{J} \sim \ell/u'$. The highest frequencies in the flow will

Table 1. Normalized correlation functions with pressure probes arranged as shown in Figure 6 (corresponding velocity correlations in brackets).

	Signals Unfiltered	Signals Filtered at St = 0.45
$\dfrac{\overline{p_0 p_1}}{\sqrt{\overline{p_0^2}}\ \sqrt{\overline{p_1^2}}}$	+0.35 (+0.03)	0.66 (0.13)
$\dfrac{\overline{p_0\ (p_1+p_2)}}{\sqrt{\overline{p_0^2}}\ \sqrt{\overline{(p_1+p_2)^2}}}$	+0.57 (+0.07)	0.83 (0.19)
$\dfrac{\overline{p_0\ (p_1-p_2)}}{\sqrt{\overline{p_0^2}}\ \sqrt{\overline{(p_1-p_2)^2}}}$	-0.07 (+0.04)	0.06 (0.02)

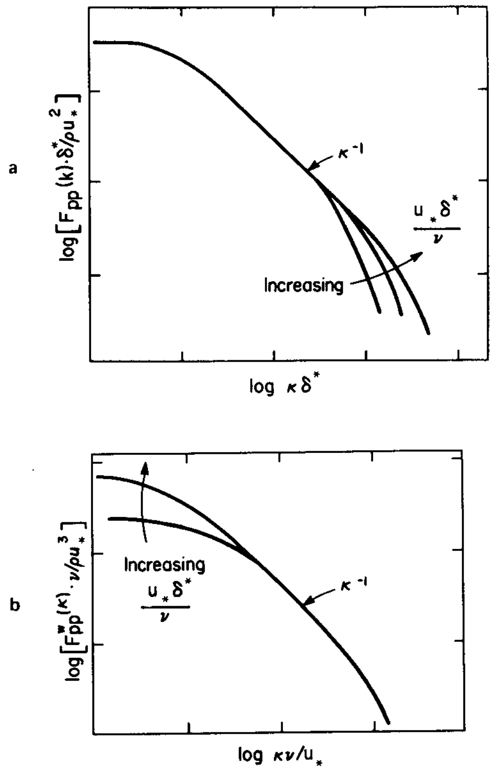

FIGURE 7. Wall pressure spectra: (a) outer scaling, (b) inner scaling.

be u_*^2/ν or u^*/h, depending on whether the wall is hydraulically smooth or rough, and there will be increasing intermittency with increasing Reynolds number. The latter effect is most interesting and is quite evident in the many observations of dye streaks in the wall layer [cf Kim, Kline, and Reynolds (1971)].

Effect of the Pressure Field on Cavitation

Whether or not the pressure fluctuations play a role in the cavitation inception process, depends on the previously cited criteria:

1) The minimum pressure must fall below a critical level.
2) The minimum pressure must persist below the critical level for a finite length of time.

The first criterion depends greatly on the yet unresolved question of intermittency and its effect on the probability density of the pressure fluctuations. At this point in time we can say that the critical cavitation index will increase with Reynolds number because larger excursions from the mean pressure are more likely. Without justification, it is hypothesized that the effect on the pressure variance will be approximated by a log-normal dependence on the Reynolds number. Detailed study of the wall pressure such as that proposed by George (1975) should aid considerably in resolving this question.

The question of time scale is more easily confronted. Since most of the energy in the pressure spectrum scales with u_* and δ it is clear that the criteria for bubble growth without appreciable tension reduces to

$$u_* T_B/\delta \le 1$$

In words, we again require a pressure fluctuation to persist for a time which is long in comparison to the response time of a typical nucleus.

Since ν/u_*^2 is the shortest time scale in a smooth wall boundary layer, all of the pressure spectrum is sampled by the nuclei when

$$u_*^2 T_B/\nu < 1$$

This criterion is especially important in view of the highly intermittant process near the wall.

For rough walls, the last criterion can be expressed in terms of the roughness height h by

$$u_* T_B/h < 1$$

Since in fully rough flow $u_* h/\nu > 1$, it is clear that the small scale criterion is more easily satisfied with rough wall experiments.

In summary, the information we have on pressure fields in turbulent boundary layers and its relationship to cavitation inception can be summarized as follows:

Significant scale effects can be expected when $u' T_B/\delta > 1$. As the ratio of T_B to the smallest time scale in the flow decreases, the scale effect would be expected to level off i.e. when $u_*^2 T_B/\nu$ or $u_* T_B/h < 1$. Further increase in the cavitation number with Reynolds number will be due to the Reynolds number dependent effects on the probabilit density of the pressure fluctuations as a result of increased intermittancy of the small scale structur The latter effect should produce a more gradual dependence of the cavitation index on Reynolds number than the former effect.

The picture, as displayed above, is plausible and perhaps even appealing, but it must be viewed simply as conjecture until definitive experimental information is made available. An important hint of the relevance of these results can be found in the work of Arndt and Ippen (1967) where it was found that the region of maximum cavitation in a rough boundary layer shifted inward with a decrease in $u_* T_B/h$. However, the change in this parameter varied only by a factor of 15 in their experiments. This will be discussed in more detail in subsequent sections.

4. CAVITATION INCEPTION DATA

A rather limited amount of experimental data have been collected under controlled conditions. The types of flows considered to date include the wake behind a sharp edged disk, submerged jets from nozzles and orifices, and smooth and rough boundar layers. There is a dearth of information relating the observed cavitation inception with the turbule parameters. Some of the earlier efforts in this direction are summarized in a paper by Arndt and Daily (1969) and by Arndt (1974b). A collation of available data is presented in Figure 8. Here the data are presented in the form of Eq. (1):

$$\sigma_c + C_p = f\,(C_f)$$

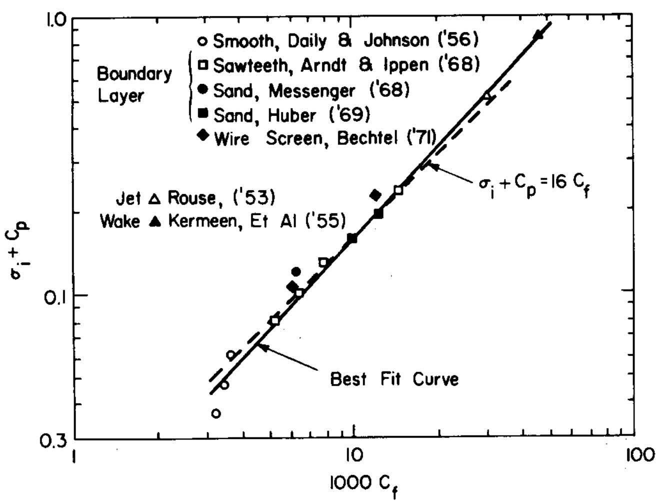

FIGURE 8. Collation of cavitation inception data.

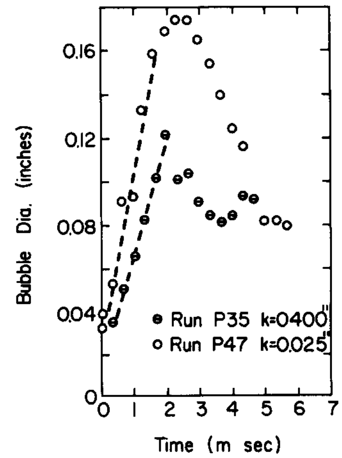

FIGURE 9. Sample bubble growth data [after Arndt and Ippen (1968)].

wherein

$$C_f = \begin{cases} 2\tau_0/\rho U^2 & \text{Boundary Layer Flow} \\ \dfrac{\overline{u_1 u_2}}{U_o} & \text{Free Shear Flows} \end{cases}$$

In this expression C_f is computed either from the measured wall shear in the case of boundary layer flows or from turbulence measurements made in the air at comparable Reynolds numbers for the case of a free jet and a wake. The measured value of C_p is only significant for the case of the disk wake and the pressure data was determined from the experimental work of Carmodi (1964). The available data seem to be well approximated by the relation

$$\sigma_c + C_p = 16\ C_f$$

which was originally proposed for boundary layer flow by Arndt and Ippen (1968). These data would seem to imply that a relatively simple scaling law already exists and would further imply that the previous discussion in this paper on turbulence effects is superfluous. This is not the case. Arndt and Ippen (1968) made observations of the bubble growth in turbulent boundary layers. Some of their results are depicted in Figures 9 and 10. Figure 9 shows sample bubble growth data. The growth rate is observed to stabilize at a constant value during most of the growth phase. Using Eq. (4), the levels of local tension are found to be quite small, of the order 20 to 100 millibar. These data correspond to observations in a rough boundary layer. Of particular interest is the fact that, in all cases, the life time for bubble growth is a fraction of the Lagrangian time scale, $\mathcal{J} = \delta/u'$. In fact growth times were observed to be of the order h/u_*. Unfortunately there is not enough experimental evidence available to completely illuminate this point. As shown in Figure 10, cavitation occurs roughly in the center of the boundary layer with a tendency for the zone of maximum cavitation to shift inward as $u_* T_B/h_s$ decreases from about 1.5 to approximately 0.1*. In the cited boundary layer experiments, C_p is negligible. Thus $\sigma_c = 16\ C_f$. Noting that p' is approximately 2.5 ρu_*^2 at the wall, we estimate that cavitation is incited by negative peaks in pressure of order 6 p'. This compares favorably with Rouse's (1953) data for jet cavitation which indicate that negative peaks of order 10 p' are responsible for cavitation.

A strong dependence on Reynolds number can be observed even in free shear flows. Figure 11 contains cavitation data for a sharp edged disk. These data were obtained in both water tunnels and a new depressurized tow tank facility located at the Netherlands Ship Model Basin. The water tunnel data are for cavitation desinence, whereas the tow tank data are for cavitation inception determined acoustically. The cross hatched data were determined in a water tunnel at high velocities by Keermeen and Parkin (1957). All the other data were obtained at relatively low velocities (2 - 10 m/sec). There is considerable scatter in these data and this is traceable to gas content effects

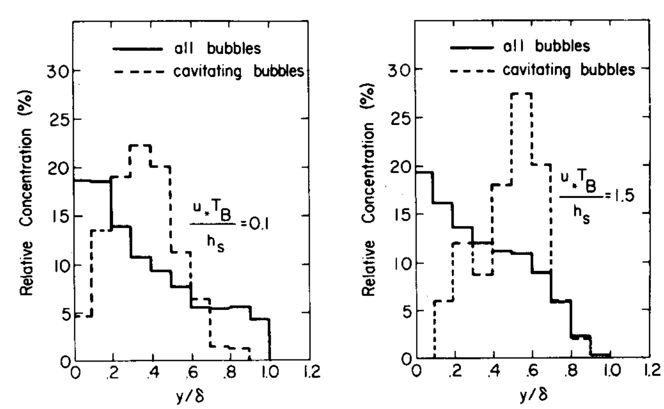

FIGURE 10. Observation of cavitation in turbulent boundary layers [after Arndt and Ippen (1968)].

*T_B was estimated from Eq. (5) using observed values of R_* reported in Arndt and Ippen (1967). For convenience, the results are normalized to equivalent sand grain roughness, h_s.

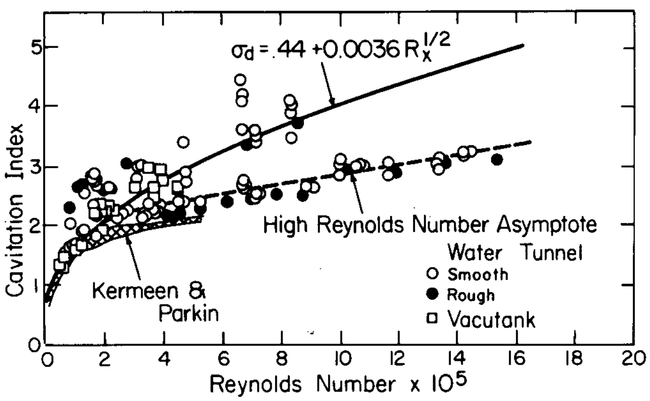

FIGURE 11. Cavitation inception data for a sharp-edged disk.

which are dominate at low velocities as will be discussed later. At low Reynolds number the data appear to be satisfied by the empirical relationship discussed by Arndt (1976):

$$\sigma_c = 0.44 + 0.0036\ (Ud/\nu)^{\frac{1}{2}} \qquad (7)$$

It was found that the tow tank data agree with this relationship at relatively high Reynolds numbers. Equation (7) was developed from a model which assumes laminar boundary layer flow on the face of the disk. It would be expected that this condition would be satisfied at higher Reynolds numbers in a tow tank than in a highly turbulent water tunnel. At high Reynolds number (and also high velocity where gas content effects are negligible), there is a continuous upward trend in the data with increasing Reynolds number. This underscores the need for further work as suggested in the introduction to this paper.

A systematic investigation of gas content effects in free shear flow was recently reported by Baker et al. (1976). Cavitation inception in confined jets, generated either by an orifice plate or a nozzle, was determined as a function of total gas content in the liquid. The results are shown in Figure 12. When the liquid was undersaturated at test section pressure, the critical cavitation index was independent of gas content and roughly equal to that observed by Rouse (1953) for an unconfined jet. When the flow is supersaturated, the cavitation index is found to vary linearly with gas content as predicted by the equilibrium theory, Eq. (6). This effect occurs even though the Lagrangian time scale is much shorter than typical times for bubble growth by gaseous diffusion. For example, in the cited cavitation data, a typical residence time for a nucleus within a large eddy is roughly 1/15 of a second. At a gas content of 7ppm and a jet velocity of approximately 10 m/s, inception occurs at a mean pressure equivalent to a relative saturation level of 1.25. Epstein and Plesset (1950) show that for growth by gaseous diffusion alone, 567 seconds is required for a 10^{-3} cm nucleus to increase its size by a factor of 10. One additional point should be kept in mind here. The local pressure within an eddy is much less than the mean pressure and highly supersaturated conditions can occur locally. Arndt and Keller (1976) also reported extreme gas content effects in their experiments with disks when the flow was supersaturated. The magnitude of the effect also depends on the number of nuclei in the flow. Gas content effects were noted only in their water tunnel experiments (where there is a healthy supply of nuclei). No gas content effects on inception were noted in the tow tank (where the flow is highly supersaturated but there is a dearth of nuclei). Thus the picture becomes more cloudy as the influence of dissolved, noncondensable gas is taken into consideration.

5. SOME REMARKS ON CAVITATION NOISE

A complete discussion on cavitation noise would be beyond the scope of this paper. Recognizing the unique features of cavitation inception in turbulent shear flows, it appears appropriate to review what is known about cavitation noise under the same circumstances.

The general features of cavitation noise were reviewed by Fitzpatrick and Strasberg (1956), Baiter (1974), and Ross (1976). The spectrum of cavitation noise can in its simplest form be defined as the linear superposition of N cavitation events per unit time. Thus we can write

$$S(f) = N\ G(f) \qquad (8)$$

The function G(f) is the spectrum of a single cavitation event. If p_b is the instantaneous acoustic pressure due to the growth and collapse of a single bubble, then by definition

$$\int_0^{\infty} G(f)df = \int_{-\infty}^{\infty} p_b^{\,2}\ dt$$

Fitzpatrick and Strasberg (1956) have shown that a characteristic bubble spectrum can be written in the form

$$G(f\tau_0) = \frac{r^2 G(f)}{\rho R_m^{\ 4}\ p_o}$$

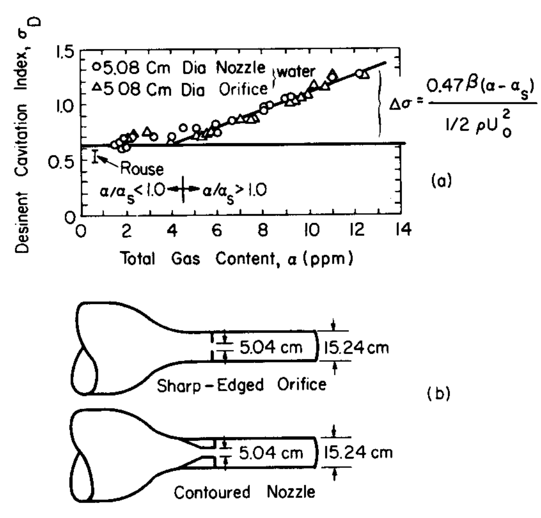

FIGURE 12. Cavitation inception in confined jets.

wherein τ_o is a characteristic bubble collapse time, R_m is the maximum bubble radius, and R is the distance to the observer. In addition, it appears reasonable to assume that N is related to the number of nuclei per unit volume, n, the velocity, the size of a given flow field, and the relative level of cavitation. Therefore we write

$$N/nU_o d^2 = f(\sigma/\sigma_c)$$

Thus a normalized version of Eq. (8) would be

$$\frac{S(f)\,\frac{r^2}{d}}{\rho U_o p_o n R_m^{4}} = f(\sigma/\sigma_c)\ G(f\tau_o) \qquad (9)$$

It is difficult to obtain appropriate scaling factors for R_m and τ_o in a turbulent shear flow. The problem is discussed briefly by Arndt and Keller (1976). Lacking more detailed information, the following assumptions can be used

$$R_m \sim d$$

$$\tau_o \sim d/U\sigma^{\frac{1}{2}}$$

If we interpret S(f) as the mean square acoustic pressure in a frequency band Δf, Eq. (9) can be written in the form

$$\frac{\left(\overline{p_a^2}/\rho^2 U_o^{\,4}\right)}{(\Delta f d/U_o)} \cdot \frac{(r/d)^2}{\sigma n d^3} = f\ (\sigma/\sigma_c)\ G(fd/U\sigma^{\frac{1}{2}}) \qquad (10)$$

Blake et al. (1977) circumvented the requirement of measuring n. They reasoned that

$$\int_0^{\infty} G(f)df = \int_{-\infty}^{\infty} p_b^{\,2}\ dt = \gamma\tau_o\ \overline{p_b^{\,2}}$$

wherein $\overline{p_b^{\,2}}$ is the time mean square of p_b and $\gamma\tau_o$ is the total lifetime of the bubble (including growth, initial collapse times and rebounding times). Further, they simply reasoned that

$$\overline{p_a^{\,2}} = N\ \overline{p_b^{\,2}}$$

or that

$$S(\tau_o f) = N\tau_o\ \gamma G(f\tau_o)$$

This results in the normalized spectrum

$$S(\tau_o f) = \frac{\overline{p_a^{\,2}}(f,\Delta f)}{\Delta f N}\ \frac{\gamma\tau_o r^2}{R_m^{\,4}\rho p_o} \qquad (11)$$

Making the same assumptions as before, we would expect that

$$\frac{\overline{p_a^{\,2}}}{(\Delta f d/U_o)} \cdot \frac{(r/d)^2}{\sigma^{3/2}} = N\ G'(fd/U\sigma^{\frac{1}{2}}) \qquad (12)$$

Blake et al. were able to determine $S(\tau_o f)$ for the case of noise due to cavitation on a hydrofoil using measured values of R_m. They assumed N equal to unity and found that Eq. (11) resulted in excellent collapse of the data.

Arndt (1978) used Eq. (12) to normalize cavitation data previously reported by Arndt and Keller (1976). These data correspond to noise from cavitation in the wake of a disk and were collected under a variety of conditions in both a water tunnel and in a depressurized towing tank. Both the level of dissolved gas and the number of free nuclei were monitored. As shown in Figure 13, the normalization is not very successful. It would appear that Eq. (10) would be more effective in taking all of the variables into account. However, n could only be measured in unison with acoustic observations in the water tunnel. Because of the nature of the laser scattering measurements used to determine n in the depressurized towing tank, these measurements had to be made separately from the acoustic measurements. The assumed form for $S(f\tau_o)$ in Eqs. (10) and (11) varies by a factor $nd^3/\sigma^{\frac{1}{2}}$. As an example, n in the depressurized towing tank appeared to be relatively constant and equal to about $15/cm^3$. Therefore the factor $nd^3/\sigma^{\frac{1}{2}}$ was found to have a maximum variation of 23 dB. This does not account for the scatter shown and one can only assume that there are other complicating factors. It should be emphasized that these data were collected under carefully controlled conditions. This underscores the fact that the current state of knowledge in this area is poor.

6. CONCLUSIONS

Cavitation inception in turbulent shear flows is the result of a complex interaction between an unsteady pressure field and a distribution of free stream nuclei. There is a dearth of data relating cavitation inception and the turbulent pressure field. What little information that is available indicates that negative peaks in pressure having a magnitude as high as ten times the root mean square

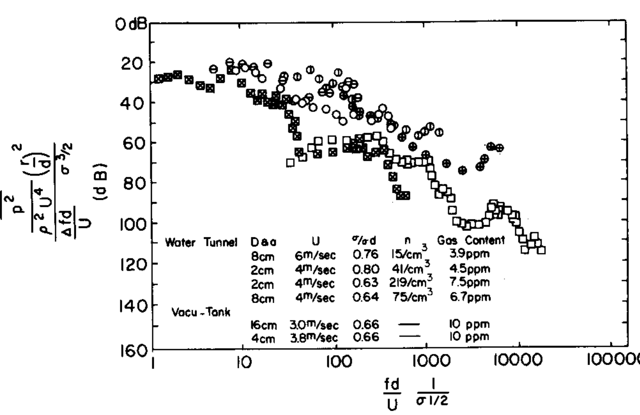

FIGURE 13. Normalized cavitation noise spectra.

pressure can excite cavitation inception. This fact alone indicates that consideration should be given to the details of the turbulent pressure field. The available evidence indicates that two basic factors related to the pressure field enter into the scale effects. First, as the scale of the flow increases, cavitation nuclei are relatively more responsive to a wider range of pressure fluctuations. Secondly, the available evidence indicates that large deviations from the mean pressure are more probable with increasing Reynolds number. This would explain some of the observed increases in cavitation index with physical scale. In view of the almost total lack of information on the statistics of turbulent pressure field (aside from some correlation and spectral data) and the potential importance of this knowledge to understanding cavitation, it is strongly recommended that careful experiments be initiated to remedy the situation. Such experiments have been proposed by George (1974b, 1975).

Direct application of the pressure field information to cavitation is unfortunately clouded by gas content effects which also increase the cavitation index with increasing exposure time. The fact that a reasonably precise scaling law for cavitation noise has not yet been found (perhaps a consequence of the lack of knowledge about the pressure field) further complicates interpretation of experiments and theory. Therefore it is also strongly recommended that the problem of the response of cavitation nuclei to turbulence receive particular attention. Such experiments have been proposed by Arndt (1978).

ACKNOWLEDGMENTS

R. E. A. Arndt gratefully acknowledges the support of the Air Force Office of Scientific Research and the Seed Research Fund of the St. Anthony Falls Hydraulic Laboratory. W. K. George gratefully acknowledges the support of the National Science Foundation under grants from the Engineering (Fluid Dynamics) and Atmospheric Sciences (Meterology) Programs and the Air Force Office of Scientific Research. Both authors are grateful to Mrs. Sandra Peterson who typed the manuscript.

REFERENCES

Arndt, R. E. A. (1974a). Cavitation inception and how it scales: a review of the problem with a summary of recent research. *Proc. Symposium on High Powered Propulsion of Ships,* Wageningen, The Netherlands, December (available from the Netherlands Ship Model Basin).

Arndt, R. E. A. (1974b). Pressure fields and cavitation. *Trans. 7th IAHR Symposium,* Vienna, September.

Arndt, R. E. A. (1976). Semi-empirical analysis of cavitation in the wake of a sharp edged disk. *J. of Fluids Engr.,* September.

Arndt, R. E. A. (1977). Cavitation and erosion: an overview. *Proc. Corrosion/77,* Natl. Assoc. of Corrosion Engineers, San Francisco, Calif.

Arndt, R. E. A. (1978). Investigation of the effects of dissolved gas and free nuclei on cavitation and noise in the wake of a sharp edged disk. *Proc. Joint IAHR/ASME/ASCE Symposium on Fluid Machinery,* Ft. Collings, Colorado.

Arndt, R. E. A., and J. W. Daily (1969). Cavitation in turbulent boundary layers. *Proc. of Symposium on Cavitation State of Knowledge,* ASME, 64-86.

Arndt, R. E. A., and A. T. Ippen (1967). Cavitation near surfaces of distributed roughness. *MIT Hydrodynamics Laboratory Report No. 104.*

Arndt, R. E. A., and A. T. Ippen (1968). Rough surface effects on cavitation inception, *J. of Basic Engr., Trans. ASME, Series D,* 249-261.

Arndt, R. E. A., and A. Keller (1976). Free gas content effects on cavitation inception and noise in a free shear flow. *Proc. IAHR Symposium on Two Phase Flow and Cavitation in Power Generation Systems.*

Baker, C. B., J. W. Holl, and R. E. A. Arndt (1976). The influence of gas content and polyethelene oxide additive upon confined jet cavitation in water. *ASME Polyphase Forum,* March.

Baiter, J. J. (1974). Aspects of cavitation noise. *Proc. Symposium on High Powered Propulsion of Ships,* Wageningen, The Netherlands, Dec.

Batchelor, G. K. (1951). Pressure fluctuations in isotropic turbulence. *Proc. Camb. Phil. Soc., 47,* 2; 359-374.

Batchelor, G. K. (1953). *The Theory of Homogeneous Turbulence,* Cambridge University Press.

Batchelor, G. K., and A. A. Townsend (1949). The nature of turbulence at large wavenumbers. *Proc. Royal Soc. (London), A199,* 1057; 238-255.

Beuther, P., W. K. George, and R. E. A. Arndt (1977a). Modelling of pressure spectra in a turbulent shear flow. *93rd Meeting Acoustical Soc. of Amer.,* State College, June.

Beuther, P., W. K. George, and R. E. A. Arndt (1977b). Pressure spectra in a turbulent shear flow. *Bulletin Amer. Phys. Soc., 22,* 12, December.

Beuther, P., W. K. George, and R. E. A. Arndt (1977c). Pressure spectra in homogeneous isotropic turbulent flow. *Bulletin Amer. Phys. Soc., 22,* 12, December.

Blake, W. K., M. J. Wolpert, and F. E. Geib (1977). Cavitation noise and inception as influenced by boundary layer development on a hydrofoil. *J. Fluid Mech., 80,* Part 4, 617-640.

Bradshaw, P., (1967). Inactive motion and pressure fluctuations in turbulent boundary layers. *J. Fluid Mech., 30,* 241.

Chan, Y. Y. (1974a). Pressure sources for a wave model of jet noise. *AIAA J., 12,* 2.

Chan, Y. Y. (1974b). Spatial waves in turbulent jets. *Physics of Fluids, 17,* 1.

Chan, Y. Y. (1976). Noise generated wavelike eddies in a turbulent jet. *ICAS* paper, No. 76-42.

Epstein, P. S., and M. S. Plesset (1950). On the stability of gas bubbles in liquid gas solutions. *J. Chem. Phys., 18,* 11, November, 1505-1509.

Fitzpatrick, H. M., and M. Strasberg (1956). Hydrodynamic sources of sound. *Proc. 1st Symposium on Naval Hydrodynamics,* Natl. Acad. of Sciences, Natl. Res. Council, Publication 515, September.

Fuchs, H. V. (1972a). Measurements of pressure fluctuations within subsonic turbulent jets. *J. Sound and Vib., 22,* 3.

Fuchs, H. V. (1972b). Space correlations of the fluctuating pressure in subsonic turbulent jets, *J. Sound and Vib., 23,* 1; 77-99.

Fuchs, H. V. (1974). Analysis of circumferentially coherent pressure fluctuations relevant to jet noise. *Colloquium on Coherent Structure in Turbulence,* Southampton.

Fuchs, H. V., and A. Michalke (1975). On turbulence and noise of an axisymmetric shear flow. *J. Fluid Mech., 70,* 1.

George, W. K. (1974a). The equilibrium range of turbulent pressure spectra, *Bull. of the Amer. Phys. Soc., 19,* 1158.

George, W. K. (1974b). Proposal to General Hydrodynamics Research.

George, W. K. (1975). Proposal to ONR, Fluid Dynamics Branch and presentation at David H. Taylor Naval Ship Research and Development Center, February.

Gurvich, A. A., and A. M. Yaglem (1967). Breakdown of eddies and probability distribution for small scale turbulence, *Phys Fluids, 10,* Supplement, S59-S65.

Heisinberg, W. (1948). Zur statistichen theorie den turbulenz. *F. Phisik, 124,* 7-12; 359-374.

Holl, J. W. (1960). An effect of air content on the occurrence of cavitation. *J. Basic Engr., Trans. ASME, Series D, 82,* 941-946.

Holl, J. W. (1969). Limited cavitation. *Proc. of the Symposium on Cavitation State of Knowledge,* ASME, June, 26-63.

Holl, J. W. (1970). Nuclei and cavitation. *Tran. ASME, J. of Basic Engr.,* December, 681-688.

Jones, B. G., R. J. Adrian, C. K. Nithianandan, and H. P. Planchen, Jr. (1977). Spectra of turbulent static pressure fluctuations in jet mixing layers, *AIAA Paper No. 77-1370.*

Keller, A. P. (1974). Investigations concerning scale effects of the inception of cavitation. *Proc. Conf. on Cavitation, Inst. of M. E.,* Heriot-Watt University, September.

Kermeen, R. W., and B. R. Parkin (1957). Incipient cavitation and wake flow behind sharp-edged disks. *Report 85-4,* Hydrodynamics Laboratory, California Institute of Tech., August.

Kim, H. T., S. J. Kline, and W. C. Reynolds (1971). The production of turbulence near a smooth wall in a turbulent boundary layer. *J. of Fluid Mech., 50,* 133.

Kolmogorov, A. N. (1962). A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number. *J. Fluid Mech., 13,* 1; 82-85.

Kraichnan, R. H. (1956). Pressure fluctuations in turbulent flow over a flat plate. *J. Acoust. Soc. Amer., 28,* 3.

Lienhard, J. H., and C. D. Goss (1971). Influence of size and configuration on cavitation in submerged orifice flows. *ASME Paper No. 71-FE-39.*

Monin, A. S. and A. M. Yaglem (1975). *Statistical Fluid Mechanics: Mechanics of Turbulence,* Vol. II, MIT Press, Cambridge.

Obukov, A. M. (1949). Pressure fluctuations in a turbulent flow. *Dokl. Acad. Nauk., SSSR, 66,* 1; 17-20.

Plesset, M. S. (1949). The dynamics of cavitation bubbles. *J. of Applied Mech.,* September, 277-282.

Ross, D. (1976). *Mechanics of Underwater Noise,* Pergamon Press.

Rouse, H. (1953). Cavitation in the mixing zone of a submerged jet. *La Houille Blanche,* February.

Tennekes, H., and J. L. Lumley (1972). *A First Course in Turbulence.* MIT Press.

Willmarth, W. W. (1975). Pressure fluctuations beneath turbulent boundary layers. *Annual Review of Fluid Mech., 7,* Palo Alto, Calif.