AZIMUTHAL MODE INTERACTION IN AN UNFORCED, HIGH REYNOLDS-NUMBER AXISYMMETRIC SHEAR LAYER

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Abstract

The dynamics of large scale azimuthally-coherent structures obtained in a high Reynolds-number axisymmetric mixing layer are presented. The structure dynamics is obtained by application of the Proper Orthogonal Decomposition (POD) to an ensemble of realizations of the streamwise velocity field obtained at 138 simultaneous positions, 3 diameters downstream of the nozzle exit. The velocity field is measured at all 138 positions simultaneously, making it possible to obtain the instantaneous coefficients of the POD modes as well as extract the large scale structure.

Pictures showing the dynamics of the structure interaction in the mixing layer demonstrate the importance of the mode-0, 3, 4, 5 and 6 azimuthal structure in both the entrainment and advection of fluid in the layer. The azimuthal structure does not appear to be a single structure but rather is made up of pairs of counter-rotating streamwise vortices and are very similar to the ribs seen in simulations of the axisymmetric layer and in plane mixing layers. It is hypothesized that recent success in mixing enhancement by multiple-mode excitation is due to the increased energy placed in these modes.

Purpose and scope

Understanding the dynamics of coherent structures in the axisymmetric mixing layer is of primary concern in many engineering applications such as noise suppression, mixing enhancement, near-field entrainment and shear layer growth. Much effort has been placed recently on control of these structures in the layer in an effort to address our inadequacies in the above areas of interest in the layer. However, there is surprisingly little knowledge of the coherent structure dynamics in the unforced, naturally occurring shear layer. With the exception of a few studies the thrust of current research has been forced excitation of the layer in an effort to control the flow.[1,2] However, using the POD it is possible to study the structure dynamics without using external forcing to fix the structures in space. The work presented in this paper, then, is an effort to take a step back and look at the dynamics of the coherent structures in the unforced, axisymmetric mixing layer with the hope that understanding the naturally occurring structure dynamics will lead to more productive control experiments.

Methods

Proper orthogonal decomposition

The present experiment was designed for the application of the Proper Orthogonal Decomposition (or POD) technique to an ensemble of realizations of the streamwise velocity field, three diameters downstream of the nozzle exit. The POD seeks the most energetic fluctuations in a random vector field and its ability to extract large scale structure in a turbulent velocity field is well

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established.[3,4,5,6,7] The structure is represented by an ordered set of orthogonal eigenfunctions, $\vartheta_i(\vec{x}, t)$, that are defined by the maximization of their normalized mean square projection on the velocity vector, $u_i(\vec{x}, t)$.[8]

The maximization is performed via the calculus of variations and the result is an integral eigenvalue equation of the Fredholm type,

$$\int R_{i,j}(\vec{x}, \vec{x}', t, t')\vartheta_j(\vec{x}', t')d\vec{x}'dt' = \lambda\vartheta_i(\vec{x}, t) \quad (1)$$

where the symmetric kernel of this equation is the two point correlation tensor

$$R_{i,j}(\vec{x}, \vec{x}', t, t') = < u_i(\vec{x}, t)u_j(\vec{x}', t') > \quad (2)$$

and λ is the eigenvalue.[8]

Solution of this equation produces the eigenfunctions and Galerkin projection of the instantaneous velocity on the basis determines the coefficients of the eigenfunctions. The velocity field can then be reconstructed via the POD and the form of this equation for the axisymmetric mixing layer, $(x_1, x_2, x_3) = (x, r, \theta)$, is,

$$\hat{u}_i^{nmf}(r, m, f) = \sum_{n=1}^{N} \hat{a}_n(m, f)\phi_i^{(n)}(r, m, f) \quad (3)$$

where $n = 1, 2, 3\ldots$ represents the discrete nature of the solution set and $\phi_i^{(n)}(r, m, f)$ and $\hat{a}_n(m, f)$ are the POD eigenfunctions and coefficients, respectively, decomposed in frequency, f, and azimuthal mode number, m.[7] Performing partial sums in equation 3, *i.e.* $N = 1, 2, 3, \ldots$, provides a way to visualize different energy-weighted views of the flow. It has been shown that setting $N = 1$ effectively filters out the small scale structure and leaves an unobscured view of the large scale, or coherent, structure in the axisymmetric mixing layer.[7]

Experiment

The flow field at 3 diameters downstream of the nozzle is representative of the fully developed mixing layer. At this position, the POD technique has been applied to realizations of the flow field made with 138 simultaneous operating single-wire hot-wire anemometer probes, *v.* figure 1. For the exit velocity of 12.5 m/s, the Reynolds number based on nozzle diameter, d, is 80,000. The free-stream turbulence intensity at the jet exit is 0.35% and the boundary layer at the jet exit was turbulent

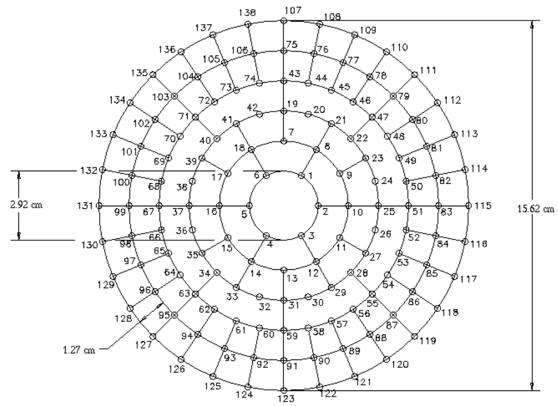


Figure 1: Spatial sampling grid for application of the POD to the axisymmetric mixing layer at $x/d = 3$ incorporating 138 simultaneous-sample, single-wire hot-wire probes.

with an approximate thickness of 1.2 mm. The mean velocity profile was flat to within 0.1%.

The sampling rate of each of the simultaneously sampling 138 hot-wires was 2048 Hz to satisfy the Nyquist criterion which must be greater than twice the 800 Hz corner frequency on the low-pass anti-alias filters. There were 300 blocks of 1024 samples producing a bandwidth of 2 Hz and a block length of 0.5 s.

The statistics of the streamwise velocity field, as measured by the sampling grid, demonstrate that an axisymmetric shear layer has been formed, *v.* figure 2. The mean and rms velocities are calculated by averaging over all azimuthal probes for each of the 6 radii in the layer. The spectral character of the velocity field is presented in figure 3. The energy distribution at the potential core, $r/d = 0.15$, is found to peak at about 100 Hz which is the natural passage frequency of the ring instability at the potential core.[2] This corresponds to a Strouhal frequency $(=fd/U_e) = 0.78$.

Results

Distribution of kinetic energy

The Hilbert-Schmidt theory which governs the solutions to equation 1 dictates that, in this case, the summation of all eigenvalues, over n, m and f produces the total streamwise kinetic energy in the flow.[8] The distribution of the kinetic energy

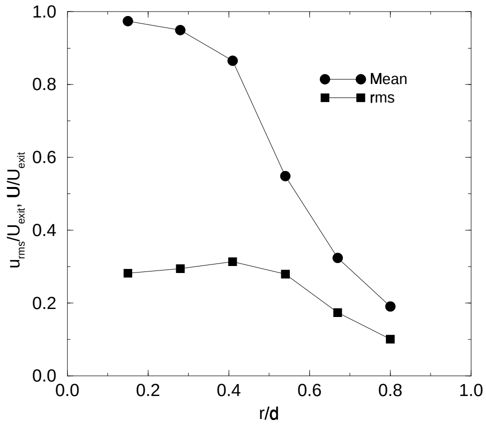


Figure 2: Mean and rms velocity in the axisymmetric mixing layer normalized by the exit velocity at $x/d = 3$.

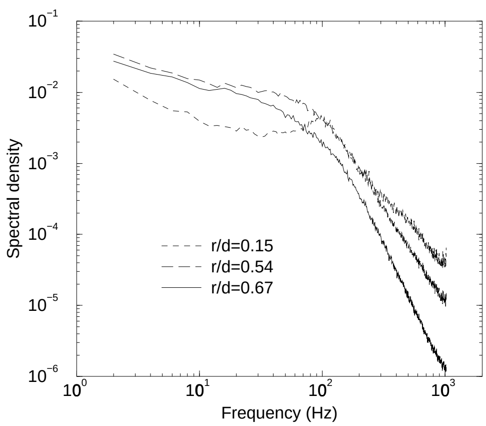


Figure 3: Spectra at the inner ($r/d = 0.15$), center ($r/d = 0.54$) and outer ($r/d = 0.67$) portions of the mixing layer at $x/d = 3$.

between radial POD and azimuthal Fourier modes can then be calculated by summing over various terms. A parameter, ξ, which is such a measure is defined by,

$$\xi = \frac{\sum_f \lambda^{(n)}(m,f)}{\sum_n \sum_m \sum_f \lambda^{(n)}(m,f)}. \qquad (4)$$

The azimuthal mode energy distribution for the first 5 radial POD modes is plotted in figure 4 for the streamwise velocity field measured at $x/d = 3$.[7] The predominance of the first POD mode (as determined by the first eigenvalue) is indicated by the fact that 67% of the kinetic energy in the

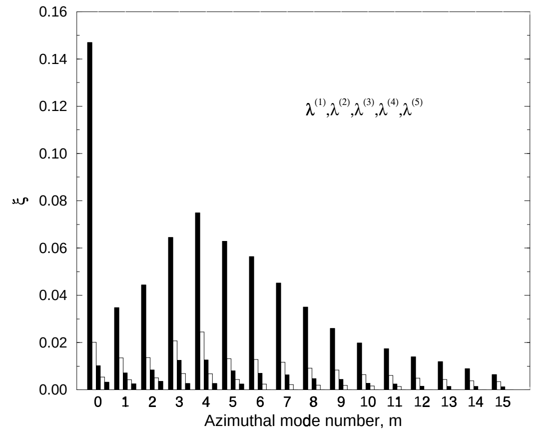


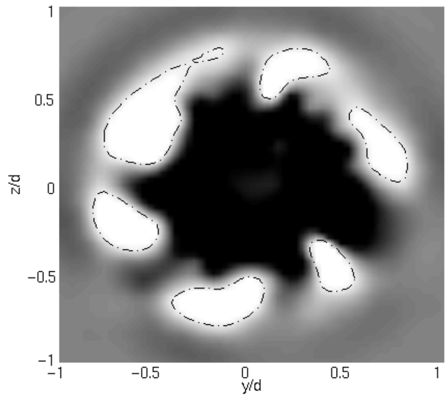
Figure 4: Azimuthal-mode kinetic energy distribution in the first 5 POD eigenvalues, from Citriniti and George (1997).

flow is contained in this eigenvalue. More importantly, the distribution of energy in azimuthal mode numbers within the first POD mode demonstrates the importance of the 0, 3, 4, 5 and 6 modes in the shear layer at $x/d = 3$. These modes have been found to be important in the description of counter-rotating streamwise vortices which are formed in the layer.[7] Thus, the fact that they contain a substantial portion of the kinetic energy in the layer suggests that they are important to the layer dynamics and specifically entrainment and growth.

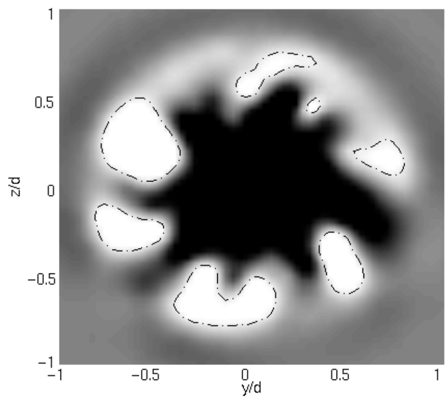
An interesting feature of figure 4 is the lack of energy in the $m = 1$, or helical, mode. This mode is quite important to the near-field dynamics in the mixing layer but by $x/d = 3$ its impact is limited. This would explain the limited success that helical-mode stimulation of the flow at the nozzle exit has on the downstream evolution of the shear layer.[9] A more attractive alternative would be to stimulate in the m=3, 4, 5 or 6 modes which are associated with more significant downstream, shear layer dynamics.

Structure dynamics

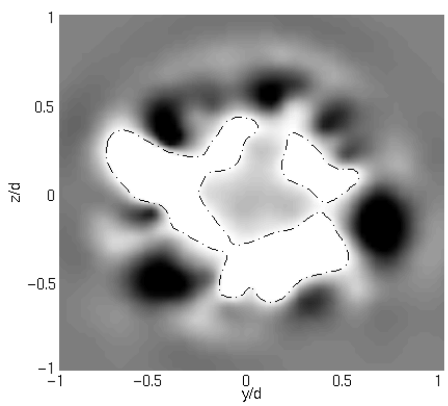
Four sequential frames of the reconstructed streamwise velocity with the mean subtracted off using the first POD mode, *i.e.* $N = 1$ in equation 3, are shown in figure 5. The time index



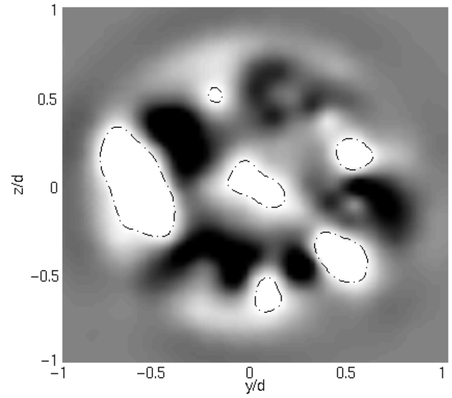
(a) $t_n = 413$



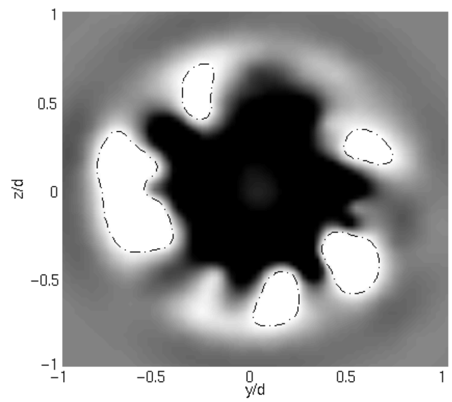
(b) $t_n = 417$



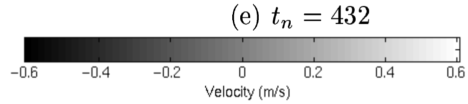
(c) $t_n = 422$



(d) $t_n = 429$



(e) $t_n = 432$



(f)

Figure 5: Two-dimensional projection of the POD reconstructed streamwise velocity field at $x/d = 3$. The temporal index is provided in the individual caption and the color bar denoting velocity above or below the local mean is given in (f) .

t_n is used to label the 1024 single images in the data blocks, the real time can be calculated by, $t = t_n \Delta t$ where $\Delta t = 0.488$ ms. The portion of the mixing layer between $0.5 < r/d < 0.8$ is found to be dominated by a 5 and 6 mode structure, *v.* figure 5a. Citriniti *et al.* have suggested that the structure causing this flow pattern are counter-rotating, streamwise vortex pairs similar to the Bernal-Roshko structures found in the plane mixing layer.[7] Further, a model has been developed describing the dynamics of the large scale structure in the mixing layer which consists of two main events. The first is the passage of the azimuthally coherent ring, or remnant of a ring *v.* figure 5c, which engulfs fluid from the irrotational outer flow. Note that high streamwise-momentum fluid is represented by white in figure 5 while low momentum fluid is black. The second is the advection of fluid by the streamwise vorticity pairs which "ride" on the induced velocity field of the rings and are stretched by the ring structures.[7]

In the movies generated from the still image reconstructions, it was found that the 3, 5 and 6 modes were usually associated with the outer portion of the mixing layer, $r/d > 0.5$, while azimuthal modes 0 and 4 were found to dominate near the potential core, where the fluid velocity is highest. The peak in the kinetic energy at the m=4 mode of figure 4 is explained because of this high velocity. The movies also show the streamwise structures "pumping" irrotational fluid from the ambient air directly to the potential core and vice versa. This motion has obvious impact on fluid entrainment and demonstrates the important affect these structures have on the entrainment field in the jet.[10]

Azimuthal "mode" structure

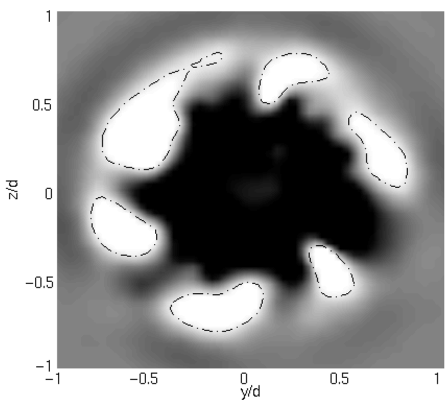
The use of Fourier analysis can provide insight into the general shape of the structure present in the flow at any instance. For instance, in figure 6 the two-dimensional projection of the reconstructed velocity field using the first POD mode and including different azimuthal mode situations is presented. The plot in figure 6a contains all azimuthal modes in the reconstruction. The highly corrugated edge of the dark structure and the 6 outer structures are all captured in each successive reduction of the azimuthal modes used in the reconstruction. In figure 6c the 6-mode structure is still evident but now in a much more ordered fashion. However, one drawback of using Fourier analysis is that the structure dynamics are no longer allowed to interact independently, as they would in the real flow. Fourier analysis has imposed a constraint on the structure motion which detracts from proper interpretation of the structure motion.

Guiding control experiments

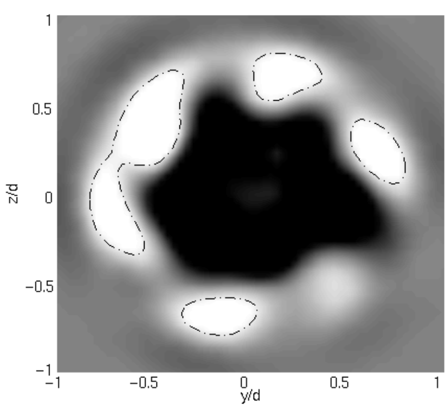
Both mechanisms involved in the coherent structure dynamics in the mixing layer, the azimuthally coherent ring and the streamwise vortex pairs, must be considered in designing control experiments. For instance, Raman and Rice have demonstrated that multiple-frequency excitation is effective at improving jet mixing and Corke and Kusek have shown how acoustic stimulation of the $m = \pm 1$ mode led to the growth of certain subharmonic helical modes.[11,12] As the results of this section show, multiple spatial mode excitation would offer the most dramatic effect on the growth of instabilities in the layer, especially if feedback control is utilized to give some indication of the streamwise structure configuration at any instant, since, as the images in figure 5 show, the structure shape is changing continuously. In this respect, the streamwise eddies should not be considered single structures which can be characterized using azimuthal Fourier modes, but rather independent, interacting structures. Treating the streamwise vortices, or ribs, as separate entities may lead to a more useful conceptual model when designing control experiments.

Glauser, using a POD based dynamical system model, has shown that stimulating the flow in the $m = \pm 5$ and ± 6 modes tends to increase the transfer of energy to lower and higher wavenumbers which is an indication that these mode numbers are important in the transition to small scale turbulence.[13] The results of the previous section indicate that stimulating the flow at these modes would tend to excite the streamwise vortex pairs. These structures must then play an important role in the transition process.[14] Also, increased layer growth and mixing can be obtained by stimulating the $m = \pm 5$ and ± 6 modes so that these streamwise structures are more energetic and can therefore advect more fluid.

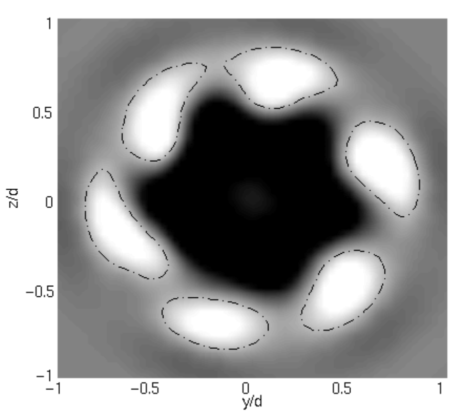
The cross-section picture in figure 5c shows the clear 4-mode structure near the potential core and the advection of fluid by the outer structures. This 4-mode predominance was usually associated with the area near the potential core and is thus associ-



(a) $m = 0 - 16$



(b) $m = 0 - 6$



(c) $m = 0, 4, 5, 6$

Figure 6: Two-dimensional reconstructed streamwise velocity field at $x/d = 3$ for $t_n = 413$ using the first radial POD mode and including the azimuthal modes listed in the individual captions. The color scale is given in figure 5

ated with the high momentum events in the layer. External forcing at this mode could then be expected to directly influence the structure most associated with the near-core large scale structure, which are the perturbed coherent rings. Thus, stimulation of this mode would tend to aid in engulfing of external fluid and directly aid layer growth.

Conclusions

Azimuthal Fourier modes 0, 3, 4, 5, and 6 have been determined to be the most important in the dynamics of the large scale structures in the axisymmetric shear layer at $x/d = 3$ The higher mode structure, modes 3, 5 and 6, are associated with the formation of streamwise vortex pairs which advect fluid into and out of the potential core of the jet, similar to the action of a side-jet. They are usually found near the outside of the layer and therefore are not convected very quickly downstream. They tend to exist for many ring passages. The 0-mode and 4-mode structures appear most often near the potential core and thus are expected to represent the perturbed remnants of the Kelvin-Helmholtz ring generated in the shear layer. They are very fast moving and are highly energetic, advecting large amounts of fluid in the short time they are in the layer.

Control of mixing in this layer would best be served by multiple azimuthal mode excitation as this would address the modes most associated with the large scale structures in the layer. Specifically, stimulating modes 3, 5 and 6 would increase energy in the structures which directly advect core and ambient fluid, *i.e.* those which control mixing. Stimulating mode-0 and mode-4 structures would also contribute to mixing but may not have as pronounced an effect because these structures are already quite energetic. Also, the mode-3, 5 and 6 structures typically span many occurrences of the mode-0 and mode-4 structures and can therefore have a larger integrated effect on the layer dynamics. Finally, it is important to note that the azimuthal Fourier mode description of the structures, while convenient, may not be useful in studying the interaction of these modes because of their fixed spatial representation of the structures.

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