Evaluating the accuracy of the LSE-POD technique in an axisymmetric shear layer.

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This is a great deal of interest in investigating the dynamics of the large energetic structures in turbulent shear flows. This can be difficult to accomplish experimentally because it is necessary to simultaneously sample the velocity over the entire region of interest at a sufficient rate to capture the structures dynamics. As a result, many previous investigation (*e.g*, Ukeiley *et al.* 1993) used measurements on a line through shear layers that yielded a two-dimensional view of the structures (*e.g.*, y and t). An exception is the investigation by Citriniti (1995) where the velocity was measured on the plane $x/D = 3$ downstream of a round jet using 138 hot-wires.

Recently, Bonnet *et al.* (1994) proposed that more extensive regions of flows could be investigated by measuring the instantaneous velocity at a few points in a region of interest and estimating the velocity at the remaining points using Linear Stochastic Estimation (LES); *i.e.*,

$$u_i^e(\cdot_n) = \sum_{l=1}^{L} A_{ij}^l(\cdot_n) u_j(\cdot_l), \tag{1}$$

where $u_j^e(\cdot_l)$ are the estimated velocities and the coefficients $A_{ij}^l(\cdot_n)$ are determined by minimizing the mean square error in the estimated velocities. Bonnet *et al.* then used the Proper Orthogonal Decomposition (POD) (Lumley 1970) to examine the dynamics of the energetic structures by projecting the combined measured and estimated velocity field onto the orthogonal basis to compute *instantaneous* coefficients; i.e.,

$$a_n^e(t) = \int u_i^e(\cdot, t) \Phi_i^{n*}(\cdot) d\cdot \tag{2}$$

and then reconstructing the velocity from the energetic modes; *i.e.*,

$$u_i^{rec,e}(\cdot, t) = \sum_{n=1}^{N_{rec}} a_n^e(t) \Phi_i^n(\cdot). \tag{3}$$

Here, $\Phi_i^n(\cdot)$ are the solutions to integral eigenvalue equation given by (Lumley 1970)

$$\int R_{i,j}(\cdot, \cdot') \Phi_i^n(\cdot') d\cdot' = \lambda^n \Phi_j^n(\cdot), \tag{4}$$

where $R_{i,j}(\cdot, \cdot')$ is the two-point velocity correlation tensor. Bonnet *et al.* showed that the velocity fields reconstructed using this approach in a shear layer were in good agreement with the results computed when the velocity was measured at all points. Ewing and Citriniti (1997) also showed the reconstructed velocity field computed by Citriniti (1995) from the 138 wire measurements could be reasonably reproduced if the velocity was measured at approximately 50 points and estimated at the remaining points (*cf.* figure 1). In these investigations, however, it was necessary to measure the full velocity field in the region of interest to verify the LSE-POD technique.

It will be shown here that the *instantaneous* coefficients computed from this measured and estimated velocity field $a_n^e(t)$ and the actual *instantaneous* coefficients from the flow $a_n(t)$ are related by

$$a_n^e(t) = \sum_{\gamma=1}^{M} H_n^\gamma a_\gamma(t), \tag{5}$$

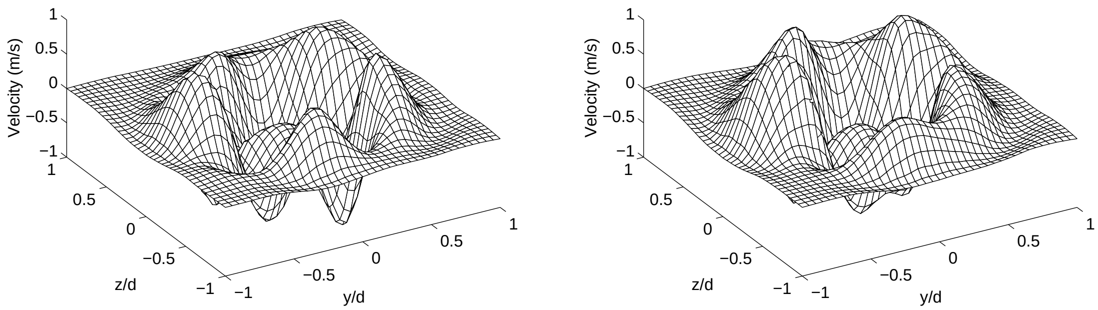

Figure 1: Estimated and actual reconstructed velocity fields using the first POD mode and azimuthal modes $0, 3, 4, 5$, and 6 in the axisymmetric shear layer. The height corresponds to the size of the instantaneous velocity.

where H_n^γ the transfer function given by

$$H_n^\gamma = \sum_{m=1}^{N} \left[\sum_{l=1}^{L} A_{ij}^l(\cdot_m) \Phi_j^\gamma(\cdot_l) \right] \Phi_i^{n*}(\cdot_m). \tag{6}$$

This transfer function only depends on statistical measures of the flow and thus can be used to determine the relationship (both amplitude and phase) between the estimated and actual coefficients without measuring the instantaneous velocity. Ideally, H_n^γ would equal 1 when $\gamma = n$ and 0 otherwise. Any deviation from these values represents an error in the estimated coefficients and hence the reconstructed field. Thus, the accuracy of the LSE-POD technic can be assessed by evaluating the transfer function.

One limitation of Citriniti's (1995) measurements is that they only include the streamwise fluctuating velocity. It would be desirable to extend these results by measuring both u and v cross-wire probes. These measurements would yield further insight into the entrainment and momentum transfer processes in the axisymmetric shear layer. It is not currently feasible to perform the experiment with 138 cross-wire probes. The transfer function outlined above will be used to evaluate whether the dynamics of the energetic structures can be accurately reproduced with the LSE-POD technique if the flow is measured with cross-wires at approximately 50 points and estimated at the remaining points.

References

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