On Accurate Experimental Measurements of the Dynamics of Large Scale Structures in Turbulent Flows

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Introduction

Quantitative measurements of the dynamics of large-scale structures, important in the turbulent mixing process, represent a significant challenge for experimentalists because in most turbulent flows the mixing is caused by three-dimensional structures or combinations of structures. Thus, it is necessary to measure the flow field; in a 2-D or 3-D domain large enough to capture the structures, with enough resolution to capture the small-scale motion to avoid spatial aliasing, and fast enough to capture the dynamics. These requirements surpass the capabilities of most current experimental approaches for flows of engineering interest. Thus, most experimental studies of large-scale structures consider either static images, phase averaged velocity fields, or the dynamics of the structures on a 1-D section through the flow. These approaches either do not capture the dynamics or provide a limited view of the dynamics. Linear stochastic estimation (LSE) has also been used expand the current experimental capabilities but the velocity fields generated using this approach are normally biased with larger velocities near the measurement points.

Recently Bonnet et al (1994) suggested using Lumley's POD to filter the velocity field estimated using LSE in order to develop an estimated low-order reconstruction of large-scale motions. Bonnet et al. found this approach could be used to accurately reproduced the dynamics of the large-scale structures for 1-D measurements.

Heretofore, the major shortcoming of the LSE-POD approach was that the accuracy of the result computed from the estimated field could only be assessed by comparing them with results computed from the full measured field. The objective of this investigation is to show that the accuracy of the velocity field reconstructed with the LSE-POD can be assessed using only the statistical information from the flow field.

Proper Orthogonal Decomposition (POD).

The Proper Orthogonal Decomposition represents the flow using orthogonal modes, Φ_i^n, that optimally represent the energy in the field. The functions are determined by solving the integral eigenvalue problem given by

$$\int R_{i,j}(\cdot,\cdot')\Phi_i^n(\cdot')d\cdot' = \lambda^n \Phi_j^n(\cdot)$$

where $R_{i,j}(\cdot,\cdot')$ is the two-point velocity correlation tensor. The dynamics of the energetic modes in the flow can then be determined by projecting the *instantaneous* velocity field measured over the region of interest onto these functions; i.e.

$$a_\gamma(t) = \int u_i(\cdot,t)\Phi_i^{\gamma *}(\cdot)d\cdot$$

and then reconstructing the velocity field from only the most energetic modes

$$u_i^{rec}(\cdot,t) = \sum_{\gamma=1}^{N_{rec}} a_\gamma(t)\Phi_i^\gamma(\cdot).$$

LSE – POD Complementary Technique.

In the LSE-POD approach, the velocity is measured at a few points and Linear Stochastic Estimation (LSE) is used to estimate the velocity at the remaining points; i.e.

$$u_i^e(\cdot_\alpha) = \sum_{l=1}^{L} A_{ij}^l(\cdot_\alpha)u_j(\cdot_l),$$

The coefficients, A_{ij}^l, determined by minimizing the mean square error in the estimated velocities only depend on statistical information from the flow. The estimated field is then projected onto the POD modes to compute estimated POD coefficients

$$a_n^e(t) = \int u_i(\cdot,t)\Phi_i^{n*}(\cdot)d\cdot$$

These coefficients can then be used to compute an estimated reconstructed field

$$u_i^{rec,e}(\cdot,t) = \sum_{n=1}^{N_{rec}} a_n(t)\Phi_i^n(\cdot).$$

Accuracy of the LSE-POD

The objective of this investigation is to develop a method of assessing the accuracy of the velocity field reconstructed using these estimated coefficients. After some manipulation it can be shown the difference between the estimated POD coefficients and the POD coefficients that would be computed from the full velocity field is given by

$$a_n^e(t) = \sum_{\gamma=1}^{M} H_n^\gamma a_\gamma(t),$$

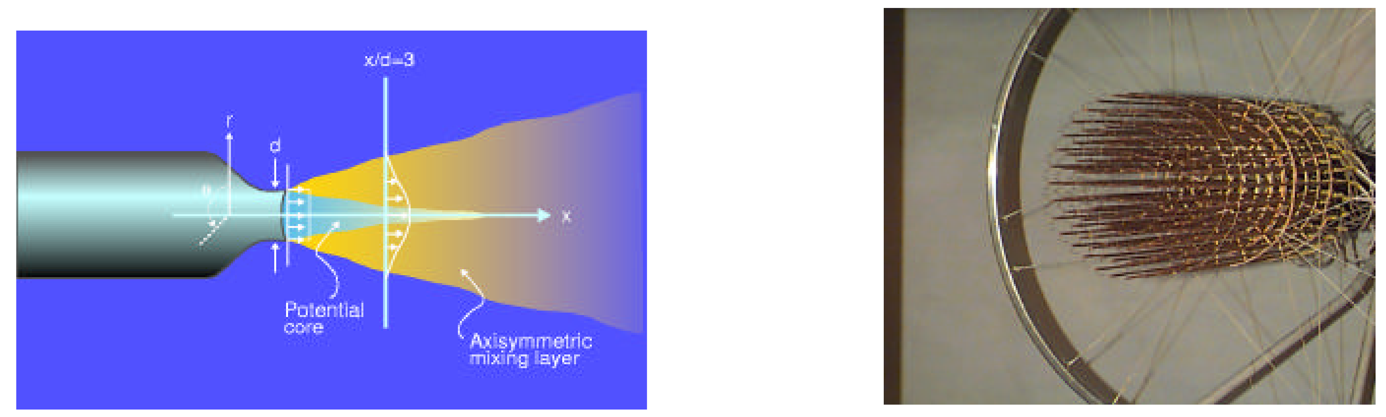

Figure 1. Experimental setup of the axisymmetric shear layer experiments with 138 hot wire probe.

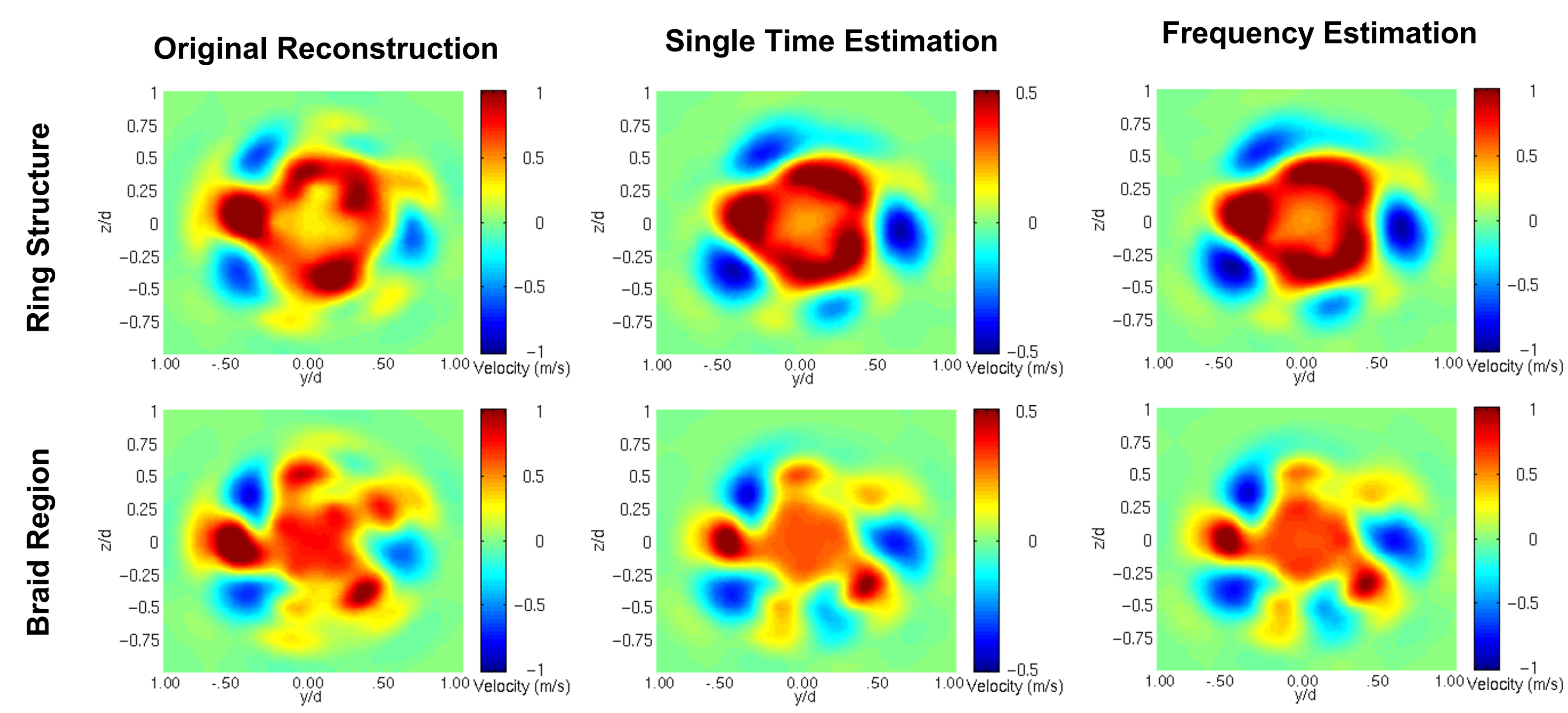

Figure 2.Contribution of the energetic motions to the fluctuating streamwise velocity at x/D=3

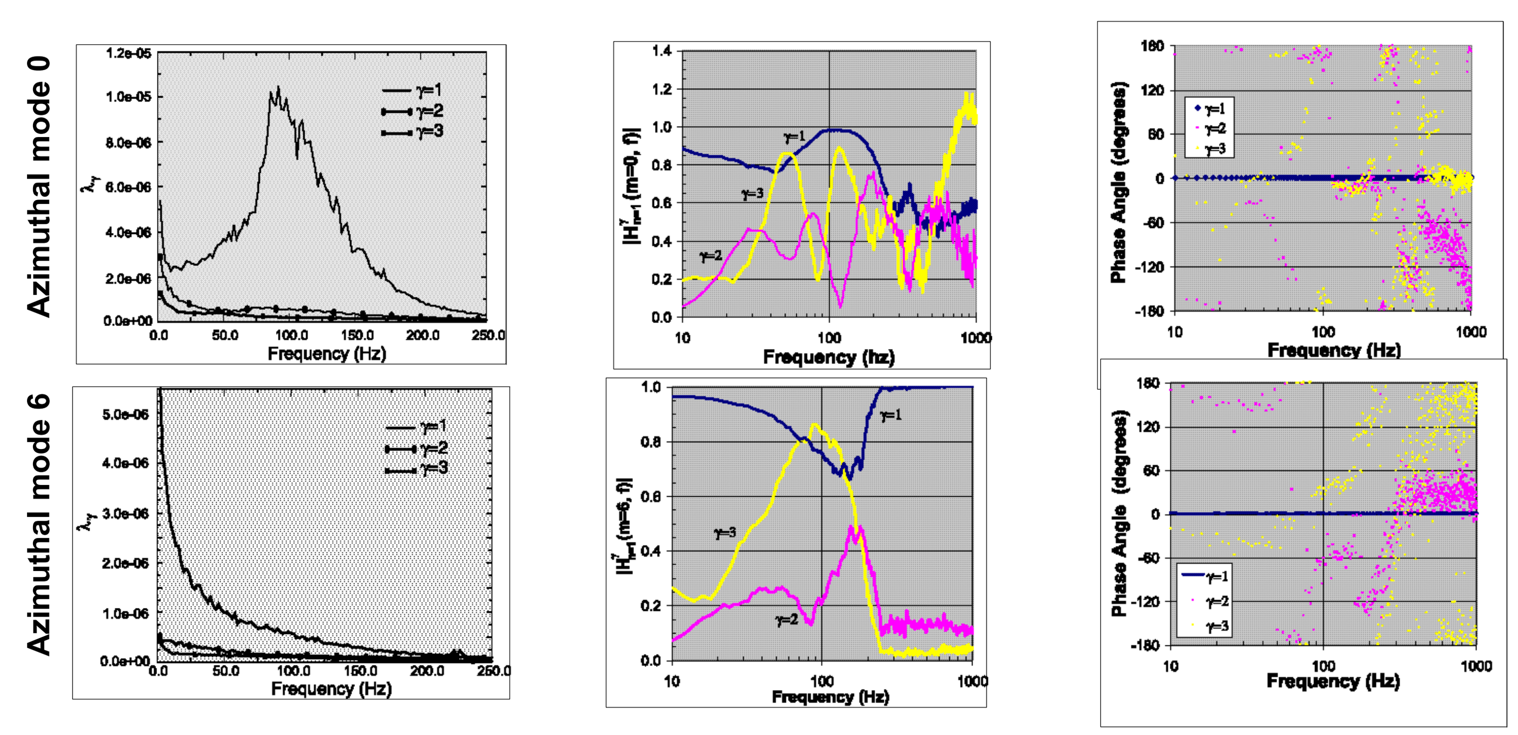

Figure 3.Magnitude and phase of the transfer function for third and fifth rings of hot-wires

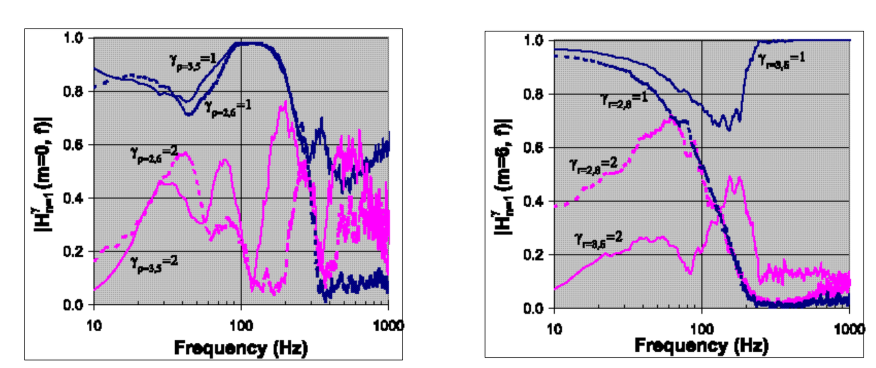

Figure 4. Comparison of Magnitude of the transfer function for two different sets of hot-wire rings.

where H_n^γ is the transfer function between the estimated and actual coefficient is given by

$$H_n^\gamma = \sum_{\alpha=1}^{N_{tot}}\left[\sum_{l=1}^{N_{tot}} A_{ij}^l(\cdot_\alpha)\Phi_j^\gamma(\cdot_l)\right]\Phi_i^{n*}(\cdot_\alpha).$$

The transfer function gives the relationship, both magnitude and phase, between the estimated coefficients and the actual coefficients for a given POD mode. Thus, it can be used to determine how much of an estimated coefficient come from the correct mode from the actual field and how much is aliased from the higher order POD modes. Of equal importance is the ability ensure the phase of the coefficient is properly estimated. It is important to recognize that the transfer function includes only the LSE coefficients and the POD functions that are determined using **only** statistical measures from the flow. Thus, the transfer function can be determined even when the full velocity field has not been measured.

Results

The accuracy of the LSE-POD approach was assessed using measurements of the velocity field on the plane 3 diameters downstream of the jet exit (Citriniti, 1996). The velocity field in the original experiment was measured using a 138 hot-wire probe shown in figure 1 that consisted in 6 rings of hot-wires. Ewing and Citriniti (1996) showed that the dynamics of the structures could have been accurately reproduced using the LSE-POD if the flow field had been measured using the 48 probes on the third and fifth ring of probe array. A comparison of the field reconstructed at two different times using the first POD mode and selected azimuthal modes are shown in figure 2. The middle column here contains the results when only data at each point in time is used in the estimation process while the right column contains the result when the data at all times, or the frequency data, is used. In both case, the estimated reconstructed fields captures the topology of the rig and braid structures. The results for the single time process are 50 percent too low.

The transfer function was computed for the frequency estimation process. The general equations given above were converted into application specific form. In particular, in this application there are individual POD modes for each frequency and azimuthal mode number. The results for two different azimuthal modes are shown in figure 3. The results in the first row are for azimuthal mode number 0 that contributes to the ring structures in the jet while the second row are for mode number 6 which dominates the `braid' region in the jet. The distribution of the energy in both these azimuthal mode numbers with frequency and POD mode is shown in the first column. In both case the first POD mode makes a dominant contribution to the energy. The magnitude of the transfer function is shown in the second column. For both azimuthal modes, the estimated coefficient contains almost all of the actual first POD coefficient in the range of frequencies that contribute most of the energy to that mode. The contribution (or aliasing) from the actual second POD mode in the field, γ=2 is relatively small but does reach as high as 30 percent of the coefficient in some regions. The contribution from the third POD mode from the actual field is larger but the size of this coefficient is typically 1-2 decades smaller than the first mode. The phase of the transfer function is in the third column. The important result here is that the phase of the contribution from the first POD mode is correct for all frequencies. These results can be used to compute an uncertainty for the reconstructed estimate velocity field.

The transfer function can be used to design LSE-POD experiments by allowing the experimentalist to assess the relative accuracy of using different sets of measuring points. For example, the transfer functions were computed for a series of different choices of hot-wire rings. A comparison of the transfer functions computed for the field that would be estimated using the hot wires on the second and sixth rings and the third and fifth rings are shown in figure 4. The results are again compared for azimuthal mode numbers 0 and 6. The results here suggest the dynamics of azimuthal mode number 6 would be estimated better when the hot wires on the third and fifth ring are used rather than the second and sixth. The contribution from the first POD coefficient from the actual field to the estimated field is larger and the contribution (or aliasing from) the second actual POD contribution is increased. The results for mode 0 are similar. The aliasing from the second mode is slightly larger when the third and fifth rings of hot-wires are used but the first POD mode is so dominant near the shedding frequency that this difference would be quite small. Thus, overall the choice of the third and fifth rings is likely the superior choice.

This approach will be used in the future to assess whether the number of hot wires can be reduced by reducing the number of wires in the aziumthal direction. It is important to note that the transfer function can only serve to compare the relative accuracy of different choices of points. It should be possible to automate this process somewhat by generating a cost function that can be used to automatically assess the merit of these choices.

References

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