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THE MEASUREMENT OF UNSTEADY FLOW WITH THE BURST PROCESSOR LASER DOPPLER ANEMOMETER

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ABSTRACT

It is shown that the burst processor LDA signal is properly interpreted as a time series and that this implies keeping tract of the time a particle is in the measuring volume. Applications of the so-called residence-time weighting technique derived from this analysis are used to obtain alias- and bias-free statistical measurements in a turbulent jet. Of particular interest are the correlations and spectra which were obtained with mean data rates well below the Nyquist frequency. Errors that result from alternative modes of signal processing are discussed and illustrated.

INTRODUCTION

The Doppler signal generated by a single particle arriving in the measuring volume can be represented [1], [2] as

$$i(t) = w(\underline{x}) \cos \underline{K} \cdot \underline{x} \qquad (1)$$

where $\underline{K}$ is the scattering wave vector, $\underline{x}$ is the location of the scattering particle at time t and $w(\underline{x})$ is a weighting function which turns on the signal when the particle enters the volume. For burst processor LDA's, the weighting function $w(\underline{x})$ is simply on or off; i.e.,

$$w(\underline{x}) = \begin{matrix} 1 & \text{inside volume} \\ 0 & \text{outside volume} \end{matrix} \qquad (2)$$

A typical LDA setup, measuring volume, and scattering geometry are shown in figures 1 and 2.

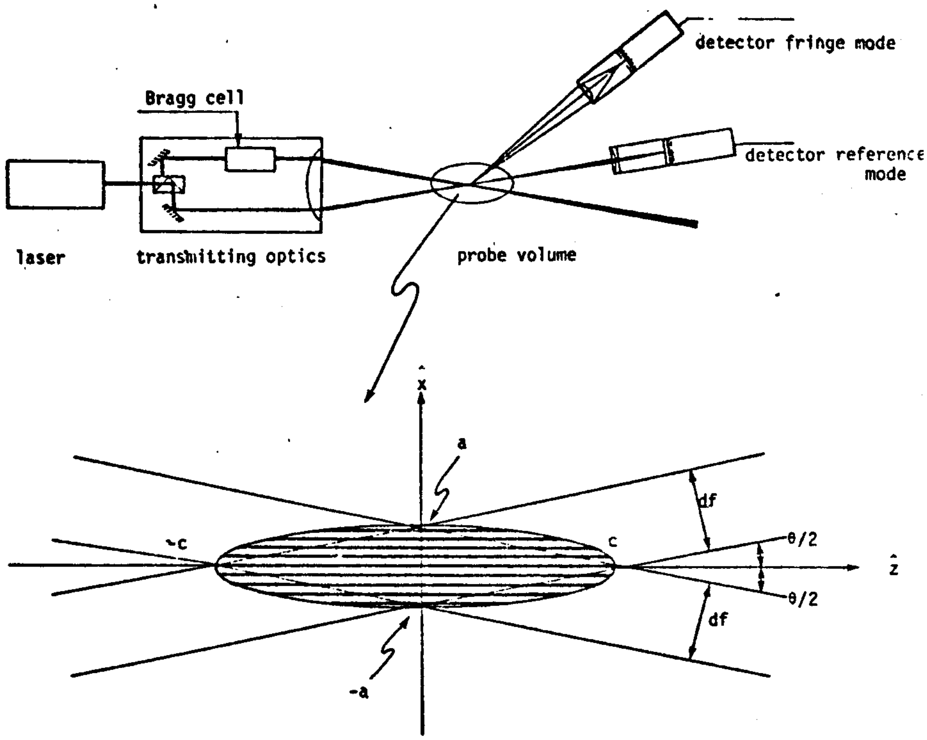
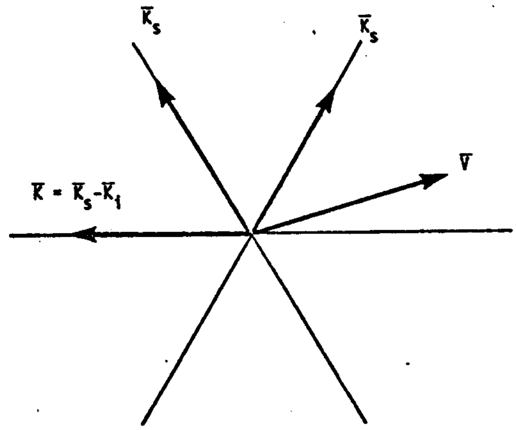


Figure 1 Optical Configuration of typical LDA.



$\omega_D = \bar{V} \cdot \bar{K}_s - \bar{K}_i$

Figure 2 Wave Vector Diagram

We can represent the signal that would be generated by randomly arriving particles as

$$u_o(t) = \int w(\underline{x})\; u[\underline{x}(\underline{a},t)]\; g_1(\underline{x},t)\; d^3\underline{x} \tag{3}$$

where

$$\underline{x} = \underline{a} + \int_0^t \underline{u}(\underline{a},t_1)dt_1 \tag{4}$$

and the statistics of $g_1(\underline{x},t)$ are given for uniform seeding by

$$\overline{g_1(\underline{x},t)} = \mu$$

$$\overline{g_1(\underline{x},t)\,g_1(\underline{x}',t')} = \mu p(\underline{x},t \mid \underline{x}',t') + \mu^2 \tag{5}$$

where $p(\underline{x},t|\underline{x}',t')$ is the probability that the particle at $\underline{x}$ at time t has moved to $\underline{x}'$ at time t' and μ is the expected number of particles per unit volume. An excellent approximation to $p(\underline{x},t|\underline{x}',t')$ for all practical purposes is:

$$p(\underline{x},t|\underline{x}',t') \to \delta(\underline{x}'-\underline{x}-\underline{U}\tau) + \mu^2 \tag{6}$$

A typical $u_o(t)$ is shown in figure (3).

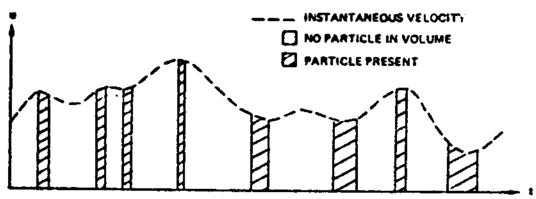


Figure 3 Typical $u_o(t)$

MEAN VALUES

It is straightforward from the above [1], [2] to show that

$$\overline{u_o(t)} = \mu\int \overline{u(\underline{x},t)} \quad w(\underline{x})d^3\underline{x} \tag{7}$$

To understand the implications of this consider the simple case where $\overline{u(\underline{x},t)}$ is independent of $\underline{x}$,t and $w(\underline{x},t) = 1$ within the volume and zero outside. Assuming this, we have

$$\overline{u_o(t)} = \mu V\; \overline{u(x,t)} \tag{8}$$

Thus the measured mean velocity is directly proportional to the desired Eulerian mean. It should not be inferred from this that one needs merely average realizations to achieve reasonable averages. Equation (8) assumes that the velocity has been measured during all of the time the particle is in the volume and the factor μV accounts for the portion of time that it is not.

To see this, assume that we are determining the mean values by time-averaging.

$$\overline{u_o(t)} = \frac{1}{T}\int_0^T u_o(t)dt \qquad (9)$$

where it is assumed that T is sufficiently long. Eqn. (8) implies that

$$\overline{u_o(t)} \approx \frac{1}{T}\int_0^T u_o(t)dt = \mu V \frac{1}{T}\int_0^T u(\underline{x},t)dt \approx \mu V\, \overline{u(\underline{x},t)} \qquad (10)$$

where $\overline{u}(\underline{x},t)$ is the information we desire. Then it follows that

$$\overline{u(x,t)} \approx \frac{1}{\mu VT}\int_0^T u_o(t)dt \qquad (11)$$

But μVT is exactly the fraction of time that the signal $u_o(t)$ is non-zero. Thus, the correct mean is given by averaging only during those periods where there is a signal.

Most real processors measure the average velocity during the burst; this implies that the velocity must be approximately constant during its traversal of the volume. Moreover, since there is only a single realization during each particle passage, then the realization must be weighted by the time the particle would contribute to the integral; that is, the residence (or transit) time. This is easily seen by approximating $u_o(t)$ as constant while there is a signal and writing the integral as a sum:

$$\overline{u(\underline{x},t)} = \frac{1}{\mu VT}\int_0^T u_o(t)dt \approx \frac{\sum_i u_o(t_i)\Delta t_i}{\sum_i \Delta t_i} \qquad (12)$$

where $u_o(t_i)$ represents the i^{th} realization and Δt_i the residence time of that particle. Note that the direction of the particle and its velocity are irrelevant; only its x-component and residence time matter.

TURBULENCE INTENSITIES

Straightforward extension of the analysis [1], [2] above leads to the following formula for the mean square fluctuating velocity

$$\overline{u'^2} = \frac{\sum_i [u(t_i)-U]^2\Delta t_i}{\sum_i \Delta t_i} \qquad (13)$$

where U is computed from equation (12).

CORRELATIONS AND SPECTRA

It is tedious but straightforward to show [2] that the autocorrelation of the signal represented by equation (3) consists of two parts: the desired autocorrelation of the turbulent velocity field and a spike at the origin.

The latter can be eliminated in practice by eliminating self products of the velocity from the analysis. It is then easy to show that remaining part yields

$$\overline{u'(t)u'(t+\tau)} = (\mu V)^{-2} \frac{1}{T} \int_0^T u'_o(t)\ u'_o(t+\tau)\ dt \qquad (14)$$

Thus, if we consider the particle to contribute to the signal <u>while it is in the volume</u>, all of the desired statistical information can be obtained.

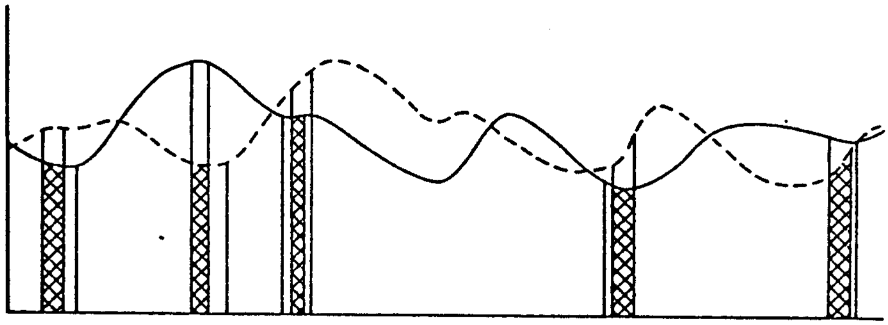


Figure 4 Computation of the autocorrelation from burst processors using only the overlap times for different particles. The middle trace is the upper trace displaced by amount τ as shown.

As a consequence, the autorcorrelation for a lag τ can be computed only when there is a particle in the volume at t <u>and</u> one at $t+\tau$. This is illustrated in Figure [4]. The interpretation of this analysis leads to the following approximation to equation (13):

$$\overline{u'(t)\ u'(t+\tau)} \equiv \frac{\sum_{i,j} u'(t_i)\ u'(t_j)\ \Delta t_{ij}}{\sum_{i,j} \Delta t_{ij}} \qquad (14)$$

where $\tau = t_i - t_j$, $i<j$ and Δt_{ij} is the overlap time of the i^{th} realization and the j^{th} realization displaced by time τ. The denominator is essentially the total overlap time corresponding to realizations of $u(t)\ u(t+\tau)$ since $(\mu V)^2 T \simeq \sum_{i,j} \Delta t_{ij}$.

Similar considerations can be applied to single and joint probability densities, and to cross-correlations of different signals. Again the algorithm must be determined by the time the one or more signals actually contribute to the appropriate time integral expression.

It is shown in reference [2] that a convenient approximation to equation (14) is given by

$$R'(\tau) = \frac{\sum_{i,j} u_i\ u_j\ \Delta t_i\ \Delta t_j}{\sum_{i,j} \Delta t_i\ \Delta t_j}\ ; \qquad \tau = |t_i - t_j| \qquad (15)$$

where Δt_i and Δt_j are the individual residence times of the i^{th} and the j^{th} particles.

EXPERIMENTAL DATA

All of the above relationships have been confirmed by experiment[3],[4]. Detailed measurements in a turbulent jet were taken with single and multiple hot-wires and a LDA tracker for comparison. In addition, the errors introduced by arithmetic averaging of uncorrected data and application of the McLaughlin and Tiederman [5] one-dimensional correction were shown to be substantial (~50%) at high turbulence intensities.

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