THE TURBULENT BUOYANT PLUME IN A STRATIFIED ENVIRONMENT

William K. George and Paul D. Beuther*

Department of Mechanical and Aerospace Engineering
State University of New York at Buffalo

In 1954 Batchelor [1] showed that similarity solutions were possible for turbulent buoyant plumes in stratified environments, if the stratification were of power law form. For the unstable case he inferred the possibility of the spontaneous generation of plumes since no source of buoyancy was required at the origin. The solutions for a stably-stratified environment were dismissed as being physically unrealistic since the buoyancy required at the origin was infinite.

In the following paragraphs we shall repeat Batchelor's analysis, and shall extend it to show that for a given power law stratification, solutions at all heights collapse to a single curve when normalized by the distance from the source and the local buoyancy flux. Moreover, it will be argued that the solutions for stably stratified environments are, in fact, physically realistic. The analysis will be substantiated by some recent measurements in axisymmetric turbulent plumes. Finally, it will be suggested that stratification is a major cause for the variety of "neutral" profiles in the literature.

The Similarity Analysis

We consider an axisymmetric plume rising in a stratified environment; z is assumed to be the vertical coordinate opposite to the gravitational acceleration and r is the radial coordinate. To within the Boussinesq approximation, the averaged equations of momentum and buoyancy are

$$W \frac{\partial W}{\partial z} + V\frac{\partial W}{\partial r} = -\frac{1}{r}\frac{\partial}{\partial r}(r\overline{wv}) + g\beta(T-T_\infty) \qquad (1)$$

$$W\frac{\partial}{\partial z} g\beta(T-T_\infty) + V\frac{\partial}{\partial r} g\beta(T-T_\infty) = -\frac{1}{r}\frac{\partial}{\partial r}(g\beta\overline{vt}) - W\frac{d}{dz}(g\beta T_\infty) \qquad (2)$$

In equation (1) the pressure gradient has been eliminated using the r-momentum equation and the streamwise gradients of the turbulent normal stress difference and the axial heat flux have been neglected. Note that the stratification of the ambient environment is expressed by the fact that T_∞

*Present address: Owens Corning Fiberglas Co., Granville, Ohio

is, in general, a function of height.

We seek similarity solutions of the type

$$\eta = r/\ell(z) \qquad g\beta(T-T_\infty) = T_s(z)p(\eta) \qquad g\beta\overline{vt} = W_sT_sh_2(\eta)$$
$$W = W_s(z)f(\eta) \qquad g\beta\overline{ut} = W_sT_sh_1(\eta) \qquad \overline{wv} = W_s^{\,2}\,q(\eta) \tag{3}$$

By eliminating V from the continuity equation and substituting the equations (3) into equations (1) and (2) we obtain

$$\left[\frac{W'_s\ell}{W_s}\right]f^2 - \left[\frac{W'_s\ell}{W_s} + 2\ell'\right]\frac{f'}{\eta}\int_0^\eta f\eta'd\eta' = -\frac{1}{\eta}\frac{d}{d\eta}\,\eta q + \left[\frac{T_s\ell}{W_s^2}\right]p \tag{4}$$

$$\left[\frac{T'_s\ell}{T_s}\right]fp - \left[\frac{W'_s\ell}{W_s} + 2\ell'\right]\frac{p'}{\eta}\int_0^\eta f\eta'd\eta' = -\frac{1}{\eta}\frac{d}{d\eta}\,\eta h_2 - \left[\frac{\ell}{T_s}\frac{d}{d_z}(g\beta T_\infty)\right]f \tag{5}$$

Since the relative dynamical effect of each term in the equations can be independent of z only if the bracketed terms are independent of z, similarity solutions exist only if $W_s'\ell/W_s$, $T'_s\ell/T_s$, ℓ', $T_s\ell/W_s^{\,2}$, and $(\ell/T_s)d(g\beta T_\infty)/dz$ are constants. The first four conditions are precisely those obtained for the neutral environment; the fifth condition is seen to arise from the stratification.

It is immediately obvious that $\ell \sim z$ from which it follows that $W_s \sim z^n$ and $T_s \sim z^{2n-1}$ are the only possible solutions where at this point n is arbitrary. The fifth constraint can then be satisfied only if the ambient gradient is also a power law; in particular, $d/dz(g\beta T_\infty) \sim z^{2n-2}$. Thus similarity solutions are seen to be possible for power law stratifications. To this point the analysis agrees with that of Batchelor [1].

A convenient evaluation of the functions $W_s(z)$ and $T_s(z)$ follows from consideration of the local buoyancy integral, the rate at which buoyancy crosses a given level z, defined by

$$F = 2\pi\int_0^\infty g\beta[W\cdot(T-T_\infty) + \overline{wt}]rdr \tag{6}$$

Using equations (3) this reduces to

$$F = [W_sT_s\ell^2]\cdot 2\pi\int_0^\infty [ft + h_1]\eta d\eta \tag{7}$$

from which it follows that $F/(W_sT_s\ell^2)$ = constant. This condition and the previously imposed constraints can be satisfied only if

$$W_s \sim F^{1/3}z^{-1/3} \quad , \quad T_s \sim F^{2/3}z^{-5/3} \tag{8}$$

These can be readily recognized as the forms for the neutral ambient case except for the fact that here F is the <u>local</u> buoyancy parameter and not the buoyancy added at the source (c.f. Turner [2]).

From the combined constraints it also follows that the local buoyancy flux and the stratification must be related by

$$F \propto z^4 [\frac{d}{dz} g\beta T_\infty]^{3/2} \tag{9}$$

The various solution regimes are summarized in the table below

Axisymmetric Plume

	$F \propto z^n$	$g\beta dT_\infty/dz \propto cz^m$	
Stable	$n < o$	$m < -8/3$	$c > o$
Neutral	$n = o$	$m = -8/3$	$c = o$
Unstable	$n > o$	$m > -8/3$	$c < o$

Note that as long as equation (8) is satisfied, and as long as the thin shear flow assumptions which led to equations (1)-(2) are satisfied, the similarity solutions are valid for both stable and unstable environments. Moreover, since the buoyancy flux is finite for all positions away from the source for the stable case and since source singularities do not necessarily affect the asymptotic validity of a similarity solution, Batchelor's disregard for the stable environment solution appears unjustified.

A similarity analysis for the two-dimensional plume can also be carried out with the result that $U_s \sim F^{1/3}$, $T_s \sim F^{2/3} z^{-1}$ where again F is the local buoyancy parameter appropriately evaluated. As for the case above, these are the forms for the neutral environment except now $F = F(z)$.

<u>Comparison With Experiment</u>

An important test of the foregoing analysis is whether or not plume data in stably stratified environments can be collapsed by equation (8) using the local buoyancy flux. Beuther [3] (see also Beuther and George [4]) presents profiles for which the environmental temperature is given by $dT_\infty/dz \propto z^{-3}$. Profiles of all velocity and temperature profiles to fourth order were taken using digitally sampled hot wire anemometry techniques. The local buoyancy flux at each height was determined according to equation (6) and the results are plotted in figure 1. The measured buoyancy flux is seen to be proportional to $z^{-1/2}$ as demanded by equation (9), so the basic constraints for a similarity solution are satisfied.

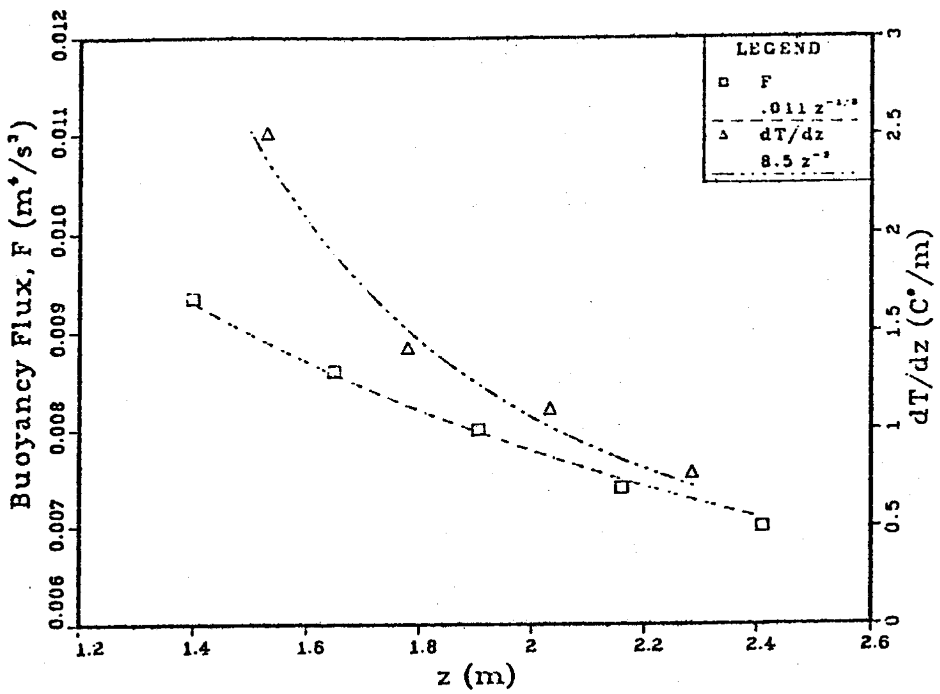

Figure 1 - Plots of experimentally determined ambient temperature gradient and buoyancy integral for axisymmetric plume in stably stratified environment.

Figures (2) and (3) plot the velocity and temperature profiles at five different heights. The profiles have been non-dimensionalized by the local buoyancy flux and the distance to the source according to equation (8). The data collapse is in excellent agreement with the theory, the scatter being well within the experimental error.

Discussion

Beuther [3] notes that increasing stratification causes the normalized profiles of both temperature and velocity to become narrower and peak at higher centerline values than their neutral counterparts. For example, for the temperature profile Beuther reports a centerline value of 10.4 which can be compared to the value of 9.1 reported by George et al. [5] for neutral conditions. Similarly, Beuther reports a value of 3.8 for the velocity compared to the value of 3.4 for the neutral case (George et al. [5]).

Since the early experiments of Schmidt [6] and Rouse et al. [7] there has been considerable confusion as to what the correct profiles are for neutral conditions (c.f. Chen and Rodi [8]). These experiments (and most which followed them) did not directly monitor the rate at which buoyancy was added at the source and used only the integrated flux determined at the height of the measurement to normalize the data. As is clear from the preceeding analysis, such results could have provided adequate collapse,

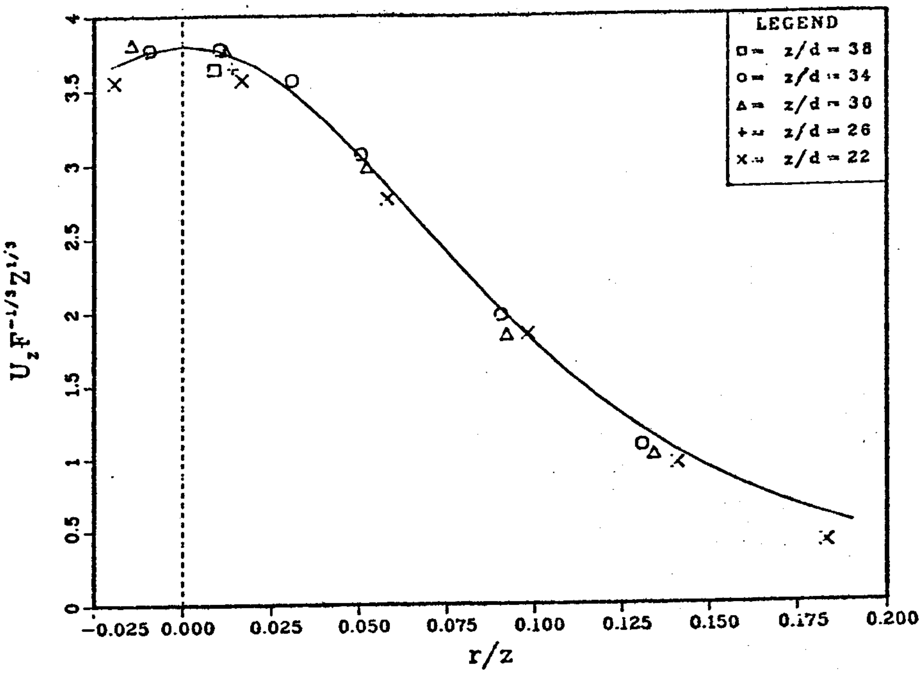

Figure 2 - Normalized mean velocity profiles.

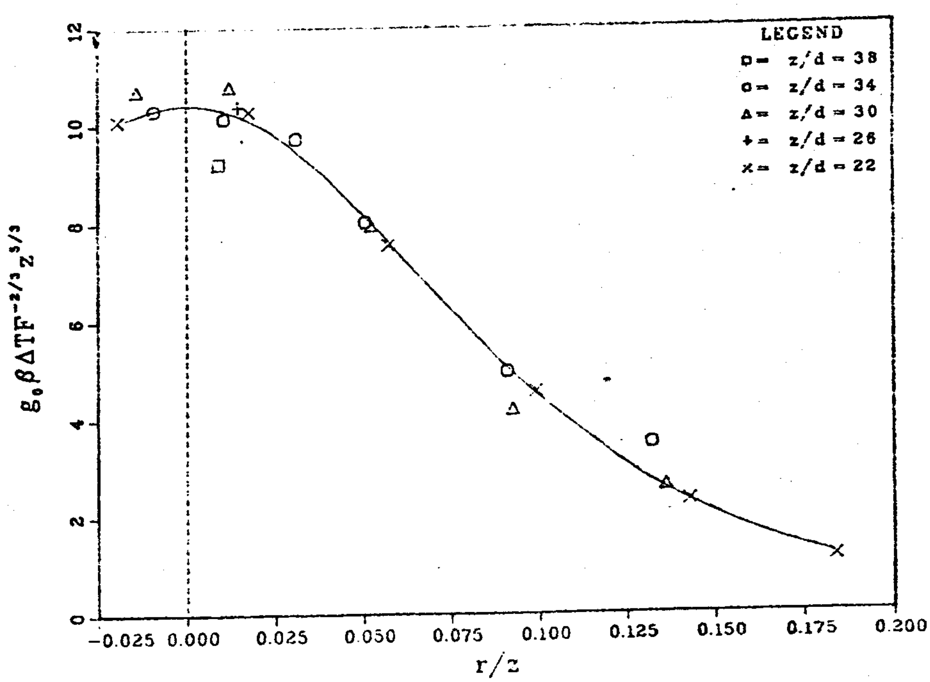

Figure 3 - Normalized mean temperature difference profiles.

even if the ambients were unknowingly stratified (as can happen very easily in these experiments). Since collapse in similarity variables was believed possible for plumes in neutral environments only, it would not be surprising if small stratification effects were ignored and if a variety of plumes in power law environments were reported as neutral.

Since the question of what constitutes a plume in a neutral environment has been raised, it is appropriate before concluding to mention the recent measurements of Ahmad [9] in our laboratory which carefully monitored the source and ambient conditions and verified that the experiment constituted a plume in a neutral environment. The data were virtually identical to those of George et al. [5].

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REFERENCES

[1] Batchelor, G.K., "Heat Convection and Buoyancy Effects in Fluids", Quart. J. Roy. Met. Soc., 80, 1954, 334-58.

[2] Turner, J.S., Buoyancy Effects in Fluids, CUP, 1973.

[3] Beuther, P.D., "Experimental Investigation of the Axisymmetric Turbulent Buoyant Plume", Ph.D. Dissertation, SUNY/Buffalo, 1980.

[4] Beuther, P.D. and George, W.K., "The Turbulent Buoyant Jet in a Uniform Environment", Proc. 7th Int. Heat Trans. Conf., Munich, Vol. II, Hemisphere, 1982.

[5] George, W.K., Alpert, R.L., and Tamanini, F., "Turbulence Measurements in an Axisymmetric Buoyant Plume", Int. J. Heat and Mass Trans., V. 20, 1977, 1145-1154.

[6] Schmidt, W.Z., "Turbulent Propagation of a Stream of Heated Air", Z. Angew. Math. Mech. 21, 1941, 265-351.

[7] Rouse, H., Yih, C.S., and Humphreys, H.W., "Gravitational Convection from a Boundary Source", Tellus, 4, 1952, 101-10.

[8] Chen, C.J. and Rodi, W., "Vertical Turbulent Buoyant Jets: A Review of Experimental Data", Oxford University Press, London, England, 1980.

[9] Ahmad, M.S., "An Experimental Study of the Axisymmetric Turbulent Buoyant Plume", M.S. Thesis, SUNY/Buffalo, 1980.