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The self-preservation of homogeneous shear flow turbulence

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Abstract. An analysis of the equations governing homogeneous shear flow shows the possibility of solutions which are self-preserving at all scales of motion, and that these solutions are dependent on the initial conditions. The appropriate velocity scale is the one obtained from the turbulence kinetic energy, $q^2/2$, while the length scale is the Taylor microscale, λ. Two cases of self-preserving flow are identified: one corresponding to constant mean shear, the other to a mean shear which is inversely proportional to time. For the first case (the only one considered in detail) the principal results of the postulated similarity are that λ is constant, while q^2 varies exponentially with time. The ratio of the turbulence energy production rate to its dissipation rate remains constant. It is also shown that the energy spectra scale over all wavenumbers with q^2 and λ, and that they have shapes determined by the initial conditions. The experimental evidence is generally consistent with the theory.

1 Introduction

The simplest type of turbulent shear flow is one in which unbounded homogeneous turbulence is maintained by a uniform gradient of mean velocity. Turbulent transport of the turbulence moments is formally zero, the turbulence energy is generated by interaction with the mean shear, distributed between components by interactions with the fluctuating part of the pressure, and dissipated through the action of viscosity. Many of the basic features of inhomogeneous turbulence are retained while the absence of diffusion simplifies theoretical treatment. Measurements in flows of this type have been widely used to study the coupling between the turbulence and the mean flow, and to validate mathematical models of the intercomponent energy transfer mechanism.

Von Karman (1937) appears to have been the first to draw attention to the importance of homogeneous shear flow in basic turbulence research. Corrsin (1963) suggested how it might be set up in a wind tunnel by using a non-uniform grid, and encouraged Rose (1966) to perform the first experiment. Later attempts to generate this type of moving equilibrium have been made by Champagne et al. (1970), Mulhearn and Luxton (1975), Harris et al. (1977), Tavoularis and Corrsin (1981, 1985), Karnik and Tavoularis (1983), Gibson and Kanellopoulos (1988), Rohr et al. (1988), and Tavoularis and Karnik (1988). The last two papers also contain useful material on all the experiments.

One of the principal difficulties in interpreting the experimental results has been to determine whether or not the shear flow reaches full development to a postulated asymptotic state which is determined by the mean shear, and not by the distance from the shear generator. A particular problem in this regard is to identify what the asymptotic state should be. The earliest studies appeared to expect a state in which the turbulence energy reached a constant value, while the consensus of the more recent work is that it continues to increase monotonically. Rohr et al. (1988) argue persuasively that the wind-tunnel experiments reasonably simulate a homogeneous shear flow evolving in time in the convected frame of reference, and provide a qualified endorsement for the semi-empirical argument of Tavoularis (1985) that the turbulence moments increase exponentially with time (or distance in the main flow direction). The work of Tavoularis and Karnik (1989) lends further support to this idea; these authors find that exponential growth is at least consistent with most of the available data.

It is the purpose of this paper to present a new theory which shows that homogeneous shear flows may achieve asymptotic states which are both self-preserving and determined by the initial conditions. The theory is an outcome of recent studies on the nature of self-preservation in turbulence. One of us (George 1989) has recently re-examined the conditions for similar solutions of the equations of motion for inhomogeneous shear flows with results that highlight the importance of the initial conditions in determining the asymptotic state of the developing flow. In a separate study of the decay of homogeneous isotropic turbulence, the same author (George 1988, 1992) found self-preserving solutions to the dynamic equation for the energy spectrum which were found which were valid at all scales of motion. In this case the velocity scale was obtained from the turbulence kinetic energy, and the characteristic length scale was shown to be the Taylor microscale. From the conditions for self-preservation, it was possible to show that the energy decay followed a power law in time, while the Taylor microscale increased with the square-root of time. It was also shown that a universal law for the energy decay cannot exist because the exponent and constants in these laws must depend on the initial conditions.

The evolution of homogeneous shear flow is now to be treated in a similar manner. As in George (1988, 1992), the starting point is the dynamical equation for the energy spectrum (Hinze 1975), this time with additional terms arising from the presence of a mean shear rate and anisotropy. Self-preserving solutions are sought for which all terms in the equation retain the same relative magnitude as the flow evolves. As in the earlier analysis of isotropic turbulence, the characteristic velocity and length scale will be shown to be those determined from the turbulence energy and the Taylor microscale.

The analysis shows that two types of self-preserving solutions are possible. The first, in which the mean shear varies inversely as time (or distance) has not been realised experimentally. The second type of solution, for which the mean shear and Taylor microscale must both be constant, corresponds closely to experimental conditions. For full self-preservation, the energy spectra must collapse to a single curve, unique to each set of initial conditions, and the integral scale must be constant. The theory is developed in the next section of the paper. This is followed by a discussion of the development of real flows in space, as distinct from the development of theoretical flows in time. The experimental evidence is then examined. This contains new data by Gibson and Kanellopoulos (1988) and Kanellopoulos (1987). It is found that the predictions of the theory are consistent with the experimental data. The Taylor microscale is indeed constant in all developed flows and the measured one-dimensional energy spectra collapse well, especially considering the probable accuracy of the data, and the degree to which the experiments simulate the theoretical flows.

2 Analysis

2.1 The spectral equations

The flow considered is one in which turbulence is produced by uniform mean shear. The mean velocity is assumed to be in the x_1 direction and to vary linearly in the x_2 direction. Thus, if K is the constant value of the velocity gradient $\mathrm{d}U_1/\mathrm{d}x_2$, the velocity U_i can be represented as:

$$U_i = K\, x_2\, \delta_{i1}\,, \tag{1}$$

and the turbulence is homogeneous in the x_2-x_3 plane.

The analysis starts from the dynamical equation for the energy spectrum tensor $\Phi_{i,k}$ in homogeneous turbulence, which is formally derived in the standard texts from the equation for the two-point velocity correlation (e.g. Hinze 1975, p. 336, Eq. 4-39. The spectrum equation is:

$$\frac{\partial}{\partial t}\Phi_{i,k} - K\, k_1 \frac{\partial}{\partial k_2}\Phi_{i,k} = \Pi_{i,k} + \Gamma_{i,k} - K(\Phi_{2,k}\,\delta_{i1} + \Phi_{i,2}\,\delta_{k1}) - 2\nu k_j k_j \Phi_{i,k}\,, \tag{2}$$

where $\Pi_{i,k}$ and $\Gamma_{i,k}$ are the Fourier transforms of the pressure-velocity and triple-velocity terms in the equations for the two-point velocity correlation.

This equation can be cast into the more familiar form of an equation for the three-dimensional energy spectrum function $E(k)$, which is defined as the average, over spherical shells of radius k, of the contracted three-dimensional spectrum. Thus:

$$E(k) = \tfrac{1}{2} \iint_{k=|\underline{k}|} \Phi_{i,i}(\underline{k})\, \mathrm{d}\sigma(\underline{k}) \tag{3}$$

where $\Phi_{i,i}$ represents the contraction of $\Phi_{i,k}$. It follows from the definition of $\Phi_{i,k}$ that the integral of $E(k)$ over all wavenumbers is equal to the turbulence kinetic energy per unit mass, i.e.

$$\tfrac{1}{2} q^2 = \tfrac{1}{2}\overline{u_i u_i} = \int_0^\infty E(k)\, \mathrm{d}k\,. \tag{4}$$

Equation (2) is integrated over spherical shells of radius k to yield

$$\frac{\partial}{\partial t} E(k,t) - K \iint_{k=|\underline{k}|} k_1 \frac{\partial}{\partial k_2}\Phi_{i,i}(\underline{k})\, \mathrm{d}\sigma(\underline{k}) = \gamma(k) - 2K \iint_{k=|\underline{k}|} Co_{1,2}(\underline{k})\, \mathrm{d}\sigma(\underline{k}) - 2\nu k^2 E(k)\,, \tag{5}$$

where

$$\gamma(k) = \iint_{k=|\underline{k}|} \Gamma_{i,i}(\underline{k})\, \mathrm{d}\sigma(\underline{k}) \tag{6}$$

and the contribution of the pressure-velocity interaction vanishes because $\Pi_{i,i}(\underline{k}) = 0$ when the fluid is incompressible. The cospectrum, $Co_{1,2}$, is defined as the real part of $\Phi_{1,2}$ and arises in this context from the fact that:

$$\Phi_{i,j}(\underline{k}) + \Phi_{j,i}(\underline{k}) = \Phi_{i,j}(\underline{k}) + \Phi^*_{i,j}(\underline{k}) = 2\, Co_{i,j}(\underline{k})\,, \tag{7}$$

where * denotes the complex conjugate.

The second term on the left side of Eq. (5) gives rise to a spectral scrambling due to the mean shear. The second term on the right side represents production of turbulence kinetic energy by the working of the Reynolds stresses against the mean velocity gradient. The remaining terms, which appear also in the equation for shear-free turbulence (George 1988, 1992), account for (a) spectral energy transfer by nonlinear interactions to the wavenumber k from all other wavenumbers and (b) viscous dissipation.

2.2 Self-preserving solutions of the spectral equations

We seek to establish the existence of self-preserving solutions of the energy spectral equation (5) for which all terms in the equation remain in relative balance. We begin by assuming solutions of the form:

$$E(k,t) = E_s(t)\, f_1(\eta) \tag{8}$$

$$\gamma(k,t) = \gamma_s(t)\, g(\eta) \tag{9}$$

$$-\iint_{k=|\underline{k}|} k_1 \frac{\partial}{\partial k_2}\Phi_{i,i}(\underline{k})\, \mathrm{d}\sigma(k) = Q_s(t)\, f_2(\eta) \tag{10}$$

$$-2\iint_{k=|\underline{k}|} Co_{1,2}(\underline{k})\, \mathrm{d}\sigma(\underline{k}) = R_s(t)\, f_3(\eta) \tag{11}$$

where $\eta = kL$ and $L = L(t)$. When Eqs. (8)–(11) are substituted into Eq. (5), the result is:

$$[\dot{E}_s] f_1 + \left[E_s \frac{\dot{L}}{L}\right] \eta f_1' + [K Q_s] f_2 = [\gamma_s] g + [K R_s] 2 f_3 - \left[\frac{\nu E_s}{L^2}\right] 2\eta^2 f_1 , \tag{12}$$

where the prime ′ denotes differentiation with respect to η. For self-preservation, all of the bracketed terms must have the same time-dependence since they depend explicitly on time. It is convenient to divide by the last term in square brackets so that the equation reduces to:

$$\left[\frac{\dot{E}_s L^2}{\nu E_s}\right] f_1 + \left[\frac{L\dot{L}}{\nu}\right] \eta f_1' + \left[\frac{K Q_s L^2}{\nu E_s}\right] f_2 = \left[\frac{\gamma_s L^2}{\nu E_s}\right] g + \left[\frac{K R_s L^2}{\nu E_s}\right] 2 f_3 - [1]\, 2\eta^2 f_1 . \tag{13}$$

It is clear from the presence of the constant coefficient multiplying the last term that self-preserving solutions are possible only if the other bracketed terms are also constant. It is easy to see from the definitions that Q_s, R_s, and E_s are all related directly to the spectrum tensor $\Phi_{i,k}$. Since in self-preserving development, each component of $\Phi_{i,k}$ reaches equilibrium with respect to the others, so that they all grow or decay together, it follows that

$$Q_s \sim R_s \sim E_s \tag{14}$$

and a single scale can be chosen for all the second-order quantities. The conditions for the self-preservation of homogeneous shear flow then reduce to:

$$\left[\frac{\dot{E}_s L^2}{\nu E_s}\right] = \text{constant} \tag{15}$$

$$\left[\frac{L\dot{L}}{\nu}\right] = \text{constant} \tag{16}$$

$$\left[\frac{K L^2}{\nu}\right] = \text{constant} \tag{17}$$

$$\left[\frac{\gamma_s L^2}{\nu E_s}\right] = \text{constant} . \tag{18}$$

2.3 Deductions from the analysis

The function E_s and the length scale L can be related directly to physical quantities by considering the energy and dissipation integrals. Consider first the kinetic energy defined by:

$$\tfrac{1}{2} q^2 = \int_0^\infty E(k)\, dk = \left[\frac{E_s}{L}\right] \int_0^\infty f_1(\eta)\, d\eta , \tag{19}$$

from which it follows immediately that we can take

$$E_s = q^2 L \tag{20}$$

without loss of generality by absorbing a constant into f_1. The rate of dissipation of turbulence energy is given by:

$$\varepsilon = 2\nu \int_0^\infty k^2 E(k)\, dk = 2\left[\frac{\nu E_s}{L^3}\right] \int_0^\infty \eta^2 f_1(\eta)\, d\eta , \tag{21}$$

and it follows from Eqs. (20) and (21) that:

$$\varepsilon \sim \nu \frac{q^2}{L^2} . \tag{22}$$

But the Taylor microscale, λ, is defined by

$$\overline{\left[\frac{\partial u_1}{\partial x_1}\right]^2} = \frac{\overline{u_1^2}}{\lambda^2} \tag{23}$$

and the energy dissipation rate is:

$$\varepsilon = 15 C \nu \overline{\left[\frac{\partial u_1}{\partial x_1}\right]^2} = 5 C \nu \frac{q^2}{\lambda^2} , \tag{24}$$

where $C = 1$ for isotropic turbulence. Equations (22) and (24) show that we can take $L = \lambda$ without loss of generality, since the turbulence quantities are assumed to be in equilibrium with respect to one another. *The length scale L is identified as the Taylor microscale* λ and $E_s = q^2 L = q^2 \lambda$ from Eq. (20). From now on λ will be used in place of L.

Integration of Eq. (11) produces the following expression for the shear stress:

$$-\overline{u_1 u_2} = \int_0^\infty \{-2 \iint_{k=|\underline{k}|} Co_{1,2}(\underline{k})\, d\sigma(\underline{k})\}\, dk = \frac{R_s}{\lambda} \int_0^\infty f_3(\eta)\, d\eta , \tag{25}$$

and, since E_s is proportional to R_s, if follows immediately that $-\overline{u_1 u_2}$ must be proportional to q^2. We denote this constant ratio by a_{12}. The rate of production of turbulence kinetic energy is:

$$P \equiv -\overline{u_1 u_2} \frac{dU_1}{dx_2} = a_{12} q^2 K \tag{26}$$

and

$$\frac{P}{\varepsilon} = \left(\frac{a_{12}}{5C}\right)\left[\frac{K\lambda^2}{\nu}\right] = \text{constant} , \tag{27}$$

where Eq. (24) has been used for ε and $[K\lambda^2/\nu]$ must be constant by virtue of Eq. (17).

Integration of Eq. (16) yields the time dependence of L (or λ) as:

$$\lambda^2 = A\nu t + B , \tag{28}$$

where A and B are constants. Two cases can be identified: one in which $A = 0$, and $\lambda = \sqrt{B} = \lambda_0 =$ constant, the other in which $\lambda \propto \sqrt{\{A\nu(t - t_0)\}}$. When $K = 0$, the second case corresponds to the decay of homogeneous and isotropic turbulence treated by George (1988, 1992). When $K \neq 0$, its time variation is constrained by Eq. (17): $K L^2/\nu =$ constant. For self-preserving solution to exist with non-zero K we therefore require:

$$\text{Case I} \quad \lambda = \text{constant} = \lambda_0 \quad K = \text{constant} \tag{29}$$

$$\text{Case II} \quad \lambda = [A\nu(t - t_0)]^{1/2} \quad K \propto [A(t - t_0)]^{-1} . \tag{30}$$

The Case II condition requires the mean vorticity to decrease with time, which is physically possible only with compression by additional mean strain, and not in pure shear flow.[1] Attention is therefore confined to Case I which correspond to the available measurements.

For constant mean shear it follows from Eqs. (15) and (29) that:

$$\frac{\dot{E}_s}{E_s} = \beta \frac{\nu}{\lambda_0^2}, \tag{31}$$

where β is the constant of Eq. (15) and is determined by the initial conditions. Integration produces:

$$E_s(t) = E_s(0) \exp\left(\frac{\beta \nu t}{\lambda_0^2}\right), \tag{32}$$

where $E_s(0)$ is the energy spectrum at $t=0$. It follows from Eq. (20), and the condition $\lambda = \lambda_0 =$ constant, that:

$$q^2(t) = \frac{E_s(t)}{\lambda} = q^2(0) \exp\left(\frac{\beta \nu t}{\lambda_0^2}\right), \tag{33}$$

where $q^2(0)$ is the initial value of q^2. Equation (33) is more conveniently written as:

$$q^2(\tau) = q^2(0) \exp(\beta' \tau), \tag{34}$$

where β' is a new rate constant:

$$\beta' = \beta\left(\frac{\nu}{\lambda_0^2 K}\right) \tag{35}$$

and $\tau \equiv K t$ is a dimensionless time. Thus for self-preservation of a homogeneous turbulent flow with constant mean shear, the turbulence energy must increase (or decrease) exponentially at a rate determined by the initial conditions and the mean shear rate.

Integration of Eq. (5) over all wavenumbers produces the equation for the turbulence energy:

$$\frac{d}{dt}\left(\tfrac{1}{2} q^2\right) = P - \varepsilon = K\, q^2 \left(\frac{-\overline{u_1 u_2}}{q^2}\right)\left(1 - \frac{\varepsilon}{P}\right). \tag{36}$$

Comparison with Eq. (34) shows that:

$$\beta' = 2 a_{12} (1 - \varepsilon/P), \tag{37}$$

which is the result obtained by Tavoularis (1985). Tavoularis *assumed* that $-\overline{u_1 u_2}/q^2$ and P/ε would be constants in homogeneous shear flow. It has now been shown that *this is a necessary condition for self-preservation*. It follows that $-\overline{u_1 u_2}$, the components of q^2, and P and ε all grow exponentially at the same rate. The principal results of the theory for self-preserving development are summarised in Table 1.

3 Experimental verification

3.1 Spatial development and the experiments

Of the two possible cases of self-preserving flow identified in the analysis, only the homogeneous flow with constant mean shear has been studied experimentally (as far as we have been able to determine). The experiments have been carried out in steady flows in wind tunnels (and in one case a water channel) so that rates of change with respect to time that appear in the governing equations considered above are replaced by space derivatives in the equations of the experimental flows. This substitution raises fundamental questions about the way in which the steady flow developing in physical space models the time-dependent flow of the analysis.

The conditions sought in the experiments were a steady mean flow with a single uniform mean shear, and turbulence homogeneous in planes perpendicular to the mean flow direction. At first it seems to have been expected that the natural state would be one in which turbulence energy production would be balanced by dissipation, convection by the mean flow as well as turbulent transport being negligible by comparison. There is, in fact, no reason to expect such a balance. As the analysis shows, exponential energy growth (or decay) will result from all but the most special choice of initial conditions.

If uniform mean shear could be associated with exact transverse homogeneity, the rate of change of turbulence energy would be equal to the difference between production and dissipation (Eq. (36)). But as Harris et al. (1977) and Tavoularis (1985) have pointed out, homogeneous conditions have not been exactly reproduced in the experiments. The difficulty arises because the kinetic energy equation for a laboratory flow which is steady in the mean and homogeneous in the transverse directions is:

$$U_1 \frac{d}{dx_1}\left(\frac{q^2}{2}\right) = P - \varepsilon. \tag{38}$$

Now, since $U_1 = U_1(x_2)$, $U_2 = U_3 = 0$, and $dU_1/dx_2 =$ constant, the left side of Eq. (38) depends on x_2 while the right side does not, on account of the postulated homogeneity. Thus, Harris et al. (1977) concluded that "*a steady rectilinear flow with constant velocity gradient plus transverse homogeneity of turbulence moments is impossible. The stationary flow cannot be homogeneous, not even transversely*".

Strictly speaking, it is necessary to consider all the terms in the energy equation. With justifiable neglect of viscous

Table 1. Self-preservation constraints for constant mean shear

Mean shear	$K \equiv dU_1/dx_2$	= constant
Length scale	$L = \lambda = \lambda_0$	= constant
Energy spectrum	$E_s(t)$	$= E_s(0) \exp\{\beta' K t\}$
Spectral transfer	γ_s	$\propto \nu E_s / K \lambda^2$
Kinetic energy (×2)	$q^2(t)$	$= q^2(0) \exp\{\beta' K t\}$
Shear stress/ Kinetic energy	$a_{12} \equiv -\overline{u_1 u_2}/q^2$	= constant
Energy production/ dissipation	$\frac{P}{\varepsilon}$	$= \frac{a_{12}}{5C}\left[\frac{K\lambda^2}{\nu}\right]$
Growth rate coefficient	β'	$= 2 a_{12} (1 - \varepsilon/P)$

[1] We are grateful to Professor W. C. Reynolds and an anonymous referee for pointing this out to us

transport the full equation is:

$$U_1 \frac{d}{dx_1}\left(\frac{q^2}{2}\right) = -\overline{u_1 u_2}\frac{dU_1}{dx_2} - \varepsilon - \left\{\frac{\partial}{\partial x_2}\left(\frac{\overline{q^2 u_2}}{2} + \overline{p u_2}\right) + \frac{\partial}{\partial x_1}\left(\frac{\overline{q^2 u_1}}{2} + \overline{p u_1}\right)\right\}. \quad (39)$$

If $\overline{u_1 u_2}$ and dU_1/dx_2 are to be uniform across the flow, the dependence of mean flow transport on x_2 must be accounted for by the cross-stream variation of the turbulent transport terms in Eq. (39), or, possibly by a non-uniform rate of dissipation. All of the evidence suggests that turbulent transport in the streamline direction is negligible, while the cross-stream turbulent transport in highly-sheared flow has been estimated by Harris et al. (1977) to be only about 3% of the mean flow convection. There was also some evidence of transverse inhomogeneity of the dissipation rate in this experiment, and in the repeat experiment of Tavoularis and Corrsin (1981), where the Taylor microscale increased slightly in the direction of increasing mean velocity. The effect on the energy balance of these departures from the ideal is small, as may be seen by rewriting Eq. (39) as:

$$U_0 \frac{d}{dx_1}\left(\frac{q^2}{2}\right) = -\overline{u_1 u_2}\frac{dU_1}{dx_2} - \varepsilon + \left\{(U_0 - U_1)\frac{d}{dx_1}\left(\frac{q^2}{2}\right) - \frac{\partial}{\partial x_2}\left(\frac{\overline{q^2 u_2}}{2} + \overline{p u_2}\right) + \frac{\partial}{\partial x_1}\left(\frac{\overline{q^2 u_1}}{2} + \overline{p u_1}\right)\right\}, \quad (40)$$

where U_0 is a constant velocity, say the value on the tunnel axis. It will be appreciated that even if the individual terms in the brackets are not negligible, their difference probably is.

Hinze (1975) has also pointed out that the basic assumptions of homogeneous turbulence with constant mean shear are inconsistent because the production of turbulence by the mean shear must simultaneously decrease the kinetic energy of the mean flow and thereby decrease the velocity gradient. In a real flow this reduction in the mean shear causes a mean velocity gradient to develop in the mean flow direction which can be balanced in the continuity equation only by the corresponding development of a cross-stream component of the mean velocity. This effect has been observed only in the water tunnel experiment of Rohr et al. (1988) where the degradation of the mean gradient as small. But it may have been large enough to contribute to the decline in the production/dissipation ratio which was an unusual feature of this experiment.

It is no doubt possible, then, to devise a wind tunnel experiment that will satisfy Eq. (36) to sufficient accuracy to test the theory, at least for scales of motion smaller than the characteristic dimensions of the facility. The remaining problem is that of ensuring that the shear flow has sufficient length to develop. In all the experiments the upstream part of the flow is dominated by the decay of the initial turbulence caused by the shear generator, and only far downstream has most of the turbulence been produced by interaction with the mean shear. For flows with moderate-to-high shear rates ε passes through a minimum. Eventually P/ε becomes a constant (except in the flow of Rohr et al. (1988) where it was found to continue to decrease). Fully-developed conditions apparently occur when the dimensionless distance (~time)

$$\tau = K\,t = \frac{x_1}{U_0}\frac{dU_1}{dx_2} \quad (41)$$

is greater than about six, where x_1 is measured from the shear generator.

In the first (pre-1977) experiments the rates of shear were too low and the development lengths were too short for τ values to exceed 3.5. Thus only the later work, starting from the highly sheared flow of Harris et al. (1977) is relevant here. In this experiment, which was repeated and extended by Tavoularis and Corrsin (1981), and again by the same authors in 1985, the maximum value of τ was raised to 13 by the use of a high shear rate. An unexpected side effect was an increase in P/ε from values near unity in the earlier experiments to approximately 1.7. Karnik and Tavoularis (1983) subsequently made measurements in three flows with τ values as high as 29 and, remarkably, P/ε values as low as 1.4. This experiment has since been described in more detail by Tavoularis and Karnik (1989). Measurements were made with and without grids or screens positioned downstream of the shear generator. The development lengths in the former cases were comparatively short, with $\tau_{max} \approx 8$, and the effects of decaying grid turbulence may not have been altogether negligible. For these reasons, only data from the experiment on flow in the unobstructed tunnel will be considered here.

Gibson and Kanellopoulos (1988) tried to obtain a developed flow with a low shear rate, not too far from local energy equilibrium. They were not entirely successful in this; their flow with $P/\varepsilon \approx 1.3$ extended to $\tau = 8$. Finally, although some of the water-channel measurements of Rohr et al. (1988) were taken at distances up to $\tau = 30$, their P/ε ratios were not constant as in the other experiments. The post-1976 experiments form the data base to test the theory. The P/ε range is from 1.3 to 1.7. Only in the case of vanishing shear, $P = 0$, have measurements been made in flows with P/ε less than unity, and it is possible that homogeneous shear flow with $0 < P/\varepsilon < 1$ cannot be set up in the laboratory.

3.2 *The conditions for self-preservation*

The theory is directly verified if it can be shown that, when scaled with the turbulence intensity and Taylor microscale, the measured energy spectra are self-similar over *all* scales for which the flow can be assumed to be reasonably homogeneous. In addition, the measured microscales must be constant. The postulated scaling for the measured one-dimensional spectra $F_{1,1}$ and $F_{2,2}$ is:

$$F_{i,i}(k_1) = \overline{u^2}\,\lambda\,\mathrm{F}_{i,i}(k_1 \lambda), \quad i = 1, 2. \quad (42)$$

The theory predicts that self-preserving flow will have fixed ratios of the component energies and of $-\overline{u_1 u_2}/q^2$. The components of the Reynolds stresses, like q^2, will increase exponentially with τ, and P/ε will be constant. The integral scales, obtained either from integration of the autorcorrelation coefficient, or from the zero-frequency intercept of the one-dimensional spectrum, must be proportional to λ and therefore also be constant. On the other hand, the turbulence length scale $l_\varepsilon \equiv (q^2)^{3/2}/\varepsilon$ and the mixing length $l_m \equiv (-\overline{u_1 u_2})^{1/2}/K$ will both increase as $\sqrt{q^2} \sim \exp(\beta' \tau/2)$, as does the microscale Reynolds number R_λ. It follows that the Kolmogorov microscale, $\eta \equiv (\nu^3/\varepsilon)^{1/4}$, will decrease as $\exp(-\beta' \tau/4)$.

3.3 *The turbulence energy and its production and dissipation rates*

All of the wind-tunnel experiments confirm that P/ε approaches a constant value as a homogeneous shear flow evolves, with *both* P and ε increasing as q^2. The dissipation rates have been variously obtained in the different experiments by assuming local isotropy and integrating velocity derivative spectra, from measured mean square velocity derivatives and the use of Eq. (24) with $C=1$ for isotropic turbulence, and by balance of the turbulence energy equation (38). The only exception to the finding that P is proportional to ε is from the water channel experiments of Rohr et al. (1988) in which P and ε both increased nearly linearly, but at different rates, to downstream values of about 1.0 and 1.7 for low and high shear, respectively. This result, which is inconsistent with weak exponential growth of the turbulence energy, may perhaps be associated with progressive degradation of mean shear in this experiment. On the other hand, this is the only one of the experiments in which the Reynolds shear stress was not directly measured, so that the production had to be obtained from the energy balance, instead of the dissipation as is more usual. In view of this, and the agreement in other respects with the other experiments, and with the theoretical predictions, this deviation should not perhaps be given too much weight.

According to Eq. (34), q^2 will vary exponentially when the flow is self-preserving. Tavoularis (1985) has already demonstrated that this is the case for the two highly sheared flow of Tavoularis and Corrsin (1981) and Karnik and Tavoularis (1983), which had β' values of about 0.23 and between 0.31 and 0.34, respectively. Figure 1 is adapted from Tavoularis's paper; a more extensive verification is presented by Tavoularis and Karnik (1989). Tavoularis and Corrsin (1981) had assumed parabolic growth, but this is indistinguishable from weak exponential growth. In the weak shear flow of Gibson and Kanellopoulos (1988) the calculated rate constant $\beta' \approx 0.0425$ was so low that it was also impossible to distinguish between weak exponential and linear growth of q^2 in the developed region. The same considerations apply to the data of Rohr et al. (1988).

3.4 *The Taylor microscale*

In all of the developed homogeneous shear flows, the Taylor microscale is nearly independent of distance downstream, as

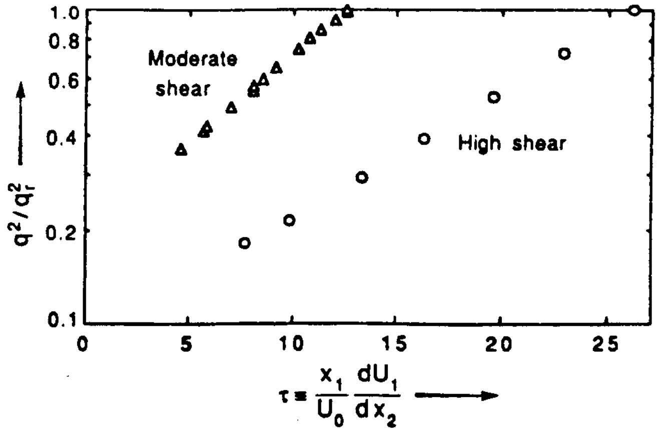

Fig. 1. Exponential downstream development of turbulence kinetic energy (from Tavoularis 1985)

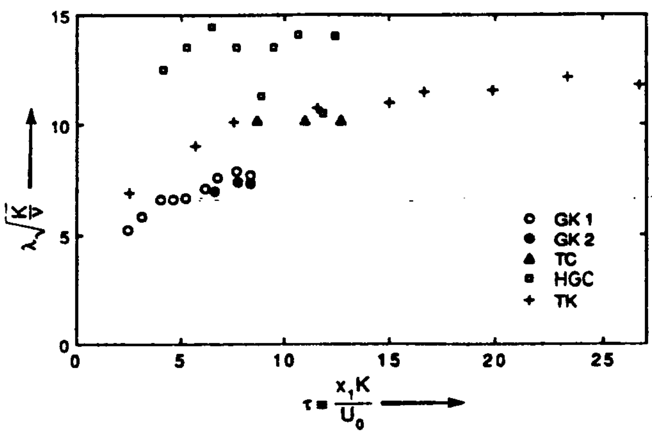

Fig. 2. Streamwise development of the Taylor microscale in different experiments: GK = Gibson and Kanellopoulos (1988) (GK1: λ calculated from the energy balance; GK2: λ calculated from the dissipation spectrum); TC = Tavoularis and Corrsin (1981); HGC = Harris et al. (1977); TK = Tavoularis and Karnik (1989)

predicted by the theory. Figure 2 is a plot of $\lambda\sqrt{(K/\nu)}$ (Eq. (17)) as a function of τ for several of the experiments. The data of Harris et al. (1977) and of Tavoularis and Corrsin (1981) show λ to have been constant in the range $6<\tau<12$ but, because the initial conditions differed in each experiment, the values of $\lambda\sqrt{(K/\nu)}$ are also different. Values obtained by Gibson and Kanellopoulos (1988) show a gradual rise to a constant value $\lambda\sqrt{(K/\nu)} \approx 7.5$ for $\tau>7$ (different symbols in this case denote different methods of calculating ε and λ). The measurements of Tavoularis and Karnik (1989), which extend to $\tau \approx 28$, are reasonably constant for $\tau>20$ where $\lambda\sqrt{(K/\nu)} \approx 12$. It is concluded that the Taylor microscale is constant in the developed flow and that the value of $\lambda\sqrt{(K/\nu)}$ is dependent on the initial conditions, as predicted by the theory. The theory is not invalidated by the slight transverse variation of λ found by Tavoularis and Corrsin (1981), which Tavoularis (1985) associates with the transverse variation of ε needed to balance the q^2 equation (as discussed previously).

3.5 The energy spectra

One-dimensional energy spectra have been documented by Tavoularis and Corrsin (1981), by Rohr et al. (1988), and in previously unpublished work by Kanellopoulos (1987). Figure 3 shows the functions $F_{1,1}(k_1)$ scaled from Fig. 14 of Tavoularis and Corrsin (1981) and replotted in Taylor similarity variables according to Eq. (42). The curves correspond to measurements made 7.5, 9.5 and 11 tunnel heights (h) from the shear generator. The uniformity of the collapse is remarkable in the data from the two downstream stations, and reasonably good overall, if the divergence of the $x_1/h=7.5$ spectrum at low wavenumbers is ascribed to the slower adjustment of the large scales to the imposed uniform shear, or to the limits imposed on the experimental model by the distance between the tunnel walls. The same comments apply to the Kanellopoulos (1987) spectra, which are also scaled according to Eq. (42) and are plotted in Fig. 4. At $x_1/h=14$ and 15 the spectra collapse on a single curve, while the upstream data at $x_1/h=12$ diverge slightly at the lower wavenumbers. Normalised spectra from these two experiments are plotted together in Fig. 5 using linear scales. (Note that Tavoularis and Corrsin plot spectra down to frequencies as low as 1 Hz, while a lower limit of about 8 Hz was imposed by the instrumentation used by Kanellopoulos). It is clear from Fig. 5 that each set of measurements produces a unique spectrum shape, entirely consistent with the emphasis laid by the self-preservation theory on the influence of the initial conditions. The comparison supports the hypothesis that, although each flow is self-preserving, no universal spectrum for homogeneous flow exists.

The spectra of both streamwise and cross-stream spectra $F_{1,1}(k_1)$ and $F_{2,2}(k_1)$ obtained by Rohr et al. (1988) from hot-film measurements in water have been rescaled in Taylor variables according to Eq. (42) and plotted in Fig. 6. Again the spectra tend to collapse on a single curve except at the lowest wavenumbers of the data at the farthest upstream position. Unfortunately, although the development length for this flow was by far the greatest of any of the experiments, the spectrum measurements were discontinued about halfway along the channel.

While the lack of collapse of all the spectra at the lowest wavenumbers might in part be attributed to the slower adjustment of the larger scales to the imposed shear, there is also reason to question whether the flow at these scales reasonably models unbounded homogeneous shear flow. Nearly all the tunnels (wind and water) were 30 cm × 30 cm square cross-section, except those of Kanellopoulos (45 × 45 cm^2) and Karnik (30 cm high × 46 cm wide). It follows that the lowest wavenumber of any possible significance for testing a model of free shear flow would be $k_1 \approx 2\pi/30 \approx 0.21$ cm^{-1}, (in fact, probably 4 to 5 times greater than this). Since λ is of the order of 1 cm for all the experiments, the data taken at very low frequencies, of the order $k_1\lambda < 0.1$, are probably not representative of unbounded homogeneous shear flow, in which case the discrepancies at low wavenumber should be ignored. Figure 7 shows the spectra of Rohr et al. (1988) on a linear plot which give a more realistic picture of the situation at low wavenumbers. It is apparent that even the low wavenumber data (to at least the largest wavenumbers for which the flow can be considered homogeneous) is consistent with the proposed scaling.

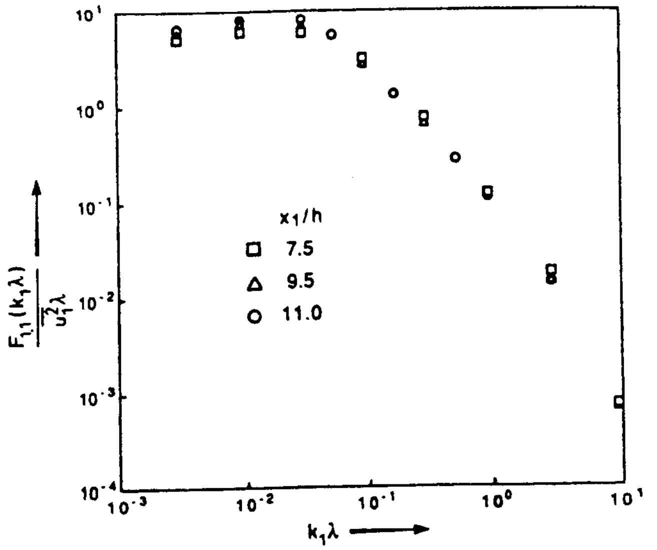

Fig. 3. Taylor scaling of the one-dimensional spectra of the streamwise turbulent velocity measured by Tavoularis and Corrsin (1981)

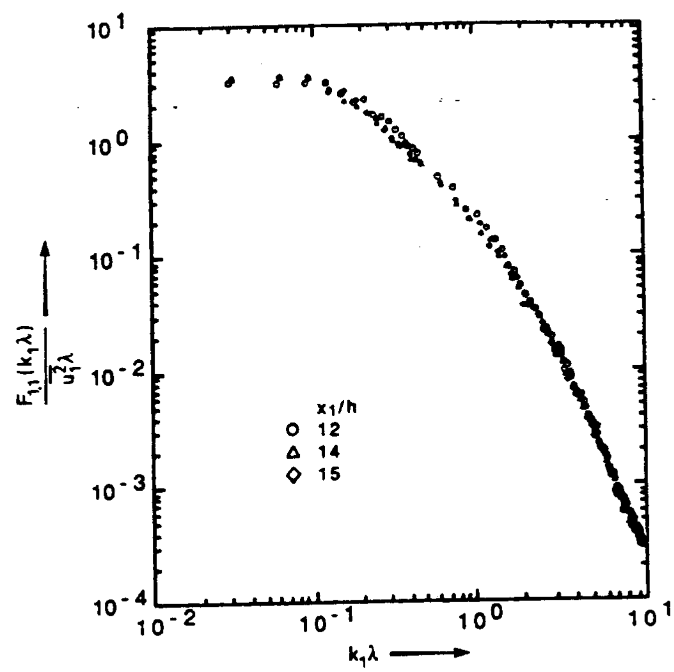

Fig. 4. Taylor scaling of the one-dimensional spectra of the streamwise turbulent velocity measured by Kanellopoulos (1987)

3.6 The integral scales

It follows from the theory that if the energy spectra are similar in a given flow, the integral scale determined from the zero-frequency intercept must obey the same scaling law. It must be admitted that there is not particulary strong support from the experimental data for this proposition. Integral scales obtained from all the flows under consideration tend to increase with increasing distance downstream, but at different, and in most cases diminishing rates. Dimensionless scales obtained from three sources are plotted against dimensionless distance (time) τ in Fig. 8. In none of these flows is full development nearly approached for values of τ less than six. Figure 8 shows that the rate of growth of the length scale $L_{1,1}$ of the streamwise velocity fluctuation is appreciable in the Tavoularis and Corrsin (1981) flow. On the other hand, Harris et al. (1977), in an earlier realization of what was nominally the same flow, report near-uniformity of $L_{1,1}$

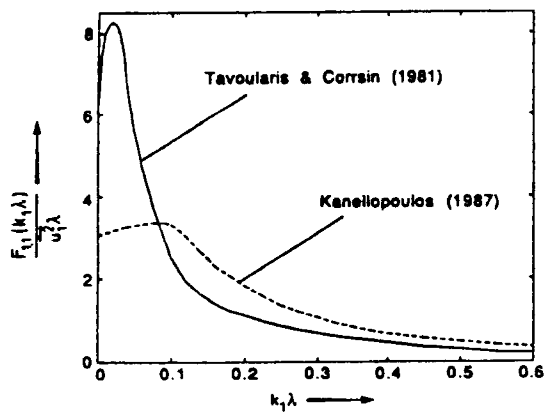

Fig. 5. Comparison on a linear plot of the normalised spectra of Figs. 3 and 4. —— Tavoularis and Corrsin (1981) $x_1/h=11$; ----- Kanellopoulos (1987), $x_1/h=14$

at the last three downstream stations (According to Tavoularis and Karnik (1989), the random experimental error in the length scales measurement was of the order of 7%). The scales estimated by Gibson and Kanellopoulos (1988) mirror the quality of the spectral collapse shown in Fig. 4, with nearly equal values being obtained at the last two stations. Figure 9, which is adapted from Fig. 7 of Karnik and Tavoularis (1983) (see also Tavoularis and Karnik 1989), shows $L_{1,1}$ increasing slowly to an inferred asymptotic value at $\tau > 29$ in the unobstructed (grid-free) flows.

These results raise the question of whether or not the measured integral scales are representative of a homogeneous shear flow developing in an infinite environment, or of one whose development is constrained by the tunnel in which it is generated. Rohr et al. (1988) expressed surprise at being able to use dimensionless time τ to collapse all of the statistical quantities from the different experiments, *except the measured integral scales*. Harris et al. (1977) note that their integral scales were nearly identical to the earlier ones of Champagne et al. (1970), even though the shear rates differed by a factor of 4. When $L_{1,1}$ is still fairly small compared to the tunnel height ($h \approx 30-40$ cm in most of the experiments), its value is strongly influenced by larger scales of motion which determine the tail of the correlation function and play an important role in determining the spectrum at low frequencies. We may doubt whether scales larger than the measured integral scales, which significantly affect these quantities, could be the same as those in the theoretical unbounded flow. The observations of Rohr et al. (1988) and of Harris et al. (1977) would seem to indicate that they were not.

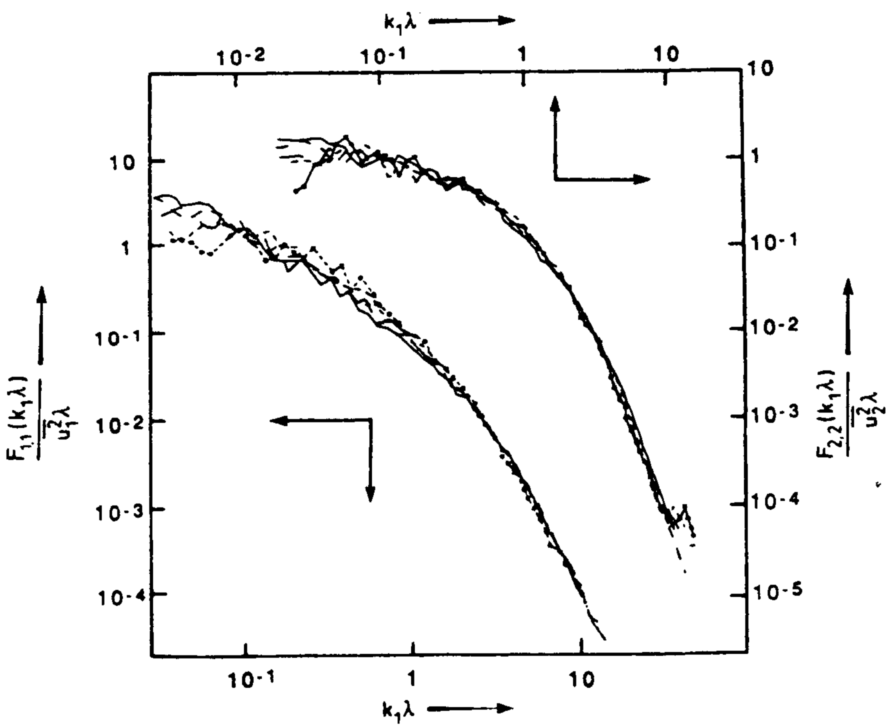

Fig. 6. Taylor scaling of the one-dimensional spectra of the streamwise and transverse turbulent velocities measured by Rohr et al. (1988)

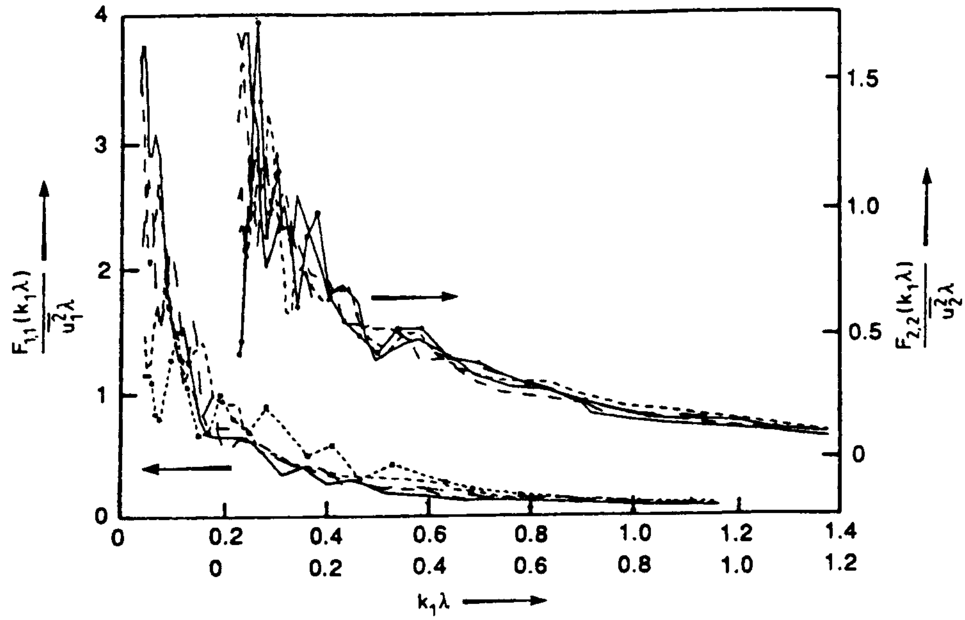

Fig. 7. Linear plot of the data of Fig. 6 (from Rohr et al. 1988)

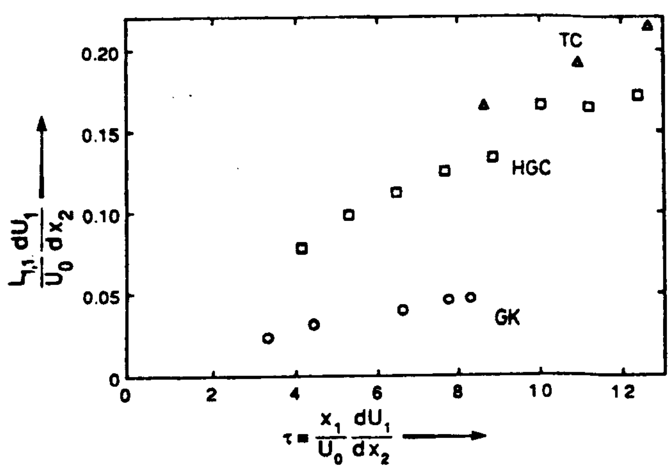

Fig. 8. Streamwise development of integral scales in three experiments; key as in Fig. 2

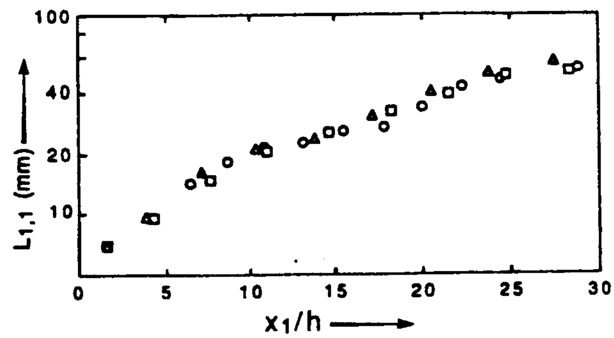

Fig. 9. Streamwise development of streamwise velocity integral scales (from Karnik and Tavoularis 1983)

3.7 *Influence of the initial conditions*

The importance of the initial conditions in determining the asymptotic character of the turbulence has already been discussed by Rohr et al. (1988) who note: "*More surprising is the continued downstream influence of the initial disturbance on the developing turbulence .... As long as the turbulence continues to interact with a constant mean shear, the data imply that the magnitude of the turbulence fluctuations should scale with the initial disturbance imposed at the inlet.*" Also of concern to these authors was the apparent inconsistency of P/ε as well as of $a_{12}(1-\varepsilon/P)$, both of which appeared to reach asymptotic values in the experiments considered, but changed from facility to facility in a manner which was not understood. Finally, the dependence on initial conditions of the velocity spectra from different experiments has been noted in Fig. 5.

The above observations are consistent with the theory. Viewed in the light of the analysis, they are no longer "surprising", but can be seen to be the natural consequences of the tendency of the flow to settle into a fully self-preserving state which is determined by the initial conditions. Unfortunately, the theory (at least in its present stage of development) can shed little light on what that dependence might be, and that only be speculating on the behaviour of the turbulence at infinite Reynolds number. It has long been believed that the energy spectra at high wavenumbers (the dissipative range) should collapse in Kolmogorov variables ε and ν, and that the extent of the collapsed region should move towards lower wavenumbers with increasing Reynolds numbers. It is easy to show that this cannot be true in general if the present theory is valid, since if the spectra collapse in Taylor variables they cannot collapse in Kolmogorov variables unless R_λ is constant. R_λ is constant only if $P=\varepsilon$, which is clearly not the case for the experiments considered here. Suppose, however, that we insist that Kolmogorov scaling must apply to the dissipative scales in the limit of infinite Reynolds number. Then we can simultaneously satisfy the conditions for full self-preservation in this limit if we also insist that $P = \varepsilon$, regardless of the way in which the flow is generated. Thus, for fixed geometry of the turbulence generator, we should expect P/ε to decrease toward unity as some appropriately defined source Reynolds number increases. Whether or not this speculation is correct can only be decided by further experimentation.

4 Conclusions

An analysis of the dynamical equations governing homogeneous shear flow has shown the possibility of solutions which are self-preserving at all scales of motion. The appropriate velocity scale is the one obtained from the turbulence kinetic energy, and the length scale is the Taylor microscale. The experimental data support the theory. Measurements in flows with adequate development lengths show the Taylor microscale to be constant and the increase in energy to be consistent with exponential growth. Collapse of the energy spectra by Taylor scaling is better than might have been expected in advance of the theory. Only in the largest scales do the spectra measured at different locations differ significantly. While it is possible that the differences at low wavenumbers are genuine, it is more likely that they arise from two limitations in the experiments: inadequate development length and the influence of flow boundaries. The development distance required for the larger scales may be much longer than the experimenters realised. Unfortunately, when full development is allowed for, and the zero-frequency intercepts of the one-dimensional spectra do tend to constant values, the larger scales become comparable with the tunnel dimensions and the theoretical assumption of unbounded flow is not satisfied. The present theory explains the observed persistent influence of the initial conditions, as well as the variations reported from facility to facility.

The theory suggests a number of possibilities for further experimentation. As for turbulence without shear, it can be shown that the spectral transfer and velocity derivative skewness depend inversely on R_λ. Since R_λ increases with distance, these quantities should decrease, the opposite of their behaviour in the absence of shear (George 1987, 1988). Such measurements appear not to have been made but they would provide an important additional test of the theory. Also of particular interest would be experiments to examine the dependence of P/ε on initial Reynolds number for fixed geometry. Finally, experiments in larger tunnels would test the arguments presented here regarding the integral scale.

Why the flow should choose to behave in a self-preserving manner remains a matter for conjecture. George (1989) has noted that turbulent flows whose governing equations admit to such fully self-preserving solutions always appear to relax asymptotically to a fully self-preserving state. In such situations, theories of local similarity appear not to apply, except possibly as an asymptotic state in the limit of infinite Reynolds number. The homogeneous turbulent shear flow considered here appears to be no exception.

Acknowledgements

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