Quantitative Measurement with the Burst-Mode Laser Doppler Anemometer*

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■The burst-mode or individual realization laser Doppler anemometer is reviewed with attention to the manner in which the data are processed. The emphasis is on making quantitatively correct measurements so that corrections to the results are not necessary. Of particular importance are the residence time weighting and the manner in which the flow is seeded. Previous analyses are evaluated, and modest extensions are proposed. The well-known criteria for analog signal processing are shown to have their counterparts for burst-mode LDA signals. A detailed analysis of the effects of counter quantization errors on the measurements is presented. Also discussed are the effects of the polydispersity of the scatterers on the measurements.

Keywords: *complex flows, instrumentation, measurement techniques, review*

INTRODUCTION

The laser Doppler anemometer (hereafter referred to as the LDA) was invented more than 25 years ago. The instrument and our understanding of it have remained relatively unchanged in the last ten years. In spite of the mature state to which our understanding and the technology have evolved, the LDA has not had a significant impact on our understanding of fluid dynamics. While in part this may stem from the fact that its unique capabilities are best utilized in industrial environments, it is in greater part due to the difficulties of properly using the instrument and in understanding what it measures.

The difficulties referred to are not problems with the use of the hardware to achieve Doppler signals. Modern integrated optics and sophisticated electronics have brought the instrument a long way from the early anemometers of the mid to late 1960s and even early 1970s. Commercial LDAs can be purchased off the shelf and made to work in a wide variety of environments by semiskilled personnel. Thus these difficulties are not related to the acquiring of Doppler signals. Rather they refer to the interpretation of the measurement as representing a fluid dynamic quantity.

The purpose of this paper is to review some of the most common difficulties in understanding the proper use of the laser Doppler anemometer. Some of the difficulties referred to are unique to the LDA, while others are common to all measuring techniques and are simply interpreted here for the LDA. It is hoped that this review will provide a checklist against which an investigator may test his or her own understanding and experiment. If the rules and guidelines that are laid down here are carefully adhered to, experiments that might have been performed incorrectly will be performed correctly, and the results will be useful to the scientific community. It is equally possible that a careful application of the principles set forth here will indicate that some experiments that have been performed are incorrect and that their results are at best unreliable. It is hoped that some experiments that might have been attempted will be found to be physically impossible and will be discarded. On the other hand, it is also hoped that an impetus for correct measurements and new experiments will have been generated.

Attention will be focused in this paper on the so-called burst-mode operation or individual realization LDA. In the succeeding paragraphs the theory of operation will be briefly reviewed and some of the more important aspects of the burst-mode signal will be highlighted. Finally, some of the problems of implementation will be discussed.

THEORETICAL CONSIDERATIONS

The Burst-Mode LDA

The basic principle behind the burst-mode LDA lies in the fact that each scattering particle generates a Doppler current of the form [1]

$$i_{sp}(t) = I(\boldsymbol{x}_p) \cos \boldsymbol{K} \cdot \boldsymbol{x}_p \tag{1}$$

where $\boldsymbol{x}_p$ is the position of the particle, $I(\boldsymbol{x})$ is a position-dependent intensity determined by the illumination of the measuring volume and the optics, and $\boldsymbol{K}$ is the scattering wavenumber vector. The position of the particle can be related to its initial position $\boldsymbol{a}$ and its velocity history $\boldsymbol{u}_{sp}(\boldsymbol{a}, t)$ by

$$\boldsymbol{x} = \boldsymbol{a} + \int_0^t \boldsymbol{u}_{sp}(\boldsymbol{a}, t_1)\, dt_1. \tag{2}$$

The scattering wavenumber vector can be defined as the vector difference in the wavenumbers of the incident and scattered light (or if there are two incident beams, by their vector difference). Thus, if as shown in Fig. 1, $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are the two beams differing

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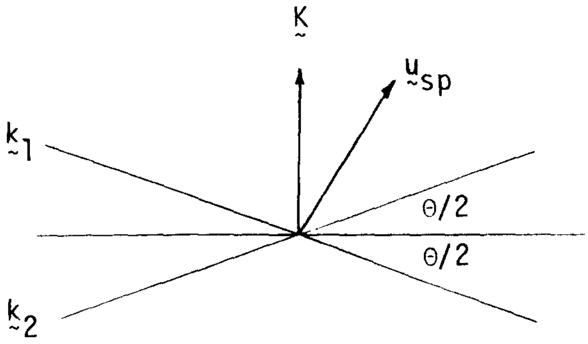

Figure 1. Illustration of scattering geometry.

by the angle Θ, then

$$K = k_1 - k_2. \tag{3}$$

The product $K \cdot x_p$ selects the component of the velocity colinear with the scattering wavenumber, say u_{sp}. Thus Eq. (1) can be written

$$i_{sp}(t) = I(x_p) \cos\left[Ka + K \int_0^t u_{sp}(a, t_1)\, dt_1\right]. \tag{4}$$

If the velocity of the particle can be assumed constant across the scattering volume, this reduces to

$$i_{sp}(t) = I(x_p) \cos\left[Ka + Ku_{sp} t\right]. \tag{5}$$

Thus the Doppler current due a single particle consists of an amplitude I that varies with the spatially varying intensity of the measuring volume, a phase Ka determined by the particle's position at some arbitrary origin in time, and a Doppler frequency defined by

$$\omega_D = Ku_{sp} = \frac{2\pi}{\lambda_i} u_{sp} \sin(\Theta/2) \tag{6a}$$

or

$$f_D = \frac{\sin(\Theta/2)}{\lambda_i} u_{sp} \tag{6b}$$

where λ_i is the wavelength of the incident light.

Note that it is sometimes useful to interpret the Doppler signal as having arisen from the particle crossing interference fringes in the flow. While not strictly correct, for most purposes the results are the same as Eq. (1).

The amplitude function I is generally close to Gaussian as shown in Fig. 2, with the result that the Doppler signal from a single particle consists of a "burst" with a near-Gaussian envelope as shown. The exact width and height of the envelope depend on where the particle enters the measuring volume and on the path it follows through it.

It is easy to see that if the frequency (or time between zero crossings) of the Doppler burst can be determined, then the particle velocity can also be determined. The purpose of the LDA burst processor is to make this determination. There are a variety of ways this can be accomplished. Commercially available counters all operate in a manner similar to that shown in Fig. 3. The Doppler burst is first amplified, and then the rising part of the envelope is used to start a clock that is stopped either after a fixed number of cycles or at the next zero crossing after the burst falls below a preset level. Since the number of cycles (or "fringes crossed") is known, the frequency of the burst can be determined.

If this were all there were to the burst-mode LDA signal, the next few sections could be omitted, and only the effects of filtering and quantization would need to be considered. Unfortunately, in many applications, the statistics determined from the particle velocities do not correspond to the usual Eulerian flow averages. This was first noticed by McGlaughlin and Tiedeman [2], who observed that in turbulent flow there appeared to be more fast particles than slow, thus biasing the statistics if normal arithmetic averages of data were computed. Buchhave [3] also noticed that frequency shift, or the absence of it, seemed also to affect the

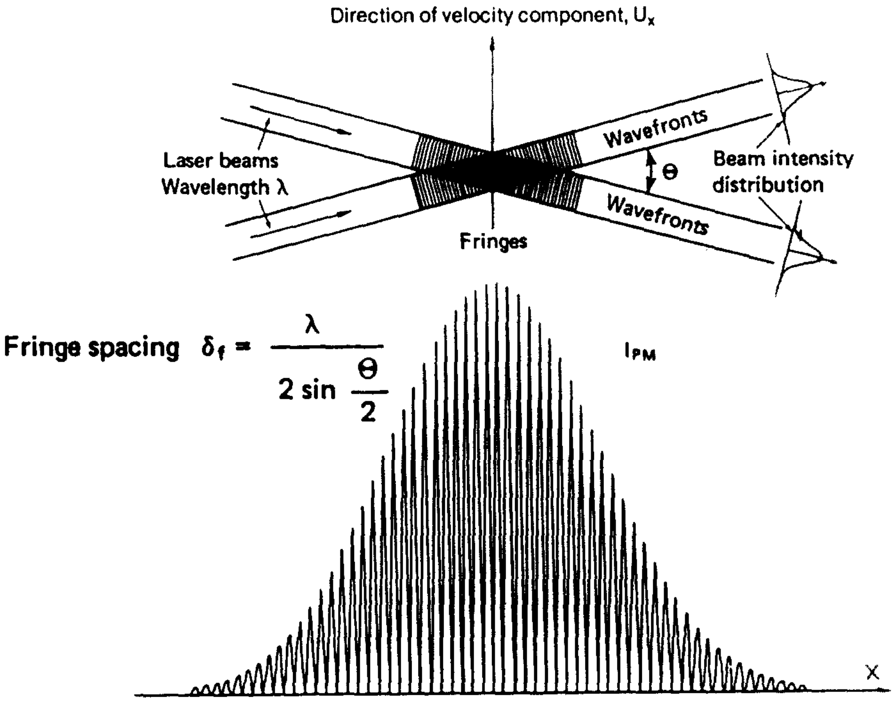

Figure 2. Typical Doppler signal from a single particle.

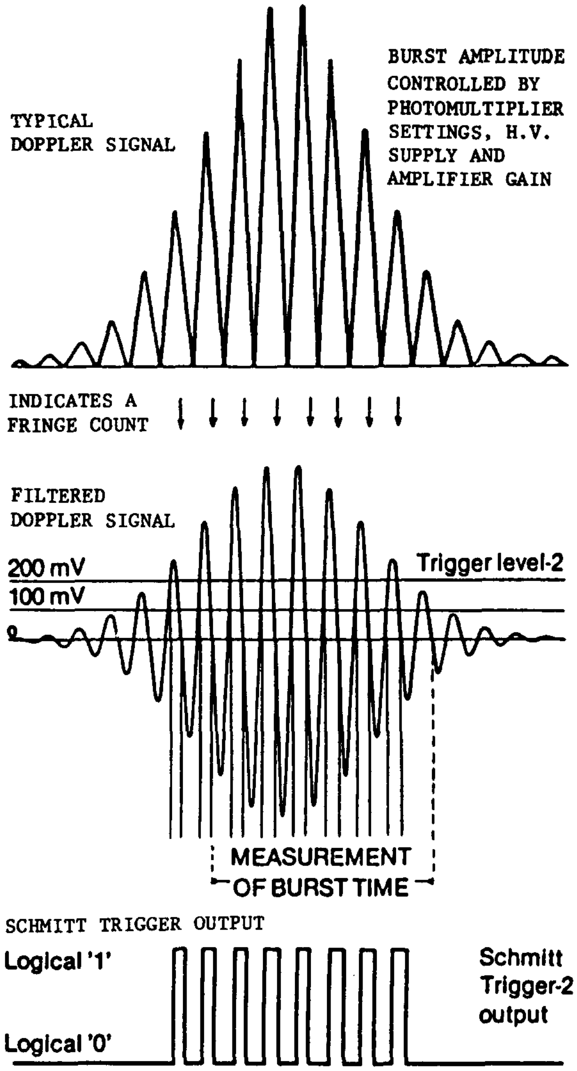

Figure 3. Operation of typical burst processor.

statistics in turbulent flows, and proposed the existence of the so-called dead angles, or directions from which measurements were ignored, thereby biasing the statistics.

The so-called bias problems referred to above were largely understood and resolved by 1975 (see papers by George [4] and Buchhave [3]). Review articles by Buchhave et al. [1] and George [5] discuss the signal characteristics in detail. Adrian [6] and Edwards [7] have also provided comprehensive and useful reviews. The results will be only briefly summarized here.

Signal Characteristics of the Burst-Mode LDA

There are two problems that are of primary importance if one is attempting to obtain physically meaningful data from a burst-mode LDA. First, rules for computing statistical quantities from the available data must be established so that results can be interpreted. For example, burst-mode signals can be processed so that the results correctly measure the statistics of the arriving particles, or they can be processed to provide information on the average characteristics of the flow in the measuring volume, the latter usually being of most interest. Second, if one is interested in the flow velocities at the measuring point, then the information provided by the LDA, which senses only scattering particles, must be related to the characteristics of the flow at the measuring point. This is a most difficult task since it must be assumed that the statistics of the particles (Lagrangian) and those of the flow at the point (Eulerian) are related. Only with the most stringent conditions on the particle distribution is a general relation possible.

The analysis of the signal produced by a burst-mode LDA begins with Eq. (1). It is possible to represent the Doppler current as a function of time by simply summing over all particles that pass through the beam [1]. The result is

$$i(t) = \iiint_{\text{all space}} i_{sp}(t, \boldsymbol{a}) g(\boldsymbol{a})\, d\boldsymbol{a}. \tag{7}$$

The function $g(\boldsymbol{a})$ is simply a function that accounts for the presence or absence of a scattering particle at initial location $\boldsymbol{a}$. Note that in spite of the fact that the integral is over all space, only those particles that pass through the measuring volume actually contribute to the integral because of the amplitude factor $I(\boldsymbol{x}_p) = I(\boldsymbol{x}_p[\boldsymbol{a}, t])$. When there is no particle in the beam, $i(t)$ is exactly zero.

If the burst processor could "track" the velocity while a particle was in the measuring volume and provide zero output the rest of the time, the resulting output would be a "velocity-like" signal, $u_0(t)$, given by

$$u_0(t) = \iiint w[\boldsymbol{x}(\boldsymbol{a}, t)] u_{sp}(\boldsymbol{a}, t) g(\boldsymbol{a})\, d\boldsymbol{a}. \tag{8}$$

The function $w(\boldsymbol{x})$, like $I_{sp}(\boldsymbol{x})$, is a position-dependent function that defines the measuring volume. Since our processor turns on only while the particle is in the measuring volume, $w(\boldsymbol{x})$ is given by

$$w(\boldsymbol{x}) = \begin{cases} 1, & \text{in measuring volume} \\ 0, & \text{outside} \end{cases}. \tag{9}$$

Figure 4 shows the velocity signal that would be generated by our hypothetical burst processor. While it might seem strange and artificial to treat what is intrinsically a digital signal (the measured Doppler frequency of the burst) as an analog signal (which is usually zero), *it is precisely this artifice that allows unbiased measurements to be taken.*

The question of how to average the data can now be addressed. If particle statistics (in particular, arrival statistics) are desired, then the Doppler frequencies measured from each burst can simply be averaged arithmetically, taking care not to count individual particles more than once. Usually, however, arrival statistics are not of interest but, instead, the time-averaged properties at the point of measurement. The simple arithmetic average will yield an incorrect result. The correct result is obtained by performing a timelike average of Eq. (8), just as though it were an analog signal [1, 4, 5]. The only difference from

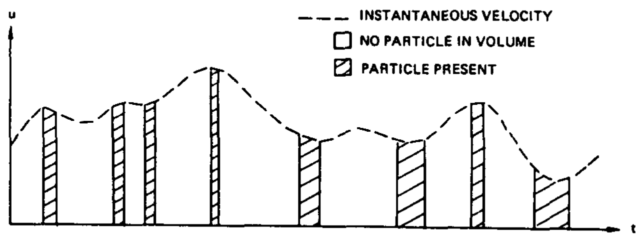

Figure 4. Hypothetical signal from burst processor.

the usual time average is that the results are counted only during those periods for which particles were in the meauring volume.

Thus, if Δt_i is the residence time of the ith particle and u_i is the velocity associated with it while it was in the measuring volume, then the mean and variance of the particle velocities are given by

$$\bar{u} = \frac{\sum_{i=1}^{N} u_i \, \Delta t_i}{\sum_{i=1}^{N} \Delta t_i} \tag{10}$$

and

$$\text{var}\ \{u\} = \frac{\sum_{i=1}^{N} (u_i - \bar{u})^2 \, \Delta t_i}{\sum_{i=1}^{N} \Delta t_i} . \tag{11}$$

Each of these results follows directly from the interpretation of the burst processor output as a time signal, nonzero only when a particle is in the measuring volume. The summation is over all particles that entered the beam and were detected in the time interval of interest.

The "pseudo" time signal $u_0(t)$ given by Eq. (8) can be used to compute other statistical quantities of interest. For example, the spectrum of the velocity can be computed by averaging the spectral estimators given by (George [5]),

$$S_T(f) = \frac{T}{(\sum_{i=1}^{N} \Delta t_i)^2} \, |\hat{u}_T(f)|^2 \tag{12}$$

where T is the total record length and where $\hat{u}_T(f)$ is the Fourier transform of the data computed as

$$\hat{u}_T(f) = \sum_{i=1}^{N} \exp\,(j2\pi f t_i) u_i \, \Delta t_i \tag{13}$$

where $j = \sqrt{-1}$, N is the number of samples arriving in time T, and t_i is the arrival time of the ith particle. Unlike equispaced time series analysis, there is no Nyquist criterion here [5].

The autocorrelation of the velocity signal can be computed by using the residence-time-weighted velocities at different time lags as in Buchhave [8]. A more computationally efficient approach is to compute the Fourier transform given by Eq. (13) for values of f = integer $\times$ $1/T$, then the spectrum from Eq. (12), and finally the inverse Fourier transform of the spectrum by an FFT algorithm to get the correlation. This was discussed in detail in Ref. 5.

Algorithms can similarly be generated for a variety of statistical quantities (e.g., probability density) by working from the analog signal analogy of Eq. (8). All such unbiased algorithms have a common feature: The velocity or indicator functions are weighted by the residence time. Even the presence of mean velocity gradients across the measuring volume does not introduce a bias (if the curvature of the velocity profile curvature is small enough) as long as the averages are residence-time-weighted [1].

A major reason why the residence-time-weighted averaging technique described above should be used is that *it is the only method that can, in general, produce an unbiased measurement of the flow properties at a point (or Eulerian averages)*, and then only if the scattering particles have been statistically uniformly seeded in space. This is because the arrival of the scattering particles at the measuring volume is, in general, strongly dependent on the flow field that brings them there. Thus the Doppler signal is not a simple time signal, but depends in a complicated way on the entire flow field and seeding distribution. Only by uniformly randomly distributing the scattering particles in space can the statistics of the particles be uncoupled from those of the flow field so that meaningful Eulerian statistics of the latter can be inferred.

It is tempting to try to analyze the LDA by assuming the characteristics of the particle arrival rate (e.g., Poisson-distributed, modulated Poisson, etc), and many have done so [9]. While these approaches may contribute useful insight into the nature of LDA signals, they cannot address the question of how to seed the flow. This is because the experimenter has no control over the arrival rate statistics, only where and when the scattering particles are introduced into the flow. The flow itself determines the rest.

It is interesting to ask what happens if there are interruptions in the data acquisition so that not all particles are included in the statistics. This can happen by accident because data are arriving faster than they can be processed, or by design (as in Capp [10]) so as to avoid overfilling the disk with statistically dependent data. Since Eqs. (10) and (11) are approximations to a time integral whose individual pieces can be separated in time, it does not matter whether the sampling process is continuous in time as long as the interruptions are due to external effects (like the computer) and not the flow. The residence time weighting must still be applied as long as it is the flow (by the particle arrival) that determines when signal acquisition is initiated and how long it characterizes events occurring in the measuring volume.

It is, of course, possible to imagine ways of interrogating the counter so that the experimenter is in control and not the flow. This might involve accepting a data point only if it arrives in some very small interval (relative to the smallest time scale of the flow). If no data point is accepted the counter is disabled until the next window of opportunity is enabled. Obviously, there will be many no-shows unless the particle arrival rate is very high. In such cases the residence time weighting is unnecessary and the data can simply be averaged arithmetically. Note that even this approach does not alleviate the seeding distribution problem.

Approximations to the Residence Time Weighting

A number of approximate corrections have been proposed to try to correct for the biased statistics that can result if the data are not weighted by the residence time as indicated above. It will be shown below that, at best, these so-called bias corrections are valid only in flows of moderate to low turbulence intensity. This is easily seen by examining the behavior of the residence time for a spherical measuring volume for which particles pass only through the center. For such a hypothetical case, the residence time is given by

$$\Delta t \cong d/|\boldsymbol{u}| \tag{14}$$

where d is the diameter of the measuring volume and $|\boldsymbol{u}|$ is the magnitude of the velocity vector. This "residence time" can now be used to compute the velocity mean by substituting it into Eq. (10) with the result that

$$\bar{u} \cong \frac{\sum_{i=1}^{N} u_i |\boldsymbol{u}_i|^{-1}}{\sum_{i=1}^{N} |\boldsymbol{u}_i|^{-1}} . \tag{15}$$

This is the three-dimensional correction suggested by McGlaughlin and Tiederman [2]. Note that it not only requires direct measurement of the velocity *vector* but also depends on some rather unrealistic assumptions about the scattering volume.

A simplified alternative to Eq. (15) can be obtained by restricting the turbulence intensity so that

$$\begin{aligned} |\boldsymbol{u}| &= [(U+u)^2 + v^2 + w^2]^{1/2} \\ &= \bar{u} \left[1 + 2\frac{u}{U} + \frac{u^2+v^2+w^2}{U^2} \right]^{1/2} \cong \bar{u} + u \end{aligned} \tag{16}$$

using the binomial expansion. Thus

$$\Delta t \simeq d/|U+u| \tag{17}$$

and a low-intensity version of Eq. (15) is

$$\bar{u} \simeq \frac{\Sigma_{i=1}^{N} u_i |u_i|^{-1}}{\Sigma_{i=1}^{N} |u_i|^{-1}} . \tag{18}$$

This is the McGlaughlin-Tiederman [2] one-dimensional correction. It has the advantage that it requires measurement of only a single velocity component. Disadvantages are that it is restricted to low turbulence intensities [say, less than 20% to ensure the validity of Eq. (16)] and it also depends on scattering volume assumptions that are unrealizable.

It should be obvious that if the turbulence intensity is sufficiently low, then all the particle residence times are nearly equal. Thus, the statistical computations can just as easily be performed by ignoring them altogether and using simple arithmetic averaging, that is,

$$\bar{u} = \frac{1}{N} \Sigma_{i=1}^{N} u_i . \tag{19}$$

Because of errors that arise in the practical determination of the residence time, this approach should always be used when the turbulence intensity is less than 5%.

In summary, there are a number of approximations that can be used to simplify the implementation of the residence time weighting or avoid it altogether. All can result in errors because of the assumptions inherent in them regarding the scattering volume. It is my opinion that a straightforward implementation of the proper residence time weighting is preferable to all of them if the turbulence intensity is sufficient to merit any of them. The measurements of Buchhave [8] and Capp [10] provide strong support for this point of view.

The Convergence of Statistics

The problem of the statistical reliability of burst-mode-averaged quantities has been reviewed in detail by George [5], and the results will only be summarized here. The question of convergence can most easily be addressed by looking at the variability of the estimator for the averaged quantity of interest. If the true mean is denoted by $\bar{u}$ and the mean estimated from N samples as u_N, then the variability of this estimator, ϵ, is defined to be

$$\epsilon^2 = \overline{\{u_N - \bar{u}\}^2}/\bar{u}^2 . \tag{20}$$

Clearly it is to be hoped that $\epsilon \to 0$ as $N \to \infty$.

If each realization (or velocity associated with a single burst) can be assumed to be statistically independent of all others, then

$$\epsilon^2 = \frac{1}{N}\left(\frac{\sigma_u}{\bar{u}}\right)^2, \quad \text{statistically independent samples} \tag{21}$$

where N is the number of samples and σ_u and $\bar{u}$ are the true rms and mean values, respectively. Thus the variability increases with the variability of the velocity itself $(\sigma_u/\bar{u})$ and inversely with the square root of the number of independent samples.

In the more general case where successive realizations may retain correlation (since the flow itself may be correlated over finite regions of time or space), the counterpart to Eq. (21) is

$$\epsilon^2 = \frac{1}{N}\{1 + 2\nu_0 T_u\}\left(\frac{\sigma_u}{\bar{u}}\right)^2 . \tag{22}$$

N is the number of realizations, which may not be independent, ν_0 is the mean data rate, and T_u is the Eulerian integral time scale of the flow velocity. Clearly if the data rate is greater then some low multiple of the inverse of the flow integral scale, the extra data points do not help to improve the accuracy of the average.

An alternative form of Eq. (22) expresses the variability directly in terms of the record length in time T, instead of the number of samples;

$$\epsilon^2 = \frac{2T_u}{T}\left(1 + \frac{1}{2T_u\nu_0}\right)\left(\frac{\sigma_u}{\bar{u}}\right)^2 . \tag{23}$$

The expected number of samples is $N = \nu_0 T$. This relation makes clear that it is really the *record length relative to the integral time scale of the flow* that is important once a minimum data rate has been achieved. Relations similar to Eq. (23) can be derived for any statistical quantity of interest [5].

It is the practice of the Turbulence Research Laboratory at the University at Buffalo to sample for fixed record lengths *in time* as opposed to a fixed number of samples. This is because if the seeding density varies widely in an experiment, as sometimes happens, the variability can be very hard to predict unless the record length is known precisely.

The consequences of Eqs. (22) and (23) to the experimentalist are very important. If only mean quantities are to be computed, then care should be taken to ensure that the data rate is sufficiently *slow* to avoid processing (and storing) the extra data to no useful purpose. In many situations this involves reducing the data rate to very low levels. (For example, in Capp's experiment [10] the optimal data rate was 2/s.) This is sometimes best accomplished by interrupting the data acquisition by disabling the counter for fixed periods between samples. Note that as pointed out above, residence time weighting is still necessary if when the counter is enabled it is the flow that determines when the data actually arrive.

Reconstruction of Time Records

In a very few applications it is necessary to reconstruct actual histories of the velocity from the burst-mode LDA. Unlike the kinds of statistical measures discussed above where low data rates were preferable, in such cases the data rate must be, on the average, faster than twice the Nyquist frequency, the highest frequency of the signal (usually considered to be the mean velocity divided by the Kolmogorov microscale). This is the exact counterpart of the digital analysis of equispaced samples in time (see Tan-atichat and George [11]).

It has been argued that such an approach does not require statistically uniform seeding in space. It is easy to see that without additional assumptions this cannot be true, since there could always exist regions of the flow from which particles never arrive, yet the flow of fluid from those areas could contribute substantially to the statistics. Thus the measurements are conditioned by the method of seeding. (Similar considerations apply to those techniques that depend on arrival time statistics.)

If the data are to be turned into an analog signal by a standard digitial-to-analog converter with a low-pass filter (as in most counter analog modules), then the data rate must be many times the Nyquist frequency of the signal (a factor of 10 is a good number) to adequately reconstruct it. Few experiments can satisfy this criterion, and as a consequence the results can be seriously in error if the data are processed as a analog signal.

There arise circumstances in practical applications where the data rate may not be high enough relative to the Nyquist frequency of the fluctuating velocity to justify treating the signal as an analog signal but are still high enough relative to the integral scale to obtain reasonable estimates of the lower order moments. Whether or not an analog signal is actually reconstructed or simply treated as a randomly sampled time series, the effective signal is a low-passed version of the true velocity, the cutoff frequency corresponding to the data rate. All such techniques must be applied with a full understanding of the approximate nature of the analysis and used with caution or, better, avoided altogether. Edwards [7] suggests that for the measurement of second-order moments the data rate should at least exceed the inverse of the Taylor microscale of the velocity, a value usually well below the Nyquist frequency.

It should be noted that is is not necessary to satisfy the Nyquist criterion for spectral analysis if the algorithms proposed in Eqs. (12) and (13) are utilized. It is a curious fact that randomly sampled data do not have the aliasing problem of equispaced data [5]. On the other hand, the convergence of spectral estimators can be very slow indeed if the data rate is too low [5, 8].

The Role of Frequency Shift in Burst-Mode Applications

Frequency shift of one or both of the incident laser beams is an essential part of any LDA burst-mode application in high-intensity turbulent flow or in any other application where the flow direction cannot be guaranteed. The reason for this is that burst-mode processors (with noise discrimination) depend on a minimum number of cycles (or fringe crossings) in order to register a measurement. A particle entering the measuring volume perpendicular to the fringes and near the center will always cross the requisite number of fringes (typically eight). On the other hand, a particle entering near the extremities of this measuring volume or entering at an angle may not cross the required number of fringes. If this is indeed the case, then that particle will not have been seen by the counter. The problem is illustrated in Fig. 5. This "dead-angle" phenomenon was analyzed in detail by Buchhave [3], and the results will only be summarized here.

The consequences of the dead-angle phenomenon are that the statistical measures of the velocity are conditioned upon the flow direction, the larger angular deviations being ignored. In a high-turbulence-intensity flow or one with reversals, the effects can be dramatic and the averages greatly in error [8]. Since the scattering volume seen by a small particle is smaller than that seen by larger particles because the signal it generates is of lower amplitude, the smaller particles will be more susceptible to this angular bias.

The solution to this problem is to move the fringes so that even if the particle enters at a high angle from the preferred direction, sufficient fringes *cross the particle* that it can always be seen. Clearly, the higher the turbulence intensity, the more probable are large angular deviations from the streamwise direction and the larger the amount of shift that will be required to ensure that no particle escapes without being measured. Figure 6 is taken from Ref. 8 and illustrates what happens to the dead angle as the shift is increased relative to the fluctuating flow. At no shift, the dead angles are seen to be at right angles to the flow direction, but as the amount of shift (as measured by the fringe velocity U_s relative to the flow speed U) is increased, the dead angles move progressively toward the downstream direction and eventually disappear altogether. This point of disappearance represents the minimal frequency shift required in a given flow application.

Rather than go through an analysis of precisely how much shift is required, there is a real temptation to simply use so much shift that there is never a problem. The consequences of this can also be quite detrimental and severe. First, as is shown later, the quantization errors in the determination of the Doppler frequency are increased as the amount of shift is increased. Second, the slower the scattering particle, the more fringes that will cross it and the greater the number that must be registered in the fringe count register. (It will be shown later that this fringe count register is often used to determine the residence time needed to obtain unbiased statistics.) If the fringe count register overflows when residence time weighting is being attempted, then the measurement will again be seriously in error.

How much shift, then, is the right amount? Clearly there must be enough shift to ensure that in a given flow situation no particle escapes without being seen. On the other hand, there must not be so much that a slow-moving particle overflows the fringe count register or that the quantization errors are unacceptable. In each and every situation the experimenter must make an intelligent choice to ensure that his or her measurement is between these limits. A useful tool in evaluating whether the correct choice has been made is the fringe count histogram as used by Capp [10] and shown later in this paper.

HARDWARE CONSIDERATIONS

The Effect of Filtering on LDA Signals

In all but the most unusual circumstances it is necessary to bandpass filter the photomultiplier signal before further processing is attempted. The purpose is to remove the low-frequency pedestal and reduce the broad-band electronic noise that can contribute stray zero-crossings. The criteria for pedestal removal and signal-to-noise optimization have been discussed elsewhere [12]. The objective here is to examine the unwanted consequences of this filtering on the data and on averages computed from them.

The frequency content of the incoming Doppler burst signal (minus noise) is entirely determined by the velocity of the fluid and the frequency shift (which is presumed steady). The finite length of the burst implies that a single velocity gives rise to a band of frequencies. The bandwidth of the burst is exactly the same as that which would airse from transit-time broadening in the continuous-mode case [12]. In the subsequent discussion the bandwidth due to this finite residence time is assumed to be

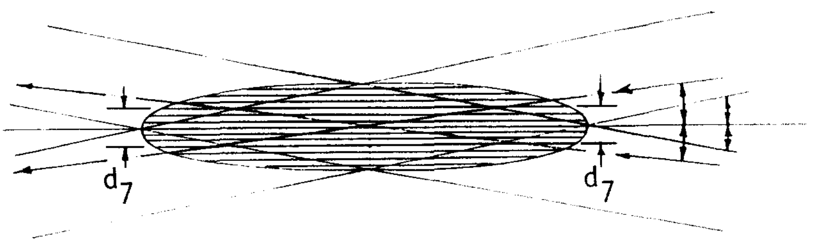

Figure 5. Illustration of existence of particle trajectories not seen by counter.

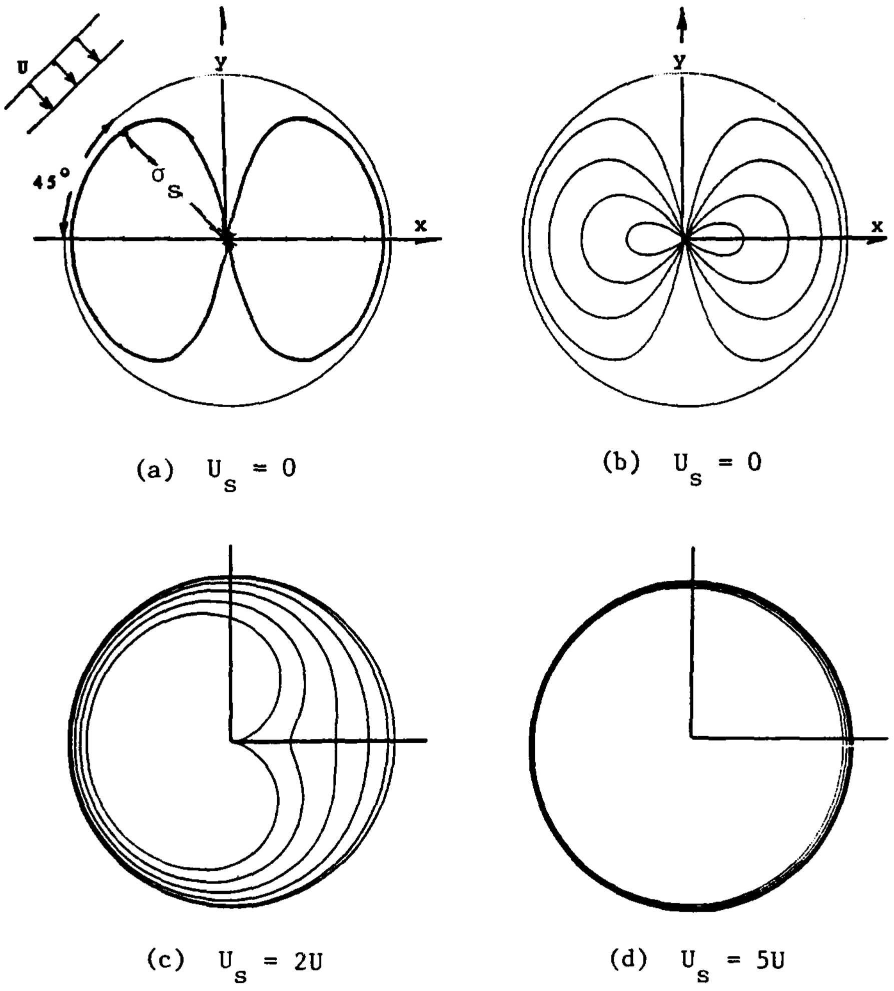

Figure 6. Measuring volume as a function of shift for fixed flow parameters showing dead angle. (From Ref. 8)

negligible compared to the overall fluctuations of the signal arising from unsteady fluid velocities. Note, however, that it is intrinsic to the Doppler burst and can adversely affect the counting process.

The bandpass filtering is often carried out by passing the Doppler current signal through high-pass and low-pass filters, the exact cutoff frequencies of which are adjustable. It should be clear that when the frequency of a burst lies outside the band of the high- and low-pass filters, it will be ignored by the counter processor. If these high- and low-pass cutoff frequencies are chosen entirely to improve the signal-to-noise ratio without reference to the statistics of the velocity itself, serious errors can be introduced.

Figure 7 illustrates how the statistics of the filter and signal can be biased by the filter. Figure 7*a* shows the probability density function (pdf) of frequencies that might arise in a typical turbulent flow. (It can be shown that this pdf is, in fact, the spectrum of the Doppler current [4].) Figure 7*b* shows the gain characteristics of the combined high- and low-pass filters, while Fig. 7*c* shows the probability density of the filtered Doppler current.

If the filtered current is now processed by an appropriate counting technique and sufficient samples are averaged (with the appropriate weighting factors), the averaged Doppler frequency will be identical to that given by the probability average

$$\langle f_D \rangle_{BP} = \int_{f_L}^{f_H} f p_{BP}(f)\, df \tag{24}$$

where $p_{BP}(f)$ is the pdf of the bandpass-filtered frequencies. The correct averaged Doppler frequency is, however,

$$\langle f_D \rangle = \int_{-\infty}^{\infty} f p(f)\, df \tag{25}$$

where $p(f)$ is *the pdf of the unfiltered frequencies* (the original pdf of the flow).

Clearly, because the parts of the integral from $-\infty$ to f_L and from f_H to $+\infty$ are missing, the computed average will be in error. Similar considerations apply to the higher moments, which must be asymptotic to

$$\langle f_D^n \rangle_{BP} = \int_{f_L}^{f_H} f^n p_{BP}(f)\, df \tag{26}$$

instead of the correct value given by

$$\langle f_D^n \rangle = \int_{-\infty}^{\infty} f^n p(f)\, df. \tag{27}$$

That the consequences of filtering can be quite severe is illustrated in Fig. 8, which shows several examples where statistics would be greatly in error. The phenomenon is exactly analogous to the clipping of an analog signal by the range limits of an analog-to-digital converter [10].

It is possible to use the above to provide rough criteria for the design and evaluation of experiments. If the pdf of the frequencies

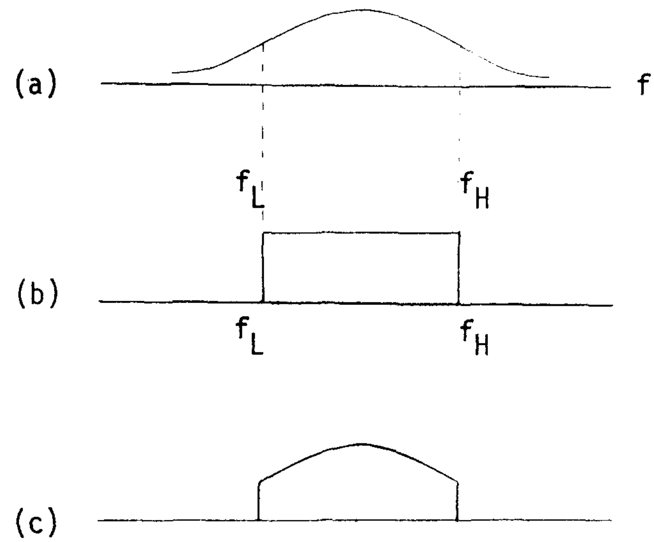

Figure 7. The effect of bandpass filtering on the probability density function of the Doppler frequency.

(and the velocity itself) is not greatly different from Gaussian and the mean value is near the center of the filter, then for measurements of the mean a minimal requirement is that

$$\frac{f_H - f_L}{2\langle f\rangle} > \frac{3u'}{U} \tag{28}$$

while for the mean square (or rms), the minimal bandwidth requirement is increased to

$$\frac{f_H - f_L}{2\langle f\rangle} > \frac{6u'}{U} \tag{29}$$

The higher the moment, the more severe these constraints will become, since the tails of the distribution are increasingly heavily weighted. Thus careful attention must be paid to what is being discarded by filtering if meaningful measurements are to be taken.

An independent consideration related to filtering, but a practical matter of great concern, is the ease with which nonsense data can be taken with the burst-mode LDA. Even when the signal quality is not adequate, it is still possible to obtain high validation rates from noise and be tricked into believing that reasonable measurements have been obtained. This is because the signal has been narrow-band-filtered so that even noise has a burstlike character. (In fact, the easiest way to make a synthetic Doppler signal is to narrow-band filter a random noise.)

There are two ways to test whether the measurement is, in fact, valid. First, *the measurement should be independent of both the high-pass and low-pass filter settings.* If it is not, then noise measurement instead of flow is probable. Second, *the measurement should be unaffected by small amounts of frequency shift other than by the precise amount of the shift change.* This can easily be checked by simply changing the amount of shift. Both of these criteria should be applied *before* data are taken to ensure that, in fact, the burst processor is measuring particles and not filtered noise.

Quantization Errors Arising from the Counting Process

Quantization errors have long been recognized as a source of noise in the analysis of analog signals. (Otnes and Enochson [14]

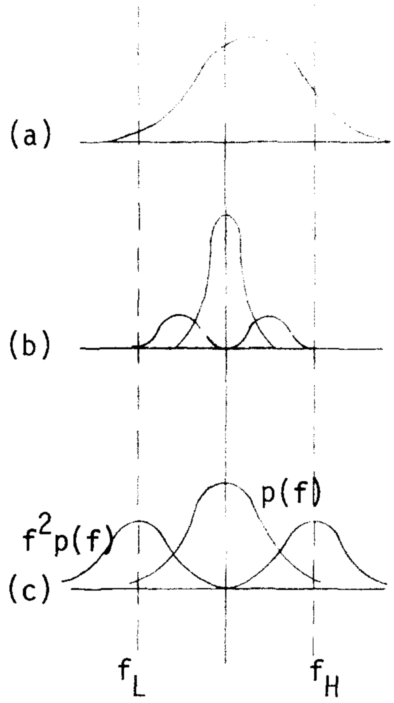

Figure 8. Bandpass filtering. (*a*) Mean and all moments will be in error; (*b*) proper filtering for mean and rms; (*c*) rms will be in error.

provide a general background on the problem as applied to analog signals, while the discussion of Tan-atichat and George [11] is more specifically directed to the problem of turbulence signals.) Unlike the direct analogy between the filtering of LDA signals and the clipping of analog signals, there is no direct analogy between the quantization of analog signals and those of laser Doppler signals. (This is because of the manner in which digitization is implemented by LDA counters.)

In the standard analog-to-digital conversion of analog signals, the signal is simply placed into the appropriate bin, one bin representing a subdivision of the total range of the signal and uniquely designated by an integer assignment. LDA counters, on the other hand, have a number of features that preclude a simple analogy. Specifically, the LDAs tend to be autoranging and make use of a mantissa-exponent representation of the quantized signal. The analysis presented below is summarized from Capp [10].

Figure 3 presents a schematic of the operation of a typical LDA counter. When the Doppler burst exceeds a predetermined level, a counting process is initiated in which the number of ticks of a crystal-controlled oscillator is counted. This process is terminated when the Doppler burst signal falls below the predetermined level or when a certain number of cycles have been achieved, typically eight. (On the Dantec 55L90a counter, for example, mode 3 corresponds to the former method of analysis, while mode 2 corresponds to the latter.) If it is presumed that the accumulators are switched off and on instantaneously, then the primary source of quantization error arises from the rate at which the clock is functioning, since the uncertainty in the measurement is between zero and the time between individual ticks. If the clock rate is denoted as f_c and the Doppler frequency plus frequency shift is denoted as f, then the rms error in the determination of the time for a fixed number of cycles can be shown to be given by $\Delta t = (3f_c)^{-1}$. The exact time for M cycles of the burst is Mf^{-1}

Therefore the relative rms error in the determination of the inverse of the Doppler frequency can be shown to be

$$\epsilon = f/\sqrt{3} f_c M. \tag{30}$$

Thus the errors arising from this source depend linearly on the Doppler frequency (plus shift) and inversely on the product of the clock rate and the number of cycles counted. For example, if the Doppler frequency (plus shift) is 1 MHz, the clock rate is 500 MHz, and the number of cycles required is 8, then $\epsilon \approx 1.4 \times 10^{-4}$, or 0.014%. For all but the very highest Doppler frequencies, clock errors are a negligible source of measurement error.

A second source of quantization error arises from the manner in which the frequency determined above is delivered to the user. Commercial counters deliver the measured frequency (or its inverse, the so-called burst time) as a digital word consisting of a mantissa and an exponent. The mantissa may have from 8 to 12 bits, while the exponent has 4 bits. In addition, the first bit is usually presumed to be 1. It is straightforward to show that the effective numbers of bits is equal to the number of bits in the mantissa plus one. The actual frequency resolution must be determined by considering the full range at which the counter is set. For example, if the counter has a mantissa of m bits and is set at a maximum frequency range of f_m, then the resolution of the device is $\Delta f = \pm \frac{1}{2} f_m/2^{m+1}$. This corresponds to an rms relative error of

$$\epsilon = f_m/2\sqrt{3}\,\sqrt{2^{m+1}} f. \tag{31}$$

Clearly the resolution errors are at a maximum when the Doppler frequency is at the lower extremities of the range. For example, if the counter is set on the 4 MHz range and has a stated resolution of 10 bits and the Doppler frequency (plus shift) is 3 MHz, then $\epsilon \simeq 8.5 \times 10^{-3}$, or 0.85%. As with the clock errors, this is seldom the most important source of uncertainty.

In the actual determination of the frequency of the bursts (or its inverse), most counters use a comparison technique to reduce the possibility of sampling a noise burst. Usually the time for eight (or in some cases, 16) cycles is compared to that determined independently for only a fraction of these (typically four or five). If the comparisons do not fall within selected limits, the measurement is discarded. The tightest limit is typically a few percent; thus this soure of error can always be greater then either of the quantization sources discussed above. Therefore, unless the burst is of extremely high quality (high signal-to-noise), it is this tolerance that establishes the minimum turbulence level in most applications.

All the mechanisms for error discussed above provide only the error in the measurement of the frequency of the burst. *If the error in the Doppler frequency is desired, then the frequency shift must first be subtracted* from the frequency so that the relative error in the measurement of the flow velocity or Doppler frequency (or rather, its inverse) is given by

$$\epsilon_D = \epsilon \frac{f_s + f_D}{f_D}\,. \tag{32}$$

Clearly, the insignificant errors above can become significant if care is not taken to avoid excessive amounts of frequency shift. Thus the experimenter operates within bounds defined by the conflicting requirements of minimizing quantization errors and avoiding dead angles, as pointed out earlier.

Implementation of the Residence Time Weighting

The residence time of a particle in the LDA measuring volume, which is essential to the accurate determination of statistics in high-intensity turbulent flows, must be determined independently of the Doppler frequency itself. Commercial counters determine the residence time by counting the number of peaks in the Doppler burst as shown in Fig. 2. Thus the residence time is given by

$$\Delta t_i = N_i/(f_i) \tag{33}$$

where N_i is the number of fringes for the ith realization. The accuracy of this determination is determined in large part by the quality of the Doppler burst itself. A significant source of measurement error can arise from this determination of the fringe count if noise is present.

It should be noted that even in a steady laminar flow the fringe count is never a single number but always a distribution. This is because particles enter and leave the measuring volume at different locations and, in effect, trace out the shape of the scattering volume, the number of fringe crossings being proportional to the distance traversed across the beam in the flow-stream direction. Also, the particles are usually polydisperse, and therefore even particles entering and leaving at the same location are "seen" differently by the LDA counter. Fortunately, this distribution of scattering volume and particle sizes does not affect the validity of the residence time theory (see Adrian [6]).

Capp [10] has used Buchhave's analysis of the measurement volume [3] to determine the fringe count distribution that would be achieved in a uniform flow with a Gaussian scattering volume distribution. The results are shown in Fig. 9*a*. Also shown are Capp's measured results using smoke particles in a laminar uniform flow facility with a single-channel LDA. Clearly evident is the effect of the polydispersity of the scattering particles. In effect, the size of the measuring volume is varying with the size of the scattering particles, since the larger particles can be detected in regions of weaker measuring volume intensity. Thus Capp's measurements represent a convolution of the fringe count distribution for a single particle (or volume) size with the probability distribution of the particles themselves. Figure 9*b* shows the same fringe count distribution measured in a turbulent jet. The distribution is substantially broader now, because the fringe count is a function of not only where the particle passes through the volume and the particle size distribution, but also the velocity direction.

Since the residence time that will be used to obtain unbiased statistics is to be determined from the fringe count and is linearly proportional to it, any uncertainty in the measurement of the fringe count will show up as errors in the statistics themselves. But the effective residence time can be determined only to within *minus* a fringe crossing, and therefore the relative error is inversely proportional to the square root of the number of fringes in the burst itself. (For example, with the Dantec 55L90a counter this number can be as small as 8 if validation is used, or as high as 255. Special versions of this counter exist in which the fringe count register has 10 bits or 1023 fringe counts possible.) Note also that some counters do not reset when the fringe count register is exceeded but simply start the register from zero and begin accumulating again, resulting in fringe count readings that can be substantially in error.

An estimate of the rms error in the statistics arising from errors in the determination of the residence time is thus inversely proportional to $\sqrt{2^p - 1}$, where p is the number of bits in the fringe count register. The exact constant of proportionality will depend

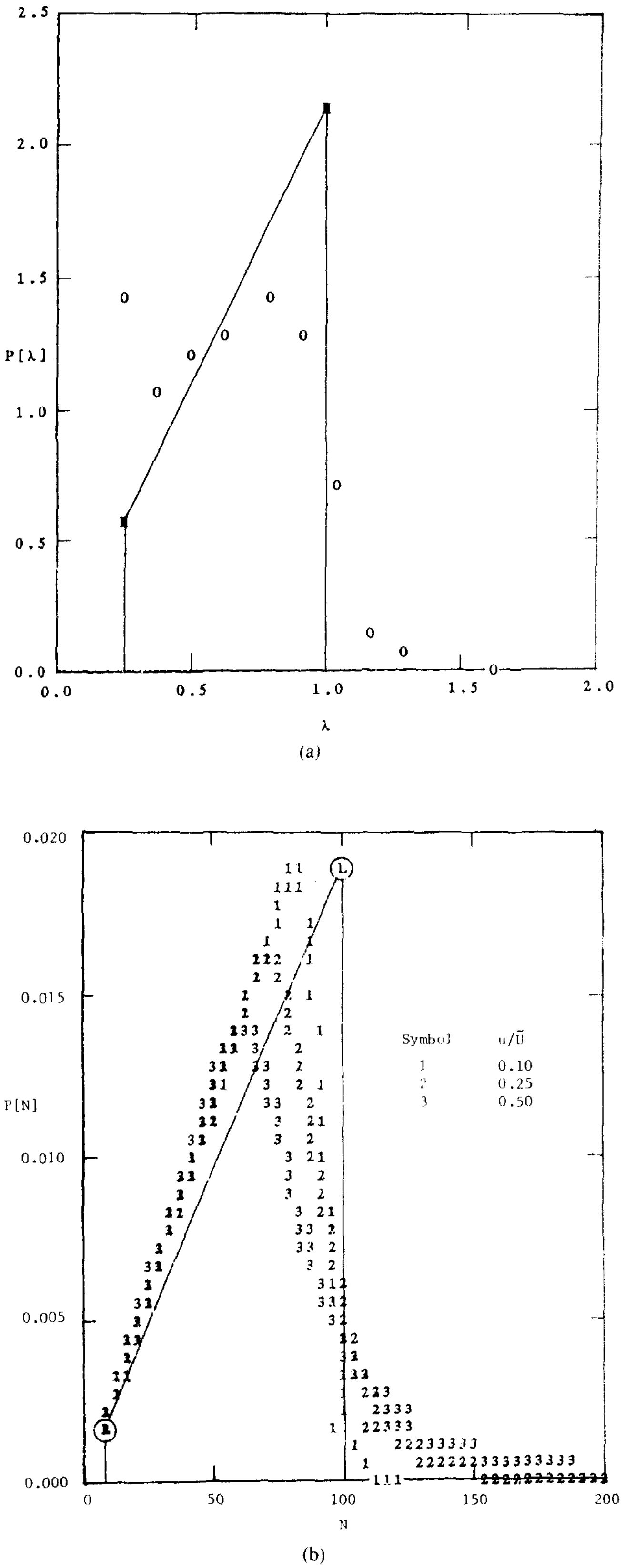

Figure 9. (*a*) Histogram of fringe count in laminar flow ($\lambda = N/N_m$, $N_m = 32$, no shift). (*b*) Fringe count histogram variation with higher turbulence intensity levels. (Mean velocity is 2 m/s for all cases. Frequency shift is 400 Hz.)

on the actual fringe count distribution. If $p = 8$, then the rms error is greater than 6%. Clearly, in most applications this is the dominant source of statistical error from the LDA. If a 10-bit fringe count register is used, this is reduced by a factor of 4 but can still be the most significant source of error in measurement of high-intensity flows. Fortunately, unless the turbulence itself is of an intensity greater than 10% (as pointed out earlier) it is not necessary to use residence time weighting of the data.

The data obtained by Capp in turbulent flow (Fig. 9*b*) indicate that because of the distribution of the fringe counts, substantial overflow of the fringe count register can occur unless care is taken to ensure that the mean fringe count is not too high. Note that this mean fringe count should also be substantially greater than the fringe count necessary to get a reading at all, or the statistics will be biased at the low end by the absence of data with low fringe counts.

Improvements to the Burst Processor

As is clear from the above, the hardware limitations of the burst-mode LDA are largely inherent in the counter principle itself. Faster clocks, more bits, and better filters will do little to improve the fundamental limits on frequency and fringe count determination.

Buchhave [15] has pointed out that the limitations on counter performance are in large part a consequence of the fact that all of the noise that is passed by the filters can affect the counting process even if the frequency content of the Doppler burst is only a small fraction of that band. It is easy to show that a substantial increase in the signal-to-noise ratio can be achieved if only the noise corresponding to the frequency content of the burst is passed.

This observation suggests that the burst should be analyzed, not in the time domain as is presently done, but in the frequency domain. If the time history of a burst could be Fourier transformed, it would be easy to obtain the frequency of interest (Doppler plus shift) from the first moment of the burst spectrum. The residence time can be shown to be inversely proportional to the square root of the centered second moment. Such a scheme would appear to substantially reduce the errors associated with both the frequency and residence time measurements.

It is relatively straightforward to implement this Fourier burst analyzer in software using bursts captured by a transient recorder. Such a "processor" is, of course, very slow. A hardware implementation of these two algorithms could prove very challenging but might certainly be worthwhile for many applications.

The Importance of the Scattering Particle Selection

There are a number of criteria that can be used to select scattering particles in a given flow situation: optical properties, ease of implementation, and flow characteristics (Adrian [6] and Durst et al. [12] have reviewed these in detail). Most often the choice is made on the basis of what is available, since one seldom has a surplus of good choices. The most important criterion, however, is the last one: the flow characteristics of the particles.

Unless there is specific interest in the velocity of the scattering particles in their own right, it is generally required that *the particles must follow the flow*, since it is the fluid velocities that are of interest. Unfortunately, there are few applications where this can be guaranteed over the entire range of mean and fluctuating quantities of interest. As a consequence, *many measurements are, in fact, only measurements of the response* *of the scattering particles to the environment in which they find themselves.* Such measurements contribute little or no information about the flow itself. Thus even though high-quality Doppler signals may have been obtainable, and even though proper analysis of these signals is carried out, the measurements are themselves worthless since they are not of fluid dynamic quantities of interest.

Whether or not the particles are tracking the fluid velocities of interest can be determined by examining the particle response time. The particle response time is a measure of how rapidly the particle can adjust to changes in the velocity field of the fluid. Since scattering particles tend to have densities that are different from those of the fluid, the inertia of the particle precludes an immediate adjustment. In simplistic terms, the viscous forces acting on the particle due to the flow over it act to accelerate (or decelerate) the particle until the flow and particle have the same velocity. A simplified equation for the response of the particle to a flow of different velocity around it is given by the Stokes approximation, which results in a first-order differential equation [1].

$$\tau_1 \frac{du_{sp}}{dt} = u - u_{sp}. \tag{34}$$

The time constant τ_1 in this equation is defined by the viscous and inertial parameters as

$$\tau_1 \frac{d_p^2}{36\nu}\left(2\frac{\rho_p}{\rho_f}+1\right) \tag{35}$$

where d_p is the particle diameter, ν is the kinematic viscosity of the fluid, and ρ_p and ρ_f are the particle and fluid densities, respectively. For example, glycerin smoke particles of 1 μm diameter in air have a time constant $\tau_1 \simeq 3.7 \times 10^{-4}$ s. Aerosol particles would have time constants substantially larger than this. Adrian [6] includes data for a variety of common particles.

The particles can be assumed to follow the flow only when the smallest flow time scale is greater than the particle response time [Eq. (35)]. Thus in order to properly estimate whether the particles follow the flow, the appropriate flow time scale itself must be estimated. In the case of a simple turbulent flow, the time scale is simply the inverse of the highest frequency fluctuation of interest. Since the effect of the particle lag is to low-pass filter the velocity fluctuations, the mean square fluctuations will be reduced by the particle lag effect if the particle response time is not less than the Taylor microscale of the flow. Therefore care must be taken to ensure that the energy removed by this filtering is negligible. Capp [10] and Nee [16] provide examples of how such estimates should be made.

In situations involving curving flows, the centripetal accelerations of the particle can be significant. In such situations the time scale of the flow defined by the inverse rotation rate must be greater than the particle time scale. This ensures that centrifugal forces on the particle can be balanced quickly by the viscous drag around the particle as it begins to slip across streamlines. A sure sign of such effects in problems with rotating machinery is the absence or scarcity of scattering particles near the hub. *Whenever a region of flow is identified in which it seems to be difficult to inject a sufficient number of scattering particles, particle lag effects should be suspected in the measurements,* and serious consideration should be given as to whether the measurements are meaningful at all.

A final point to be considered regarding the scattering particles is how they are placed in the flow. Buchhave et al. [1] point out that *unless the flow is uniformly seeded everywhere, a conditional measurement is being made.* In other words, only the fluid from regions that have been seeded will contribute to the measurements, and the rest will be ignored. Unless all fluid entering the measurement volume can be assured an equal chance of having been seeded, the measurements will not be representative of the velocity at that point. Sometimes this is desirable—for example, if one only wishes to measure inside an intermittent turbulent jet interface. Most of the time it is not, and unless the flow is seeded everywhere the measurements will be meaningless or at best difficult to interpret. This places a heavy responsibility on the experimenter who cannot seed the "world" to prove that his or her measurements have not been biased by the manner in which the flow has been seeded.

SUMMARY

It has been suggested that the primary problems in using the burst-mode LDA lie not in the hardware but in the manner in which it is used. It has also been suggested that there are circumstances in which it should not be used at all (for example, if particles cannot be found that track the flow). It should be obvious that if the trouble has been taken to overcome the experimental difficulties so that meaningful measurements can be taken, then at the very least care should be taken to process the data correctly. If the procedures outlined here are followed, the burst-mode LDA can make a substantial contribution to our understanding of complex flows.

Both Drs. P. Buchhave and S. P. Capp provided critical analyses of this manuscript. This and their collaboration over the past decade in the development of these ideas are gratefully acknowledged. The efforts of Mrs. E. Graber in typing the manuscript and Mr. S. Woodward in preparing the figures are also appreciated.

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NOMENCLATURE

a	component of $\boldsymbol{a}$ aligned with $\boldsymbol{K}$, m
$\boldsymbol{a}$	initial position of scattering particle at arbitrary origin in time, m
d	diameter of hypothetical (spherical) measuring volume, m
d_p	particle diameter, m
f	Doppler frequency plus shift, Hz
f_L	low-frequency filter cutoff, Hz
f_c	clock frequency, Hz
f_D	Doppler frequency (circular), Eq. (6b), Hz
f_H	high-frequency filter cutoff, Hz
f_m	upper limit of counter range, Hz
f_s	shift frequency, Hz
$g(\boldsymbol{a})$	random function accounting for presence or absence of a scattering particle at position $\boldsymbol{a}$, 1/m^3
i_{sp}	current generated by a single scattering particle, H
I	amplitude of Doppler current, H
$\boldsymbol{k}_1$, $\boldsymbol{k}_2$	wavenumbers of incident laser beams (Fig. 1), 1/m

K	magnitude of $\boldsymbol{K}$, 1/m
$\boldsymbol{K}$	scattering wavenumber, defined by Eq. (3), 1/m
m	number of cycles in burst that are counted, dimensionless
N	number of realizations, dimensionless
N_i	number of fringes crossed by ith particle, dimensionless
$p(f)$	probability density function (pdf) for Doppler frequency, 1/Hz
$p_{BP}(f)$	bandpass pdf for Doppler frequency, 1/Hz
S_T	spectral estimator, defined by Eq. (12), $m^2/(s^2\ Hz)$
t	time, s
t_1	integration variable in Eqs. (2) and (4), s
T_U	flow integral time scale, s
$\bar{u}$, U	mean velocity, m/s
u'	rms fluctuating velocity, m/s
$\boldsymbol{u}$	flow velocity, m/s
$\boldsymbol{u}_i$	velocity of ith particle, m/s
u_i	streamwise component of velocity of ith particle, m/s
u_{sp}	component of $\boldsymbol{u}_{sp}$ aligned with $\boldsymbol{K}$, m/s
u, v, w	components of fluctuating velocity, m/s
$\hat{u}_T$	velocity transform defined by Eq. (13), m/(s Hz)
$w(\boldsymbol{x})$	three-dimensional function describing scattering volume, dimensionless
$\boldsymbol{x}_p$	particle position, m
$\langle\ \rangle$	expected value or mean

Greek Symbols

Δt_i	residence time for ith particle, s
ϵ	variability, defined by Eqs. (20) and (31), dimensionless
ϵ_D	variability of Doppler frequency measurement, Eq. (32), dimensionless
λ_i	wavelength of incident laser beam, m
ν	kinematic viscosity
ν_0	data rate, 1/s
ρ_f	fluid density, kg/m^3
ρ_p	particle density, kg/m^3
σ_u	standard deviation or rms fluctuating velocity, m/s
τ_1	particle time constant, defined by Eq. (35), s

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