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THE NATURE OF TURBULENCE

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Abstract

Turbulence research has always been torn between two objectives: the need to find engineering solutions to the myriad technical problems presented by turbulent flows, and the desire to gain a scientific understanding of the turbulence phenomenon. Because of these sometimes conflicting, sometimes complementary objectives, it is easy to lose a sense of perspective about fundamental turbulence research, where it has been, and where it is headed. In this paper, the history of the development of our ideas about turbulence is briefly traced with particular attention to self-preservation, local similarity, and coherent structures. Recent research on chaos in non-linear dynamical systems will be examined, and shown to provide a conceptual framework for the understanding of turbulence. In particular, it will be suggested that the attractors for the Navier-Stokes equations are the flows themselves. From this perspective, it will be argued that turbulence is the counterpart of the strange attractor, consequences of which are the observed tendencies toward self-preservation and coherent structures.

1 Introduction

The turbulent motion of fluids has captured the fancy of observers of nature for most of recorded history. From the howling winds which paralyze continents to the babbling brooks which fascinate adult and child alike, the omnipresent turbulence both challenges our quest for authority over the world around us and delights us with its unending variety of artistic forms. From billowing clouds to falling leaves, from cigarette plume to the first swirl of creme into waiting coffee cup, turbulence constantly competes for our attention. From the almost subconscious observation of exhaust jets on a frosty day to being willingly hypnotized by the licking flames in an open fire, turbulence by its handiwork immeasurably enriches the lives of even those who cannot comprehend its mysteries. And as the poem below shows, even for those who try, the distinction between art and research is often difficult to make.

SONNET TO TURBULENCE
by
S. Corrsin
(For Hans Liepmann on the occasion of his 70th birthday, with apologies to Bill S. and Liz B.B.)

Shall we compare you to a laminar flow?
You are more lovely and more sinuous.
Rough winter winds shake branches free of snow,
And summer's plumes churn up in cumulus.

How do we perceive you? Let me count the ways.
A random vortex field with strain entwined.
Fractal? Big and small swirls in the maze
May give us paradigms of flows to find.

Orthonormal forms non-linearly renew
Intricate flows with many free degrees
Or, in the latest fashion, merely few —
As strange attractor. In fact, we need Cray 3's.

Experiment and theory, unforgiving;
For serious searcher, fun ... and it's a living!

This is a paper about the mechanical mysteries of turbulence, and an attempt to place in perspective our efforts to understand them. Before beginning this quest, however, I would first like to direct your attention to the people who study turbulence — the turbulence community, if you will — and their reasons for doing so. Of these, only two are primary: scientific curiosity and engineering need. It is tempting to identify the former with the "fun" referred to in the last line of Corrsin's poem, and the latter with "making a living". However, as we all know well, engineers can have fun too, and (in the modern world) scientists can make a living.

One of the curious characteristics of turbulence researchers is that, unlike researchers in many other fields, they frequently wear two hats — scientist and engineer. On the one hand they can perform seemingly useless experiments to elucidate some esoteric feature of turbulence which has no foreseeable engineering application. On the other, they tinker with the constants of turbulence "models" to improve engineering prediction even though the models can be connected with the physics of the flow in only the most primitive manner. Witness, for example the recent review by Frisch and Orszag [1] which moves from eddy viscosity to fractals in scarcely more than a page.

Unfortunately, there is a tendency by some who concentrate only on one aspect of this dual mission to disparage the efforts of those who focus on the other — to the detriment of all! An unfortunate consequence of this is that the rest of the world (and sponsors, in particular) get the idea that the field is in disarray with little or no consensus as to how to proceed. While in part this factionalism may be driven by the ever-increasing competition for funds, I suspect that in greater part it results from a failure to appreciate that there really are the TWO reasons for studying turbulence — and they are not necessarily complementary, at least *in this generation.*

Certainly a case can be made that we don't know enough about turbulence to even start to consider engineering problems. To begin with we have fewer equations than unknowns in any attempt to predict anything other than the instantaneous motions. Moreover, we won't be able to perform the latter for engineering problems until at least several generations of computers have come and gone. And even if we could perform a simulation of real flows, we would be overwhelmed by the amount of data, especially in the absence of real criteria for selecting from it in a single lifetime what is important. Further, even the things we think we understand — like the Kolmogorov similarity of the small scales and the Law of the Wall — have never really been tested in controlled experiments in the limits of high Reynolds number because we have failed to invest in the large scale facilities required to do so. That most would accept these particular examples as fact instead of just theory is perhaps more due to the time lapsed since they were proposed and found to be in reasonable agreement with a limited data base, than that they have been subjected to experimental tests over the range of their assumed validity. (This point was made rather forcefully by Long and Chen [2] in their controversial paper.)

The counter argument is that airplanes must fly, weather must be forecast, sewage and water management systems must be built, and society needs ever more energy-efficient hardware and gadgets. Thus, no matter the inadequate state of our knowledge, we have the responsibility as engineers to do the best we can with what we have. Who, considering the need, could seriously argue with this? That — almost incredibly — some do, can only be understood as a response to the impatience sometimes expressed by those attempting to meet to such needs about those more focused on increasing the state of our fundamental knowledge. B. Melville Jones [3] [1] wonderfully captured the essence of the problem when he said,

> A successful research enables problems which once seemed hopelessly complicated to be expressed so simply that we soon forget that they ever were problems. Thus the more successful a research, the more difficult does it become for those who use the result to appreciate the labour which has been put into it. This perhaps is why the very people who live on the results of past researches are so often the most critical of the labour and effort which, in their time, is being expended to simplify the problems of the future.

It seems self-evident then that, once personalities, questions of personal self-worth, and limited resources are stripped aside, there must of necessity be at least two levels of assault on turbulence. At one level, the very nature of turbulence must be explored. At the other, our current state of knowledge — however inadequate it might be — must be stretched to provide *engineering solutions* to *real problems.* The great and not so self-evident danger is of being deceived by the successes and good fortune of the latter into a sense of complacency about our knowledge of the former.

While it may be difficult to place a price tag on the cost of our limited understanding of turbulence, it requires no imagination at all to realize that it must be enormous. Try to estimate, for example, the aggregate cost to society of our limited turbulence prediction abilities which result in inadequate weather-forecasts alone. Or try to place a value on the increased cost to the consumer of the need of the designer of virtually every fluid-thermal system —from heat exchangers to hypersonic planes— to depend on empiricism and experimentation, with the resulting need for abundant safety factors and non-optimal performance by all but the crudest measures.

There are some who may argue that our quest for knowledge about turbulence should be driven solely by the insatiable scientific curiosity of the researcher. Whatever the merits of this argument, it is impossible to consider the vastness of the applications and not recognize a purely financial imperative for fundamental turbulence research. The problem is, of course, that the cost of our ignorance is not confined to a single large need or to one segment of society, but is spread across the entire economic spectrum of human existence. If this were not the case, it would be easy to imagine federal involvement at the scale of a Star Wars or a Super-Collider to advance more rapidly our understanding. By failing ourselves to recognize clearly the need and nature of what we do in turbulence research, we have settled for far, far less than a reasonable allotment of the national and corporate research budgets.

However, enough for now of the turbulence wars, and the endless debate over the relative merits of fundamental research and engineering. Enough also of public needs and policy. Since this is an Engineering Conference, it is safe to assume that the members of the audience are well-versed in and committed to the engineering aspects of turbulence research. Therefore, with your indulgence, I would like draw your attention now away from the *nature of turbulence research*, and don my other hat – of pure scientist – to lead you on a brief visit to the Never, Never Land of Ideas about the *Nature of Turbulence* itself. The tour will, by the very nature of the subject, be a very personal one since no two peo-

[1] I am grateful to F.Culick of Cal Tech for calling this to my attention.

ple could be in complete agreement about something about which we know so little. It will be up to you to distinguish between what you wish to consider as fact and to dismiss as fantasy. Thus, like two artists painting the same scene, the pictures we paint will as much reflect our own personalities and histories, as the facts. And, like real works of art, both my picture of turbulence and yours might enable others to see things that they would have otherwise missed.

We will begin by examining what I believe to be the three great ideas which should most influence our view about the nature of turbulence: First, the idea of self-preservation and the tendency of turbulent flows toward self-preserving states; second, the recognition of coherent and organized structures in turbulent flows; and third, the recent discovery of the chaotic nature of deterministic non-linear systems. Then I shall attempt to show how I believe these ideas fit together to tell us something about the nature of turbulence. Finally, I'll try to speculate as to where this might be taking us.

Note that I shall not discuss what many consider to be the fundamental problem of turbulence — the closure problem. From the perspective of this tour, the closure problem is an engineering problem, and does not bear directly on the nature of turbulence itself. The inverse is not true, of course, since our attempts to deal with it should depend crucially on our understanding of the nature of turbulence. It is probably worth pointing out that most closure attempts to-date use very little of even our present understanding beyond the simple recognition that if you have more of a quantity in one place than in another, *on the average* the turbulence tries smooth out the difference. (Lumley [4] chides those who rail against gradient transport models for ignoring the "on the average", thereby missing the whole point.) This observation does not imply that such turbulence models are particularly simple, since even the higher level single point models can become bewilderingly complex as they try to satisfy the constraints of rational mechanics, cf. Lumley [5]. As one who has spent a significant portion of his career trying to improve the data base on which such models must depend, I hardly need to defend the fundamental character of work on the closure problem or the priority it deserves. Nor, therefore, do I need to apologize for arguing that it contributes little to our understanding of the nature of turbulence.

2 Self-preservation

The first clue to the nature of turbulence is offered by the observation that in every case where the averaged equations of motion admit to self-preserving (or similarity) solutions, the flows observed in nature appear to settle into these states. Moreover, in many cases where the equations do not admit to such solutions, the flow tries to settle into some kind of local similarity. Examples of the former include the familiar self-preserving solutions for jets and wakes, while the latter would include the Law of the Wall and Law of the Wake for turbulent boundary layers, and the Kolmogorov similarity for the dissipative scales of high Reynolds number turbulence. (Almost every book on turbulence includes most of the flows mentioned; cf. Tennekes and Lumley [6], Monin and Yaglom [7], and Hinze [8].)

The term self-preservation is used here to mean that the averaged flow properties in space or time can be collapsed by characteristic scales which depend on only a single variable. For example, if the mean velocity is given by

$$U = U(x,y), \tag{1}$$

then self-preservation would imply that there exists a length scale, $\delta = \delta(x)$, and a velocity scale, $U_s = U_s(x)$ so that

$$U = U_s f(y/\delta) \tag{2}$$

An important consequence of the definition above is that the equations of motion are reduced by one-dimension. Thus for flows which are planar or axisymmetric in the mean, the averaged equations are ordinary differential equations. The consequences of self-preservation are thus similarity solutions to the governing equations. Figure 1 illustrates the self-preservation of the velocity profile in an axisymmetric buoyant plume.

Recently, I have explored a somewhat different definition by insisting first that all of the terms in the averaged equations go up and down together at the same relative location (like y/x above), and then seeking appropriate scaling functions for each term which accomplishes this, v. ref. [10]. While this assumption leads to the same the kind of profiles assumed above, there are some important differences which will be discussed briefly later.

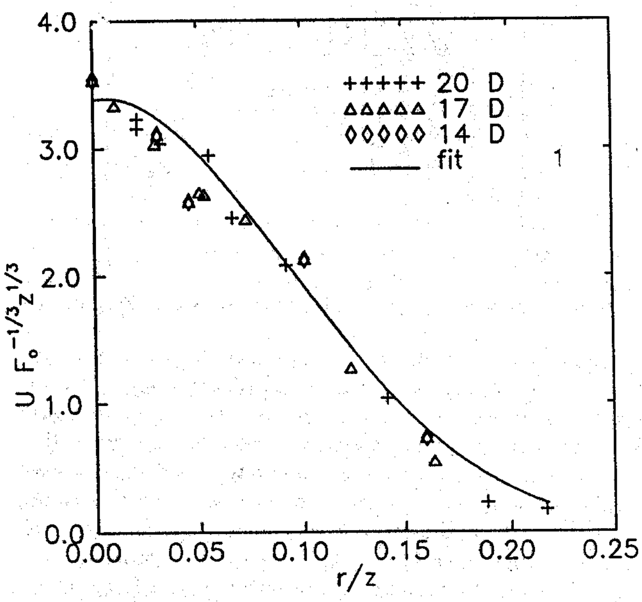

Figure 1: Velocity profile in an axisymmetric plume (ref 9).

The second kind of self-preservation is called *local self-preservation* or *local similarity*. The principal difference is that local self-preservation does not lead to a reduction of the order of the governing equations, and similarity solutions are not possible. Perhaps the best example of local self-preservation is the experimental observation that for many types of boundary layer flows in the near-wall region, the velocity moments can be collapsed with only the friction velocity (defined from the wall shear stress as $u_\tau = \sqrt{\tau_w/\rho}$) and the kinematic viscosity, ν. This Law of the Wall for the mean velocity profile is thus given by

$$U = u_\tau f(y u_\tau/\nu) \tag{3}$$

This is only a local self-preservation since it is only valid in the wall region. Another well-known example (illustrated in Figure 2) is that of the so-called Kolmogorov similarity of the small scale turbulent motions in high Reynolds number turbulence for which all statistical quantities scale with the rate of dissipation of turbulence kinetic energy, ϵ, and the kinematic viscosity, ν.

Now it should be noted that not everyone who works in turbulence believes in self-preservation. And even those who do, don't necessarily agree on the details of what they believe. Most who do believe follow the view perhaps best presented by Townsend [11], [12] that the self-preserving states attained are truly universal. For example, all wakes, regardless of how generated, should asymptotically achieve the same self-preserving state (when normalized by the drag and free stream speed). In the same vein, all high wavenumber turbulent spectra should look alike (when plotted in Kolmogorov

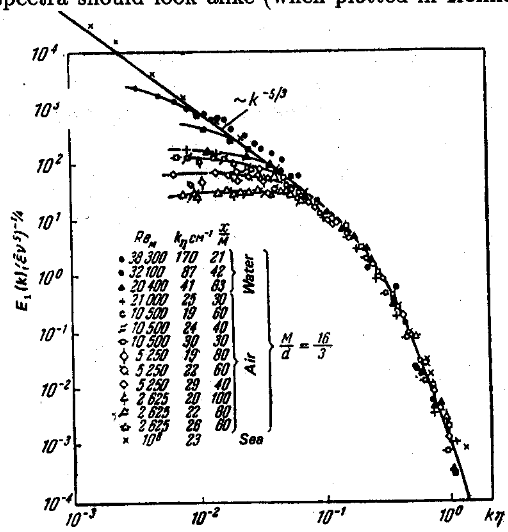

Figure 2: Velocity spectra in Kolmogorov variables (ref. 7).

variables), and all wall layers should have the same same velocity profiles (when plotted in wall variables), regardless of how the flows were generated. Probably the most articulate defender of this view at present is Narasimha, e.g. ref. [13].

The alternative view of self-preservation mentioned above is both new *and* controversial — so much so in fact that some will argue that I'm the only one who holds it! (In fact, most of the papers where I've discussed it have yet to be published because the journal reviewers keep rejecting them.) Regardless, it follows from the rather different approach described briefly above where one begins with the equations of motion, and asks what kinds of scaling yield similarity equations in which all the terms go up and down together. Much to my surprise, when such self-preserving forms can be found, the solutions depend forever on the initial conditions (for example, the initial Reynolds number or the shape of the wake generator). The universal profiles above, and the Kolmogorov similarity are only achieved at *infinite* Reynolds number.

As often seems to be the case in turbulence, the data provide some support for both views. While there does appear to be some dependence on Reynolds numbers, for example, most argue that it is just experimental error. This argument is countered by arguing that the range of variation of initial conditions over which experiments have been performed is just too small to see much variation. This is understandable, of course, since the theory suggesting such a dependence is rather new, and the experimentalists haven't yet designed experiments to look for it. On the other hand, some argue that the observed variations in the self-preserving states simply prove that there can be no such thing. (Usually, however, the proponents of this view are uninformed of the new alternative to universal states.)

Now the basic ideas of self-preservation have been around for a very long time (at least 60 years). So why, you may ask do I suddenly believe them to be so important to understanding the nature of turbulence? The full answer will have to wait for the concluding section of this paper, but for now another question may give a clue. The question is: Presuming that turbulent flows do have this tendency toward self-preservation, WHY? Certainly the instantaneous equations admit to no self-preserving solutions. (The inviscid equations can, however, be shown to be scale invariant, v. ref. [1]). Thus, this tendency to settle into self-preserving states is only an average property of the turbulence, and it reduces the effective number of dimensions of the equations governing the averaged motion. How does the turbulence know it is supposed to behave this way? For example, how does the flow from an orifice know it is supposed ignore the possibility of spreading uniformly in space (like an inverse sink flow), and organize itself into the thin shear layer that is a self-preserving jet? Even if we allow for the separation at the edge of the orifice as a partial explanation of why the flow starts as a jet, it is still not obvious why it stays that way.

It is my view that self-preservation is itself a clue to some deeper understanding of why turbulence does what it does? In other words, if we would understand the nature of turbulence, we must be able to explain *Why self-preservation — of any kind?*

3 Coherent structures: real or imaginary?

A second clue to the nature of turbulence comes to us from the presence of coherent structures in many turbulent flows. While the means for quantifying these structures are at times difficult to understand, the occurrence of characteristic patterns of motion in many turbulent flows is obvious to even the most casual observer. Some of the most common examples include the rolls of clouds overhead, the wind-streaks along the surface of highways in a snow-storm, and rain-marked gusts in a thunderstorm. The famous sketch by Leonardo da Vinci of the turbulent flow from a pipe (see the paper cover of ref. [6]) is replete with a variety of characteristic structures. In spite of the human mind's ability to quickly recognize visual patterns, the problem of quantifying the existence and character of coherent motions in turbulence has proved to be a major and continuing challenge.

The idea of using statistical measures to look for highly correlated disturbances underlying an apparently random turbulent field can largely be attributed to Townsend and his students in the 1950's. They postulated the existence of big turbulent eddies to explain the small, but persistent, correlation between velocity fluctuations at widely separated points and the large scale intermittency of turbulent shear flows. This idea was given mathematical rigor a decade later with the proposal by Lumley [14] to use proper orthogonal decomposition (POD) techniques to find and characterize these structures. Nearly concurrent with this development was the attempt by Kovasznay and his students [15] to investigate coherent structures experimentally by using conditional sampling techniques. It was, however, two relatively simple applications of flow visualization which captured the attention of the fluid dynamics community, and buried forever the old idea of turbulence as a rather disorganized jumble of many interacting degrees of freedom.

The first of these came from the dye and hydrogen bubble studies of the near wall region of a turbulent boundary layer by Kline and his students in the late 1950's and the subsequent decade. Their photographs not only made it clear that this region of the flow was not quiescent as previously believed (and as implied by its old name, the laminar sublayer), but very active (hence its new name, the viscous sublayer). More than this, the visualization studies showed clearly the existence of highly coherent streaks near the wall which persisted for nearly 1000 times the viscous length scale with an average spacing of more than 100 times it. This was not only unexpected, but almost inconceivable, to a scientific community which had been weaned on a mixing length proportional to the distance from the wall. A intensive hunt was begun using all of the techniques described above to further document the character of the wall region and its relation to the rest of the boundary layer. The findings still defy complete explanation today and remain an active area of research. (Kline [16] provides a comprehensive collection of current efforts and viewpoints.)

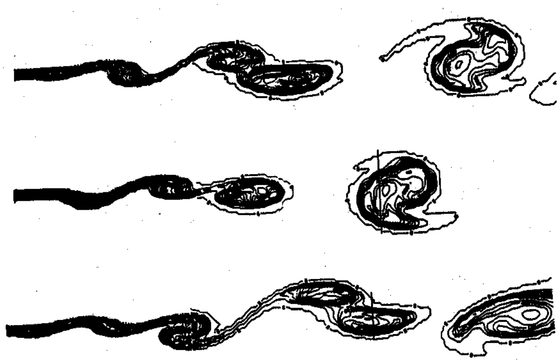

Figure 3: Numerical simulation of vortex roll-up in a shear layer (ref 18).

While the existence of structure in the wall layer revised our view of turbulent boundary layers, it did not significantly influence thinking about the nature of turbulence away from surfaces. That did not happen until more than a decade later when another flow visualization event captured the imagination of fluid dynamicist and non-fluid dynamicist alike, the publication of the now famous photograph by Brown and Roshko [17] of vortex roll-up in a turbulent free shear layer (similiar to that shown in Figure 3). It had been preceded by the forced jet mixing layer experiments of Crow and Champagne [19] in which the flow could be clearly seen to organize itself into large scale puffs. Regardless of one's opinion about the existence of coherent structures before, from this point forward these structures were recognized by most as being a fundamental part of turbulence.

These two early experiments unleashed a flurry of research efforts to identify coherent structures in a variety of turbulent flows. Flows of all types were artificially forced, conditionally sampled, and visualized by all sorts of exotic techniques. (Cantwell [20] and Hussain [21] provide excellent reviews of the results of these efforts.) While there remains even today considerable debate as to how to interpret the various results, there is a general consensus that coherent *vortical* structures are present in most turbulent flows. Not at all clear yet is their dynamical role; or presuming they have one, how to incorporate them into improved turbulence models. In fact, this debate has not subsided since the very beginning. Originally, some (myself among them) subscribed to Townsend's view that, however visible, the role of coherent structures was largely a passive one. Others (e.g. Liepmann [22]) argued that coherent structures were the dominant determinants of the turbulent energy, and that one couldn't begin to *model* turbulence without explicitly accounting for them. The truth, as we are beginning to understand it now, appears to be that both views were partially right and partially wrong. (Isn't this always the case?)

My own efforts in coherent structures through these last two decades were focused, not on the methods described above and being used by most, but rather on attempts to apply proper orthogonal decomposition techniques to the determination of coherent structures. From the early 1970's when I was collaborating with Lumley, we realized that the POD held forth the possibility (perhaps the only one) of capturing objectively the full kinematical behavior of coherent structures. The problem was that the POD needed prodigious amounts of experimental data to properly capture this behavior — billions of cross-spectral data at hundreds of points! The first experiment we designed was for the viscous sublayer, and built off of an earlier much less ambitious effort by Bakewell and Lumley [23]. I changed universities shortly after the experiment was underway, and after great effort it was completed by Herzog [24] — more than 15 years after its inception. The second application of the POD was initiated in the axisymmetric jet mixing layer when I came to Buffalo in the mid- 70's, and was completed more than a decade later by Glauser [25], [30]. Both these experiments broke new ground in terms of experimental complexity and sheer amounts of data; and were it not for the parallel development of on-line computer acquisition and processing, might never have been completed. (In fact, one of the greatest difficulties was in keeping sponsors and proposal reviewers believing the experiments were possible at all [2]) The advent of the full Navier-Stokes computer simulation of turbulence has opened many new possibilities for further application of these

[2] I have more than a few personal heroes out there who never lost the faith, but foremost among them were Dr. George K. Lea of NSF and Lt. Col. Lowell Ormand, formerly of AFOSR.

techniques (cf. Moin [26] and Moser and Moin [27]).

Now these POD applications were always viewed with suspicion by those using conditional sampling techniques (and still are today, by some!), largely I believe because of a failure to appreciate what the POD is. (Truth to tell, we didn't always explain it too well – in part, because our own understanding was evolving.) In simplest terms, the POD decomposes an instantaneous random field of space and time into the deterministic modes (or eigenfunctions) which can represent it with the smallest number of terms; in fact, optimally in a mean square sense. (See George [28] for an introduction to the POD and how it can be interpreted physically.) For a homogeneous or periodic field, the optimal functions are the harmonic ones, so the POD produces the familiar Fourier representation in these cases. In the more interesting inhomogeneous case, the eigenfunctions must be determined from an eigenvalue problem involving the full space-time two point correlation tensor – hence, the need for experimental data. In this case, the lowest order eigenfunctions correspond rather loosely to the largest coherent structures present. What is most interesting however, is that by studying how the eigenfunctions change in space and time, and how they interact with each other, we can see for the first time what the dynamical role of the coherent structures is.

This use of the experimentally obtained eigenfunctions to investigate dynamical behavior is just now beginning with the first published results due to Aubry et al. [29] using Herzog's and Moin's viscous sublayer eigenfunctions. By using only the lowest order mode in the cross-stream direction and a few modes from the other two directions, they were able to obtain dynamical behavior of a synthetic sublayer which displays the principal features observed in real flows by the flow visualization and conditional sampling techniques. Another effort by Glauser and George [30] used only qualitative arguments to construct the life-cycle illustrated in Figure 4 for pairs of vortex ring-like structures in the axisymmetric jet mixing layer which were destabilized by their interaction, the net result of which was to produce smaller scale vorticity , perhaps also rings. From these two examples, it is clear that the gap between the POD and coherent structures appears to be disappearing, if it is not gone already. And with the disappearance is the confirmation that there really are coherent structures in turbulence, and that they are a very important reason why turbulence does what it does.

From this very brief summary, it should be obvious that I believe coherent vortical structures to be an essential feature of any attempt to define the nature of turbulence. While recognizing them and defining their role is important, an equally important question is: "Why do they arise in the first-place?" A partial answer might be that they arise from an instability of the velocity profile much in the matter of transitional instabilities. While operationally useful, this too begs the question? The more fundamental question is: Why does the turbulence seem to want to organize itself into these highly coherent motions? As will be seen later, my own view is that this question is closely related to the question asked at the end of the previous section about self-preservation.

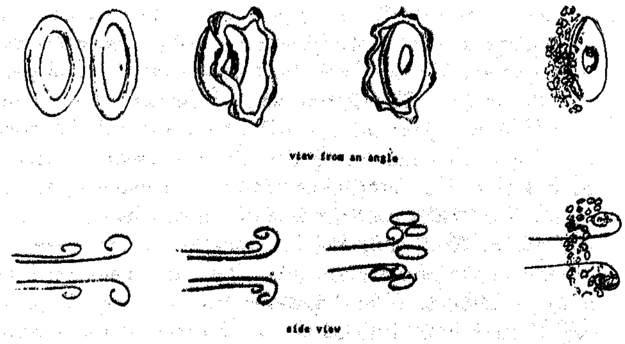

Figure 4: Schematic of proposed life-cycle in jet mixing layer (ref 30).

4 Chaos: the key to understanding?

The last clue we shall consider is *chaos*. The so-called "theory of chaos" has emerged from the subculture of mathematicians, physicists, and fluid mechanicians who nourished it in almost anonymity to the most dramatic influence on scientific thinking since quantum mechanics and relativity. The tale of how this happened was beautifully captured by Gleik [31] in the best-seller that should be required reading for every student of mechanics, and especially of turbulence [3]. That something referred to as *chaos* should have significance to a field named *turbulence* can hardly be a surprise to anyone. In fact, a considerable portion of the early interest in chaos arose from an interest in how transition to turbulence occurs.

Chaos, or deterministic chaos as it is sometimes referred to, is the seemingly unpredictable behavior of purely deterministic solutions to non-linear equations. (Note that the word random has not been used, although "seemingly random" would be equally appropriate.) Prior to chaos, it was commonly believed that a necessary prerequisite for obtaining "random-looking" solutions was either intimidating and intractable equations (like those governing fluid motion), or random boundary conditions often believed to "trigger" the complex behavior of turbulent flows. The big discovery of the chaos pioneers was that perfectly normal and solvable equations *which just happened to be non-linear* can produce solutions which have all the unpredictability of turbulence. While this seemed to come as a major surprise to most of the scientific community, it was less surprising to at least the turbulence school of thought in which I have my roots. At Johns Hopkins in the 1960's, for example, both Corrsin and Kovasznay spent a great deal of time challenging us to understand the characteristics of various kinds of non-linear systems, and always inferred some analogous relation between such behavior and turbulence. Even with this background and the firm belief that at the core the world really was governed by nonlinear behavior, I confess to have been awed by the actual consequences of that non-linear behavior as it has been revealed over the past ten years or so by the chaos community. The full consequences for scientist, engineers *and especially educators of both* are only beginning to be understood. [4]

My own favorite example of chaos (and one I require my

[3]It is required in the turbulence course I teach at UB.

[4]See, for example, the recent interview of F. Moon [32]

students to investigate first hand) is the logistic map explored first in detail by May [33]. Consider the simple non-linear equation given by

$$x_i = r x_{i-1} (1 - x_{i-1}) \tag{4}$$

for $0 \leq x \leq 1$. The equation is often used by ecologists to try to account for the changing population of a particular species from year to year for a given reproduction rate r. It is easy to show with a hand calculator that if the value of r is too small for a given initial value of x, then the limiting value of x is zero, i.e. extinction. For values of r above this critical value, but less than 3. the value of x stabilizes at value determined by its initial value. For $3 \leq r \leq 4$, however, no stable solution is achieved. For r near 3, after an initial transient, the solution simply oscillates between two values. As r increases, it switches abruptly to oscillate among four values, and then changes equally abruptly to oscillate among eight values. Above $r = 4$, however, the solution varies chaotically (and hence the name), never returning to any previous value.

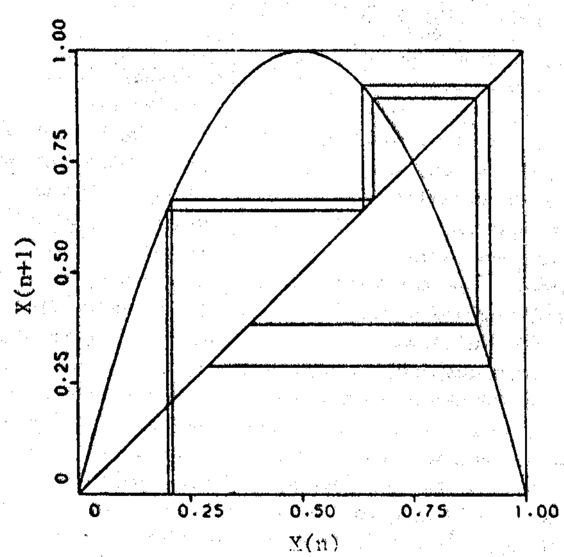

Figure 5: Logistic map (from ref 37).

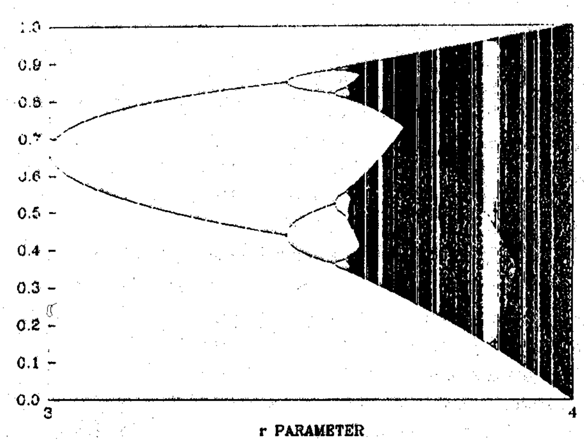

Figure 6: Bifurcation diagram for logistic map (from ref 37).

In spite of this apparently random behavior in the chaotic regime (It's, of course, not random since it's perfectly reproducible!), there is a pattern to even this solution as shown in Figure 5, where the values continue to trace square-like figures. But these "squares" never close since the values never repeat! As if this wern't mystery enough, within the chaotic regime are windows of order in which the whole route to chaos repeats itself. In fact, no matter how large the magnification, each ordered region also goes chaotic and in these chaotic regions are windows of order, and so on. This is illustrated in Figure 6 which shows the bifurcation diagram for the system, and makes clear the islands of order with chaos.

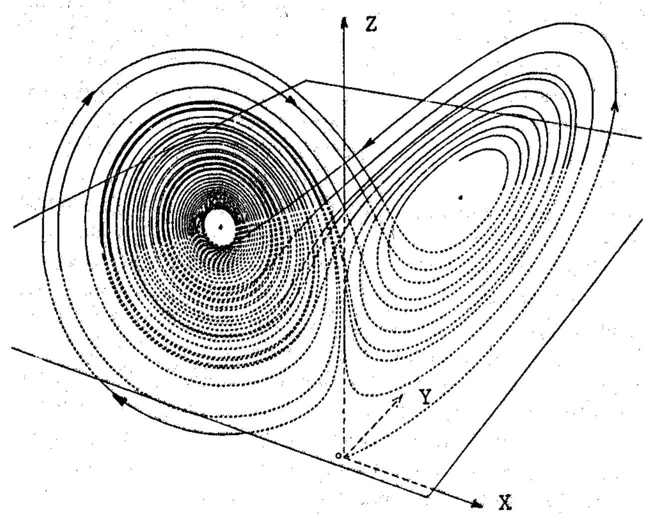

Figure 7: The Lorenz attractor (ref 37)

There are many other systems of equations — algebraic and differential — which give rise to the same basic patterns of behavior: simple limiting solutions yielding to periodic solutions which in turn yield to chaotic ones. For time-dependent equations, phase-plane plots of the chaotic solutions are characterized by *strange attractors* like the now-famous butterfly attractor of Lorenz shown in Figure 7. Over the past decade or so, many time-dependent non-linear and chaotic systems have been analyzed, and all ultimately end up producing the kind of confined chaotic solutions observed above, often with spectacularly artistic strange attractors, see for example Ruelle [34]. There has been a great deal of attention focused over the past decade on the various routes to chaos, and the attempts to apply the results to transition-to-turbulence of fluid dynamics are both interesting and controversial, v. Morkovin [35]. For our purposes here, however, we are only interested in the chaotic part of the problem, and particularly the *strange attractor.*

Schuster [36] lists numerous properties of strange attractors; among them:

- (a) It is an attractor, i.e. a bounded region of phase space to which all sufficiently close trajectories from the so-called basin of attraction are attracted asymptotically for long enough time.
- (b) It has a sensitive dependence on the initial conditions so that points initially close together do not remain so.

- (c) It is *structurally stable* (i.e. small changes in the parameters change it continuously) and *generic*.

- (d) It is characterized by *fractal* dimension, i.e. it is not space-filling. [5]

Now let us step back from the terminology of chaos and strange attractors, and try to describe the above characteristics from a more familiar perspective. First, note the tendency of solutions to non-linear equations to become "turbulent" all by themselves, without any effort to make them so. Second, these "turbulent" solutions are not just random, but fall asymptotically into certain patterns which depend on the initial conditions. Third, these "self-preserving" states have a great deal of "coherent structure" to them, which is clearly evident when "visualized" in phase space. Finally, as a consequence of the fractal nature of the attractor, "similar" structures are observed at any level of magnification. It is clear that the terminology of the preceding sections (and of the turbulence community for the past 20 – 30 years) fits nicely to this task.

The purpose of the last paragraph has not been to argue that turbulence and chaos are the same thing, nor should we expect them to be. The kind of chaos we have been talking about applies to algebraic maps and dynamical systems where time is the only independent variable. Turbulence, on the other hand, is both a temporally and spatially varying phenomenon. Thus, at most the behavior of dynamical systems can give us clues as to what the nature of turbulence is. In the next section, we shall try to tie all the clues together.

5 The real nature of turbulence

There are many other clues to the nature of turbulence which we could include and discuss in some detail, were space and time permitting. In the context of this paper probably the most important are the observations by Screenivasan and Mendiveau [38] that the turbulent non-turbulent interfaces and the dissipative scales of turbulence appear to have a fractal character. As these authors have noted, a fractal description does not preclude a fluid mechanical description of these phenomena by vortex elements, since fractals can only describe the geometry of turbulence, and not its dynamics.

The concepts of chaos and strange attractor discussed briefly in the preceding section have considerable appeal in trying to understand the nature of turbulence. However, since chaos has traditionally been examined in phase space in the traditional manner of dynamical systems theory, it is not at all clear what we should expect in a spatially and temporally varying system. It is probably for this reason that attention has largely been focused on the "Lagrangian Turbulence" arising from particle motions in fluids (not necessarily turbulent flows, either), and on the behavior of flows in time at a single point. (Chevray [37] provides an interesting review of these efforts.) Therefore it is to this problem that I would like to address my attention. In particular,

- Do the Navier-Stokes equations give rise to chaotic solutions, i.e. solutions which are deterministic but appear random?

- Do the Navier-Stokes equations have strange attractors associated with them?

- And if they do, what should we expect them to look like, and how would we represent them?

Recognizing that the first step in any application of the scientific method is the formation of the hypothesis, I offer the following: I suggest that the clues we have examined are strong indicators that the turbulence is the manifestation of the chaotic tendencies of the Navier-Stokes equations. Moreover,

- The Navier-Stokes equations do indeed have attractors, and *they are in fact the flows we realize.*

- *The strange attractors of the Navier-Stokes equations are the turbulent flows we see.*

Now if one insists on defining an attractor only as a construction in phase space the above makes no sense. However, if one is willing to consider a more general definition, then turbulent flows have counterparts to all of the properties we noted about strange attractors. From the sensitive dependence of the instantaneous motions on initial and boundary conditions, to the relatively insensitive dependence of the self-preserving and entraining "average" flows they seem to fall into; from the generic flow patterns we so quickly recognize and name (e.g. jet, wakes, etc.), to the elusive coherent structures we see but cannot unambiguously identify; from the low order representation by orthogonal functions, to fractal geometry which dictates self-preserving forms — all of the features of strange attractors have their counterpart in the turbulent motions around us. Thus turbulence must be the natural manifestation of the non-linearity of the Navier-Stokes equations, and the baffling features we see are the characteristics of its strange attractor in a given environment.

If all of this makes sense, we have succeeded in identifying *turbulence* with a large class of non-linear and chaotic phenomena, and vice versa. And in doing so have provided the answers to many of the questions we posed earlier. For example, turbulence tends to be self-preserving because its attractors have fractal dimension. Or, thin shear layers develop as a consequence of the particular strange attractor corresponding to the equations and gross boundary conditions. Thus from this viewpoint, there is nothing particularly unique about nature of turbulence.

Now unless you haven't been thinking while you read the above, you've probably noticed that by answering these questions in this manner, we haven't really answered them at all! We have only moved them into a much larger category of things we don't understand about nature. For example, Why do non-linear systems give rise to strange attractors? Or from another discipline, Why is space-time in the large governed by general relativity? Equally, Why does the world in the small obey quantum mechanics? Obviously, regardless of how far we push this, there will always be a point at which we must answer as Richard Feynman [39] was fond of saying: "It's crazy that the world behaves this way, but that's the way it

[5] The subject of *fractals* was recognized by Mandelbrot as being relevant to turbulence, and could have been included as a section in this paper. However, it is primarily of interest in this context as being a property of the strange attractor.

is!" The challenge, of course, is to know when we've reached this point.

Suppose that we have not reached that point in our understanding of non-linear phenomena, and that there remain principles to be uncovered which explain why chaos occurs, and what the nature of the attractor will be. It is my own belief that beneath it all lies at least a variational principle which if in our grasp will solve (or remove entirely) the turbulence (closure) problem by giving us directly the form of the attractor. For example, one might hypothesis that the flows we see represent the minimum rate at which entropy can be produced for a given set of boundary conditions. One cannot, of course, infer from my own lack of success in proceeding from this to a solution of the turbulence problem, that someone else using this or an alternative hypothesis will *not* succeed.

While it may be easy to dismiss this kind of thinking as a useless exercise because of the difficulties inherent in it, let us not fail to recognize the enormous practical implications should it succeed. And spurred on by this hope, let us keep alert as we continue to explore the nature of turbulence and as we tackle our engineering challenges. Maybe if we get really lucky, we won't fail to recognize what we've found *if we should trip over it!*

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