The decay of homogeneous isotropic turbulence

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A new theory for the decay of homogeneous, isotropic turbulence is proposed in which truly self-preserving solutions to the spectral energy equation are found that are valid at all scales of motion. The approach differs from the classical approach in that the spectrum and the nonlinear spectral transfer terms are *not* assumed *a priori* to scale with a single length and velocity scale. Like the earlier efforts, the characteristic velocity scale is defined from the turbulence kinetic energy and the characteristic length scale is shown to be the Taylor microscale, which grows as the square root of time (or distance). Unlike the earlier efforts, however, the decay rate is shown to be of power-law form, and to depend on the initial conditions so that the decay rate constants cannot be universal except possibly in the limit of infinite Reynolds number. Another consequence of the theory is that the velocity derivative skewness increases during decay, at least until a limiting value is reached. An extensive review of the experimental evidence is presented and used to evaluate the relative merits of the new theory and the more traditional views.

I. INTRODUCTION

The ideas of similarity and self-preservation have played an important role in the development of turbulence theory for more than a half-century. The traditional approach to the search for similarity solutions in turbulence has been to assume the existence of a single length and velocity scale, then ask whether and under what conditions the averaged equations of motion admit to such solutions (cf. Townsend[1] and Tennekes and Lumley[2]). When shown to exist, such solutions have been assumed to represent universal asymptotic states, retaining no dependence on initial conditions other than generic ones (e.g., jets, wakes, plumes, etc.).

There has been considerable debate over the years as to whether or not real turbulent flows actually converge to such universal solutions, and if so, precisely which experiments best represented them. It was, in fact, the apparent failure of the von Kármán–Howarth[3] similarity analysis of decaying isotropic turbulence to adequately describe the turbulence behind a grid that led Batchelor[4] to propose the now commonly accepted view of turbulence as an intrinsically multilength scale phenomenon, describable only by local similarity laws at the energy and dissipative scales, respectively. Incorporated into the latter was the Kolmogorov[5] proposal for the near statistical independence of the dissipative motions from the larger scales, and the idea of a universal equilibrium range.

The advances over the past decade in chaos theory with its strange attractors and fractal properties have given new impetus to examining the role of self-preservation ideas in turbulence. Recently, George[6] showed that the averaged equations of motion for a number of flows admit to more general similarity solutions that retain a dependence on both the Reynolds number and the initial conditions. These types of solutions are particularly tantalizing in view of the similar properties of strange attractors in dynamical systems theory (see Schuster[7] and George[8]). This paper reexamines the possibility of self-preserving solutions to the averaged spectral equations from this more general perspective. The existence of solutions for isotropic turbulence that are both self-preserving at all scales of motion and dependent on the initial conditions will be shown to be possible.

The results will be seen to have been anticipated by the analysis of Barenblatt and Gavrilov[9] and by the experimental results of Ling and Wan,[10] and are closely related to the general theory of intermediate asymptotics and similarity solutions of the first and second kind (see Barenblatt[11]). An important contribution of the present work is the recognition of the role of the derivative skewness in limiting the range of validity of the initial-condition-dependent solutions, and in sorting out many of the apparent inconsistencies in the experimental observations in decaying turbulence. Specifically, it is argued that there is an initial decay region that is determined by the initial conditions and in which the nonlinear transfer terms (as measured by the velocity derivative skewness) are increasing in importance during decay. As the initial Reynolds number of the flow is increased, the von Kármán–Howarth solution (in which the energy decays as t^{-1}) is approached asymptotically as the limiting solution at infinite Reynolds number. Two different kinds of $t^{-5/2}$ decay are identified: one in which the nonlinear terms are of increasing importance throughout the decay and one in which they are negligible, the latter corresponding to the true final period of decay. Finally, it is suggested that for finite Reynolds numbers, the nonlinear terms reach saturation so that a second region of decay is entered in which the derivative skewness is nearly constant (or decreases slowly) and which may or may not be self-preserving. The theory is substantiated by a careful examination of the experimental data for decaying turbulence.

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II. THE DYNAMICAL EQUATIONS

The spectral energy equation for a homogeneous isotropic turbulence is given by Batchelor[12] as

$$\frac{\partial E}{\partial t}=T-2\nu k^2 E, \tag{1}$$

where E and T are functions of both the wave number k and time t. The three-dimensional energy spectrum function $E(k,t)$ is defined as the integral over spherical shells of radius $k=|\mathbf{k}|$ of the contracted velocity spectrum tensor, i.e.,

$$E(k)\equiv\frac{1}{2}\iint_{|\mathbf{k}|=k}\phi_{ii}(\mathbf{k})d\sigma(\mathbf{k}), \tag{2}$$

where

$$\phi_{ij}(\mathbf{k})=\iiint_{-\infty}^{\infty} e^{i\mathbf{k}\cdot\mathbf{r}}\langle u_i(\mathbf{x})u_j(\mathbf{x}+\mathbf{r})\rangle d\mathbf{r}. \tag{3}$$

Hereafter, the three-dimensional spectrum function $E(k,t)$ will simply be referred to as the spectrum.

It follows from the definitions that the integral of the three-dimensional spectrum function can be integrated to obtain the turbulence kinetic energy, i.e.,

$$\frac{3}{2}u^2\equiv\frac{1}{2}\langle u_i u_i\rangle=\int_0^{\infty}E(k,t)dk. \tag{4}$$

The spectral energy transfer $T(k,t)$ arises directly from the transformed convective terms of the equations of motion. Following Batchelor,[12] the Fourier transform of the two-point triple-velocity correlation is defined as

$$\Gamma_{ij,l}(\mathbf{k})\equiv\iiint_{-\infty}^{\infty}\langle u_i(\mathbf{x})u_j(\mathbf{x})u_l(\mathbf{x}+\mathbf{r})\rangle e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}. \tag{5}$$

From the isotropic relations and incompressibility, $\Gamma_{ij,l}(\mathbf{k})$ can be related to a single scalar function $\gamma(k)$ by

$$\Gamma_{ij,l}(\mathbf{k})=i\gamma(k)(k_ik_jk_l-\tfrac{1}{2}k^2k_i\delta_{jl}-\tfrac{1}{2}k^2k_j\delta_{il}). \tag{6}$$

The transformed inertial terms in the spectral equations can be shown to be given by

$$T(k)=4\pi k^6\gamma(k). \tag{7}$$

Thus $T(k,t)$ contains the essential nonlinearity of the Navier–Stokes equations, and represents the transfer of energy from all other wave numbers.

The spectral energy equation can be integrated over all k to yield the energy equation for the turbulence as

$$\frac{d}{dt}\left(\frac{3}{2}u^2\right)=-\epsilon, \tag{8}$$

where ϵ is the rate of dissipation of turbulence energy per unit mass given by

$$\epsilon=2\nu\int_0^{\infty}k^2E(k,t)dk \tag{9}$$

and where the fact has been used that the net spectral transfer over all wave numbers is identically zero, i.e.,

$$\int_0^{\infty}T(k,t)dk=0. \tag{10}$$

III. SIMILARITY ANALYSIS OF THE SPECTRAL EQUATIONS

Self-preserving forms of the spectrum and spectral transfer functions are sought for which

$$E(k,t)=E_s(t,*)f(\eta,*) \tag{11}$$

and

$$T(k,t)=T_s(t,*)g(\eta,*), \tag{12}$$

where

$$\eta=kL \tag{13}$$

and

$$L=L(t,*). \tag{14}$$

The argument $*$ is included to indicate a possible dependence on the initial conditions. *Only those solutions of the spectral energy equation will be sought for which the coefficients of all terms have the same time dependence so that all terms maintain the same relative balance as the turbulence decays.* (Note that this is, thus, a more restrictive class of self-preserving solutions than those usually considered in which the only assumption is that the profiles collapse with a single length scale.)

Differentiating Eq. (11) yields

$$\frac{\partial E}{\partial t}=[\dot{E}_s]f(\eta,*)+\left[\frac{E_s\dot{L}}{L}\right]\eta f'(\eta,*), \tag{15}$$

where the overdot denotes time differentiation and prime differentiation with respect to η.

Substituting into the spectral equation (1) leads immediately to the transformed equation

$$[\dot{E}_s]f(\eta,*)+[E_s\dot{L}/L]\eta f'(\eta,*)$$
$$=[T_s]g(\eta,*)-[\nu E_sL^2]2\eta^2f(\eta,*). \tag{16}$$

(Note that the bracketed terms are functions of both t and $*$.)

It is convenient to divide by $\nu E_s/L^2$ so that the transformed equation reduces to

$$\left[\frac{\dot{E}_sL^2}{\nu E_s}\right]f+\left[\frac{L\dot{L}}{\nu}\right]\eta f'=\left[\frac{T_sL^2}{\nu E_s}\right]g-[1]2\eta^2 f, \tag{17}$$

where the dependences on t, η, and $*$ have been suppressed for now.

Now, since the coefficient of the last term is time independent, self-preserving solutions of the type sought here are possible only if the other bracketed terms are also time independent; i.e.,

$$[\dot{E}_sL^2/\nu E_s]=\text{const}, \tag{18}$$

$$[L\dot{L}/\nu]=\text{const}, \tag{19}$$

and

$$[T_sL^2/\nu E_s]=\text{const}. \tag{20}$$

From Eq. (19) it follows immediately that

$$L^2=2A\nu(t-t_0), \tag{21}$$

where the constant of proportionality has been chosen for convenience as $2A$, and must be determined from other considerations. Also, the dependence on t_0 can be eliminated by an appropriate choice of origin in time; so, hereafter t_0 will be assumed zero with no loss in generality. Equation (18), together with Eq. (21) reduces to

$$\dot{E}_s t/E_s=p, \tag{22}$$

where p is a constant. This can be integrated to yield

$$E_s/E_{s1}=[t/t_1]^p, \tag{23}$$

where E_{s1} and t_1 denote an arbitrary reference state. Thus the spectrum undergoes a power-law decay (assuming p to be negative). The third equation, Eq. (20), can be satisfied if and only if

$$T_s\sim\nu E_s/L^2, \tag{24}$$

or using Eq. (21),

$$T_s\sim t^{-1}E_s. \tag{25}$$

IV. THE LENGTH SCALE AND NONLINEAR TRANSFER TERMS

From Eq. (4), it follows by substitution that the energy integral can be written as

$$\frac{3}{2}u^2=[E_sL^{-1}]\int_0^\infty f(\eta)d\eta. \tag{26}$$

Since the integral is time independent,

$$E_s\sim u^2L. \tag{27}$$

It follows from Eqs. (25) and (27) that

$$T_s\sim\nu(u^2/L). \tag{28}$$

From Eqs. (21), (23), and (27), a decay law can be immediately obtained as

$$\frac{u^2}{u_0^2}\sim\left[\frac{t}{t_0}\right]^p\left[\frac{L_0}{L}\right]\sim\left[\frac{t}{t_0}\right]^{p-1/2}, \tag{29}$$

or

$$u^2\sim t^n, \tag{30}$$

where

$$n=p-\tfrac{1}{2} \tag{31}$$

and either p or n must be determined. Thus the kinetic energy also undergoes a power-law decay.

It remains to relate L to a physically observable length scale. This can be accomplished by considering the rate of dissipation of turbulence energy given by Eq. (9). In similarity variables, this becomes

$$\epsilon=[\nu E_sL^{-3}]2\int_0^\infty \eta^2 f(\eta)d\eta, \tag{32}$$

or using Eq. (27),

$$\epsilon\sim\nu u^2L^{-2}. \tag{33}$$

For isotropic turbulence

$$\epsilon=15\nu\left\langle\left[\frac{\partial u}{\partial x}\right]^2\right\rangle=15\nu\frac{u^2}{\lambda^2}, \tag{34}$$

where λ is defined to be the Taylor microscale, Batchelor.[12] By comparing Eqs. (33) and (34), the similarity length scale L is readily recognized to be the Taylor microscale λ, i.e.,

$$L\sim\lambda. \tag{35}$$

Thus, from Eq. (21),

$$\lambda^2=2A\nu t. \tag{36}$$

The coefficient A can be related to the decay law exponent n by Eqs. (8) and (34) with the result that

$$\lambda^2=-(10/n)\nu t, \tag{37}$$

which is the result obtained by von Kármán and Howarth.[3] (Note that this result does not depend on self-preservation, but only on a power-law energy decay, Batchelor.[12])

A consequence of Eqs. (35) and (28) is that the scale function for the spectral transfer is given as

$$T_s=\nu(u^2/\lambda)=R_\lambda^{-1}u^3, \tag{38}$$

where R_λ is the Reynolds number defined from the Taylor microscale and varies as

$$R_\lambda=u\lambda/\nu\sim t^{(n+1)/2} \tag{39}$$

using Eqs. (30) and (36). The constant of Eq. (28) has been chosen as unity by absorbing a factor into g with no loss of generality. The result of Eq. (38) will be seen below to represent the principal point of departure of the analysis presented here from the earlier analyses of von Kármán and Howarth[3] and Batchelor,[4] where T_s was *assumed* to equal u^3. Note that, for $n<-1$, viscous effects increase during decay, whereas for $n>-1$, they decrease. Since the latter is inconsistent with the idea of a turbulence that is decaying in the absence of an energy input, an upper bound on the decay exponent must be $n\leq-1$, with equality possible only at infinite Reynolds number where the dissipation is exactly zero. This interpretation of $n=-1$ as an infinite Reynolds number limit will be seen to play an important role in the development of the ideas presented below.

What determines the value of the coefficients and the decay exponent? Except at infinite Reynolds number and for the final period of decay which are discussed below, this question cannot be answered in general except to say that they must be determined by the initial conditions. However, since these coefficients directly enter the spectral equations, they are closely related to the shapes of the spectrum and spectral energy transfer. This can be easily seen by substituting into Eq. (17) the appropriate form of the bracketed terms. From Eqs. (22), (31), (37), and (38), it follows that

$$n(10f+g-2\eta^2 f)+5\eta f'+5f=0. \tag{40}$$

Clearly, the shapes of the functions f and g satisfying Eq. (40) will depend on the value of n. Thus turbulence generated in different ways must be expected to have a different spectral shape if its decay constant is different, and vice versa.

In summary, self-preserving solutions to the spectral energy equations are possible and have the following characteristics:

(i) The characteristic length scale for the entire spectrum is the Taylor microscale λ.

(ii) The Taylor microscale increases as the square root of time, i.e., $\lambda \sim t^{1/2}$.

(iii) The spectrum and spectral transfer functions collapse when plotted as $E(k,t)/u^2\lambda$ vs $k\lambda$ and $\lambda T(k,t)/\nu u^2$ vs $k\lambda$.

(iv) The energy undergoes a power-law decay, i.e., $u^2 \sim t^n$, where n is a constant.

(v) The turbulence Reynolds number characterizing the motion is R_λ.

(vi) The constants of proportionality and exponents are imposed externally by the initial conditions, and should be expected to vary from flow to flow.

It is important to note that there is nothing in the theoretical development to this point (or subsequently) that suggests the possibility of a single universal self-preserving state.

V. THE VELOCITY DERIVATIVE SKEWNESS AND THE INERTIAL TERMS

The velocity derivative skewness is defined as

$$S \equiv -\frac{\langle(\partial u/\partial x)^3\rangle}{\langle(\partial u/\partial x)^2\rangle^{3/2}} \tag{41}$$

and can be related to the spectral energy transfer by (Tavoularis *et al.*[13])

$$S = -\frac{3\sqrt{30}}{14}\frac{\int_0^\infty k^2 T(k)dk}{[\int_0^\infty k^2 E(k)dk]^{3/2}}. \tag{42}$$

Thus the velocity derivative skewness is a direct measure of the importance of the inertial terms in the dynamical equations. By substituting the similarity forms of the preceding analysis, it follows that

$$S = \frac{3\sqrt{30}}{14}\frac{[\nu u^2\lambda^{-4}]\int_0^\infty \eta^2 g(\eta)d\eta}{[u^2\lambda^{-2}]^{3/2}[\int_0^\infty \eta^2 f(\eta)d\eta]^{3/2}}. \tag{43}$$

This in turn implies

$$S \sim R_\lambda^{-1}, \tag{44}$$

or

$$SR_\lambda = \text{const} = \gamma(*), \tag{45}$$

where the constant of proportionality is a function of the initial conditions. Alternately, Eq. (44) can be rewritten using Eqs. (36) and (30) as

$$S \sim t^{-(n+1)/2}. \tag{46}$$

Thus, for $n < -1$, the velocity derivative skewness *increases* throughout the decay, while for $n = 1$, it is constant.

The value of the constant $\gamma(*)$ in Eq. (45) can be related to the decay exponent n and the spectral shape by substituting for the spectral transfer using the spectral energy equation. From Eqs. (40) and (45), it follows after some manipulation that

$$\gamma(*) = SR_\lambda = \left(\frac{30}{7}\right)\left(\frac{n-1}{n}\right) - \frac{4}{35}\int_0^\infty \eta^4 f(\eta)d\eta. \tag{47}$$

Since both the decay exponent n and the spectral shape depend on the initial conditions, so must γ.

Both Eqs. (44) and (47) are important consequences of the proposed theory of self-preservation, and provide a crucial experimental test of its validity. These results can be contrasted with that of Kolmogorov,[5] which requires that the velocity derivative skewness itself be a constant (see Batchelor[12]), or the modified theory of Kolmogorov[14] in which the velocity derivative skewness increases with Reynolds number. Neither of Kolmogorov's theories suggests a separate dependence on initial conditions. It is important to note that, while Kolmogorov's theories make statements about the derivative skewness of turbulence in general, the theory presented here only argues that the skewness increases during decay (when $n < -1$) for a given set of initial conditions, and then only for isotropic turbulence! It seems likely that this increase in skewness *during decay* cannot continue indefinitely, since the steepness of the velocity gradients would increase without bound. Thus it is suggested that there exists an upper bound on the velocity derivative skewness, which when achieved, prescribes the limit of validity of the fully self-preserving region, after which another region of decay is entered that is not necessarily self-preserving. It will be argued below that this upper bound on the derivative skewness is, in fact, the Kolmogorov value that is achieved in the limit as the Reynolds number characterizing the initial conditions becomes infinite.

Note that for the special case $n = -1$, both the von Kármán–Howarth and Kolmogorov results are recovered, but with the additional possibility of dependence of the remaining coefficients on the initial conditions. It has already been argued (Ling and Wan[10] and Barenblatt and Gavrilov[9]) on empirical grounds that $n = -1$ is the appropriate limiting solution at infinite Reynolds number. Theoretical justification for this idea is provided below by arguing that the Kolmogorov similarity law must also be satisfied in this limit. A consequence of $n = -1$ is that both R_λ and S are constant throughout the decay, thus, there can be no bounds on the limit of validity of the solution, as in the case for which $n < -1$. An interesting possibility (Speziale and Bernard[15]) is that it is also this $n = -1$ solution to which the turbulence evolves after the derivative skewness achieves its maximum value and the initial self-preserving state is no longer viable.

VI. COMPARISON WITH PREVIOUS THEORIES OF SELF-PRESERVATION

The key assumption of the earlier von Kármán–Howarth analysis (see also Monin and Yaglom[16]), is the arbitrary choice, $T_s \sim u^3$, which can be contrasted with that obtained here in Eq. (38) in less arbitrary fashion. The earlier choice dictates that the energy decay as inverse time and that the turbulence Reynolds number be constant during decay. Thus both of these important results (which have presented so much difficulty to the turbulence community) follow not from the similarity analysis, but from the assumptions that went into it.

Batchelor,[4] approaching the self-preservation analysis from the correlations functions using the von Kármán–Howarth equation, in essence makes the same assumption by arguing that the triple-correlation function $k(r)$ defined by

$$\langle u^2(x)u(x+r)\rangle = u^3 k(r) \tag{48}$$

can be written in self-preserving form as

$$k(r) = \tilde{k}(r/L). \tag{49}$$

The full implications of this can be seen by writing the entire triple correlation in self-preserving form as

$$\langle u^2(x)u(x+r)\rangle = u^3 k(r) = K_s(t)\tilde{k}(r/L). \tag{50}$$

In effect, Batchelor has also arbitrarily chosen

$$K_s = u^3, \tag{51}$$

which *assumes* that the velocity triple correlation is independent of Reynolds number. A further consequence of this assumption is that a factor of R_λ is left in the equation, from which it follows (as in the von Kármán–Howarth analysis) that the Reynolds number must be a constant for self-preservation at all scales. Unlike von Kármán and Howarth before him, Batchelor then argues that this can only occur in the limit as R_λ goes to zero, the final period of decay (see below).

If an analysis of the von Kármán–Howarth equation is carried out using Eq. (50) leaving K_s to be determined by the analysis (as in the spectral analysis presented above), the result is

$$K_s \sim \nu(u^2/\lambda) \sim R_\lambda^{-1} u^3. \tag{52}$$

Thus the triple-moment term retains the Reynolds number dependence and no factor of R_λ remains in the equation. Most importantly, true self-preservation is possible at all scales of motion, independent of R_λ. Physically, the nonlinear transfer terms continually adjust themselves to maintain the relative balance of the remaining terms.

It should be noted that the solutions found in this paper do not rule out the possibility of the von Kármán–Howarth type of universal self-preservation; the difference is that the initial assumptions do not dictate them. Specifically, if for some reason u^2 decays as t^{-1}, then R_λ is a constant during decay as is the derivative skewness. Why the turbulence might decay as t^{-1} can be debated, but at least the choice does not give the illusion of having been deduced from the analysis!

It is instructive to examine why the choice $T_s \sim u^3$ ($K_s \sim u^3$) might have been made and what physics it implies. If it is assumed (as in Monin and Yaglom[16]) that, at some moment during decay, the turbulence spectrum can be completely characterized by two parameters, say u and λ, then it follows from dimensional analysis that $E_s \sim u^2\lambda$ and $T_s \sim u^3$ and a von Kármán–Howarth analysis is indicated. What kind of turbulence would be characterized only by its energy and a single length scale? Only one which is completely independent of its initial conditions, precisely the conditions previously believed to be necessary for a flow to be asymptotically self-preserving. [Asymptotic refers to times (or distances) sufficiently far removed from the generation so that not all of the details of generation are important.]

As shown by the analysis above, independence of initial conditions is not essential for a flow to achieve a self-preserving state. However, because of the dependence on initial conditions, there is no single universal solution to which all flows must be asymptotic, at least at finite Reynolds number (see Sec. VII). Unfortunately, self-preservation has often been confused with the existence of a single universal state. Clearly, such is not necessarily the case. It has recently been argued by George[6] that most turbulent shear flows relax to a self-preserving state determined by their initial conditions, and not to a single universal state as previously believed. The same possibility apparently exists for isotropic turbulence.

Before leaving this section, it is useful to note the virtually identical conclusions of Barenblatt and Gavriolov,[9] who recognized that Eq. (50) led to initial-condition-dependent solutions. Barenblatt[11] identifies these as "self-preserving solutions of the second kind," and distinguished them from the universal "self-preserving solutions of the first kind." (This distinction does not seem appropriate in light of the arguments of the next section since the von Kármán–Howarth solutions are recovered as the infinite Reynolds number limit of the solutions identified herein.) Moreover, he argues that the former have a limited range of validity and introduces the term "intermediate asymptotics" to describe them, thus, anticipating the maximum derivative skewness argument put forth earlier and explored in detail below.

VII. THE RELATION TO KOLMOGOROV'S THEORY OF LOCAL SIMILARITY

Before comparing Kolmogorov's theory for the local self-preservation of the dissipative scales of motion to that proposed here, it is worth examining the precise conditions under which it can be expected to hold. For a given set of initial conditions of isotropic turbulence, the governing equations make it clear that the spectrum at all wave numbers must be a function of the kinetic energy $3u^2/2$, the rate of dissipation ϵ, and the kinematic viscosity ν. In the limit of infinite Reynolds number, it can be argued that the spectrum at low wave numbers makes no contribution to the dissipation. Also, in this limit, the spectrum at high wave numbers contains no energy. Thus, in the limit of infinite Reynolds number, the high-wave-number spectrum must be determined by ϵ and ν (which is Kolmogorov's proposal[5]). Similarly, the spectrum at low wave numbers can only depend on u^2 and ϵ, the latter entering only because it is equal to the spectral energy flux from the low- to the high-wave-number regions (and then only at infinite Reynolds number). (This was apparently first recognized by Batchelor.[4]) It is clear from the above that, strictly speaking, neither scaling can be entirely correct at finite Reynolds numbers since the high-wave-number region will always contain some of the energy while the low-wave-number region contributes some of the dissipation, thereby invalidating the arguments on which the scaling is based. Note that it follows that the familiar assumption that $\epsilon = u^3/l$ can be only approximately true at finite Reynolds numbers if l is to taken to be a real length scale of the flow.

In view of the above, it is reasonable that the high- and low-wave-number scaling laws might be approximately valid at very large, but finite, Reynolds numbers. In other words, each might be assumed to be the leading terms in an expansion about the infinite Reynolds number limit. If so, then there must be a region where both scaling laws are valid and they can be matched. This matching in the limit of infinite Reynolds number yields the familiar $k^{-5/3}$ inertial subrange (Tennekes and Lumley[2]). Thus the existence (or nonexistence) of a $k^{-5/3}$ range in a particular experimental spectrum provides a clue as to whether or not the Kolmogorov or von Kármán–Howarth scaling laws might approximately describe the appropriate spectral regions. Certainly, if there is no $k^{-5/3}$ range, then there is little reason to believe that the assumptions for either scaling law would be even approximately satisfied.

It is clear from the above that Kolmogorov's theory is at best an approximation for turbulence at finite Reynolds number. Thus it is quite unlike the theory of self-preservation proposed here, which places no restriction on Reynolds number. Therefore it is only in the limit of infinite Reynolds number that these two theories must be compatible. This requirement for compatibility in the limit of infinite Reynolds number can, in fact, be used to determine the decay rate exponent in this limit, as will be shown below. The spectrum in Kolmogorov variables is given by

$$E(k)=\nu^{5/4}\epsilon^{1/4}\tilde{f}(k\eta_K), \tag{53}$$

where η_K is the Kolmogorov microscale defined by $\eta_K = \nu^{3/4}/\epsilon^{1/4}$. It has been generally believed that the spectrum $\tilde{f}$ is universal in that it is independent of how the turbulence is generated (see Batchelor[12] and Monin and Yaglom[16]). However, if the spectrum for isotropic turbulence, in fact, collapses in Taylor variables u and λ as required for full self-preservation, then the spectrum in Kolmogorov variables must be related to it by

$$\nu^{5/4}\epsilon^{1/4}\tilde{f}(k\eta_K)=u^2\lambda f(k\lambda,*), \tag{54}$$

so that

$$\tilde{f}(k\eta_K)=15^{-1/4}R_\lambda^{3/2}f(k\lambda,*). \tag{55}$$

Thus the Kolmogorov scaled spectrum appears to be directly dependent on R_λ, contrary to the hypothesis that it should not be. Moreover, since $f(k\lambda,*)$ is determined by the initial conditions, so must be $\tilde{f}(k\eta_K)$, and the Kolmogorov spectrum cannot, therefore, be universal, even at high wave numbers, at least at finite Reynolds numbers.

Despite the above, it is easy to show that the two theories are not necessarily incompatible. Lin[17] showed the equivalence between the von Kármán–Howarth theory and that of Kolmogorov. In particular, he showed that Kolmogorov scaling was consistent with full self-preservation of the type proposed by von Kármán and Howarth only if the energy decayed as t^{-1} and the length scale increased as $t^{1/2}$. For the special case where $n=-1$, the theory of full self-preservation presented here yields an R_λ that is constant during decay [see Eq. (39)]. When R_λ is constant, η_K/λ is also constant so that both η_K and λ evolve together, with the result that the spectra scaled in Kolmogorov variables and Taylor variables are equivalent. Thus, for the special case of $n=-1$, all three theories (including the one proposed here) are equivalent.

A heuristic argument for a decrease in the energy decay rate with increasing initial Reynolds number can be made by noting that energy is removed from a given wave number in the energy-containing range by both the nonlinear spectral transfer and by the direct action of viscosity. If the spectral transfer is relatively insensitive to the Reynolds number (as is usually assumed in spectral closure models, see Lesieur[18]), then the rate at which energy is removed from a given spectral component decreases with increasing Reynolds number since the viscous term is less. In the limit of infinite Reynolds number, no energy is removed by direct viscous action on the energy-containing scales, and the decay rate is entirely determined by the spectral transfer. It follows immediately that, if the turbulence settles into a self-preserving state in which the energy decay can be represented by a power law, then that power must vary with initial (or grid) Reynolds number and approaches an asymptotic value in the limit as this Reynolds number becomes infinite. It is interesting that this is the same limiting solution proposed by Speziale and Bernard[15] for infinite time.

The theory of full self-preservation proposed herein can only predict that there is a decay exponent, that it does not vary during decay (at least as long as the nonlinear transfer can continue to increase), and that it is determined by the initial conditions. It is entirely consistent that this dependence on initial conditions vanishes in the limit as the Reynolds number characterizing the initial conditions (usually the grid mesh Reynolds number) becomes infinite. If so, then the decay exponent must approach some asymptotic value. The applicability of the Kolmogorov theory in the limit of infinite grid Reynolds number can be interpreted *to require* that this limit be $n_\infty \to -1$ as $R_M \to \infty$, where R_M is a Reynolds number characterizing the initial turbulence (like the grid Reynolds number). It is not hard to imagine that the shape of the spectrum appropriate to this limit might also be independent of the details of the initial conditions (at least at high wave numbers). In fact, it could even be the same as for other turbulent flows. If so, all of the apparent points of conflict between the competing theories vanish and, in fact, the theories are seen to be complementary.

Before leaving this section, it is interesting to examine the behavior of the derivative skewness in the limit of infinite Reynolds number if $n \to -1$. First note from Eqs. (39) and (46) that the closer n approaches the limiting value, the slower the variation of R_λ and the less rapid the increase of the derivative skewness during decay. In the limit of infinite Reynolds number, neither R_λ nor S will vary at all during decay. By expressing the integral of Eq. (47) in terms of the Kolmogorov spectrum using Eq. (55), the resulting limiting value of the derivative skewness can be obtained as

$$S_\infty = -\frac{12}{7}\sqrt{15}\int_0^\infty \tilde{k}^4 \tilde{f}(\tilde{k})\,d\tilde{k}. \tag{56}$$

If this limiting spectrum is independent of Reynolds number (as in Kolmogorov's[5] theory), then this limit is a constant. On the other hand, if the limiting spectrum retains a Reynolds number dependence (see Champagne[19]) or a dependence on the other initial conditions (see George[6]), then the limiting value of the derivative skewness will reflect this. Thus, even though the derivative skewness increases during decay for finite Reynolds numbers, the theory proposed here can still be consistent with previous theories when the Reynolds number increases without bound.

VIII. INVARIANTS OF THE DECAY

There have been several attempts to establish theoretically the existence of integral invariants of the decay of homogeneous, isotropic turbulence. These all take the form

$$\int_0^\infty r^m B_{LL}(r)\,dr = I_m, \tag{57}$$

where I_m is the invariant and is thus presumed constant throughout the decay. The first of these was due to Loitsiansky,[20] who argued that the fourth integral moment of the longitudinal velocity correlation should be an invariant of the motion, i.e., $m=4$. Necessary conditions were the existence of the integral which required that $B_{LL}(r) \to 0$ at least as fast as r^{-5} as $r \to \infty$ and that the two-point triple correlation $B_{LL,L}(r) \to 0$ faster than r^{-4}. Subsequently, Proudman and Reid[21] and Batchelor and Proudman[22] showed that these conditions were not satisfied in general (except in the final period of decay which will be discussed in the next section). In particular, they showed that, because of the influence of the pressure coupling terms in an incompressible flow, $B_{LL,L}(r) \to r^{-4}$ as $r \to \infty$, even when the turbulence was generated at the initial instant so that the velocity correlations rolled off exponentially.

Saffman[23] considered a field of turbulence generated at an initial instant by a distribution of random impulsive forces. Such a distribution was shown to correspond at the initial instant to a turbulent field for which all vorticity correlations roll off exponentially with r. By a series of arguments paralleling those of Batchelor and Proudman, Saffman was able to show that the second integral moment of the velocity correlation was an invariant of the motion, i.e., $m=2$.

The interrelation of self-preservation and the existence of an integral invariant have been realized from the beginning (see Batchelor[4] and Saffman[24]). Unfortunately, none of the measured decay rates, except in the so-called "final period of decay" (see discussion below), were consistent with both self-preservation and the proposed invariants. However, as pointed out by Monin and Yaglom,[16] there is no reason to believe that idealized models based on assumed behavior of turbulence in an infinite environment should be able to describe real turbulence, particularly that behind a grid in a wind tunnel.

It has already been established in the preceding section that the turbulence behind a grid might be self-preserving at all scales. Moreover, this self-preservation has been shown to be consistent with the spectral energy equation for the flow. Therefore it makes sense to reverse the usual question and ask: What integral invariant of the decay corresponds to the observed self-preserving state of the turbulence? Since, from self-preservation, if $\xi = r/\lambda$,

$$B_{LL}(r) = u^2 f_L(\xi), \tag{58}$$

it follows that

$$[\lambda^{m+1}u^2]\int_0^\infty (\xi^m) f_L(\xi)\,d\xi = I_m. \tag{59}$$

The integral over ξ is independent of time, and therefore constant throughout the decay. Hence the entire integral is an invariant only if

$$[\lambda^{m+1}u^2] \sim \text{const}. \tag{60}$$

From Eqs. (30) and (36), it follows immediately that

$$t^{(m+1)/2} t^n \sim \text{const}, \tag{61}$$

which implies that

$$m = -2n - 1. \tag{62}$$

Thus the Loitsiansky and Saffman invariants correspond to energy decay rates of $n = -5/2$ and $n = -3/2$, respectively. For the von Kármán–Howarth value of $n = -1$, the corresponding integral invariant has $m=1$, which, if the arguments of the preceding section are correct, should characterize only a hypothetical isotropic decaying turbulence at infinite Reynolds number. It is important to note that there is no reason to expect that either m or n will be universal constants and independent of the initial conditions, except possibly in the limit of infinite "grid" Reynolds number. In this same context, it will be interesting to see whether an idealized model of the turbulence at the initial instant can be shown to be consistent with the fractional powers that characterize most experiments. Such a model would probably recognize the unique vortical character of grid-generated turbulence, and would most probably attribute the differences behind geometrically similar grids to the changes in these vortical structures with Reynolds number.

IX. A $t^{-5/2}$ REGION AND THE FINAL PERIOD OF DECAY

During the final period of decay or at low-turbulence Reynolds numbers, an additional constraint can be added to the spectral equations considered earlier; in particular, the constancy of the Loitsiansky integral. Batchelor and Proudman[22] argue that, while the Loitsiansky integral is not an invariant of the decay (as originally proposed), it is at least constant during the final period of decay where the nonlinear terms are negligible. As noted above, the constancy of the Loitsiansky integral determines the energy decay exponent so that $u^2 \sim t^{-5/2}$.

Thus the $t^{-5/2}$ decay law is not new having been previously derived theoretically and confirmed experimentally (Batchelor and Townsend[25] and Bennett and Corrsin[26]). (Batchelor[4] even used similar arguments.) *What is important here is that it has been derived without assuming the negligibility of the spectral transfer terms (or inertial terms).* By contrast, the widely accepted analysis (Batchelor and Townsend[25] and Batchelor[12]) was based on linearized equations of motion. (Note that Bernard[27] recently has studied the behavior of the nonlinear Sedov[28] solution in the final period of decay, thus anticipating in part the result here.) This difference is important since Bennett and Corrsin,[26] while confirming experimentally the predictions of the theory with regard to the $t^{-5/2}$ law and $\lambda^2 \sim t$, showed that the inertial terms (as measured by the velocity derivative skewness) were certainly not negligible and, in fact, increased in importance throughout the decay. This trend does not reverse to yield the roll off expected from the linearized analyses until $R_\lambda < 2$ (Tavoularis *et al.*[14]). Thus the analysis here would appear to have resolved the paradox by eliminating the necessity of neglecting the inertial terms.

There remains, however, another paradox. In arguing that the Loitsiansky integral was constant during the final period of decay, Batchelor and Proudman[22] resorted to the linearized dynamical equations to argue that viscosity caused the correlation function to roll off exponentially, thereby ensuring the constancy of the integral. The apparent consistency of the theory presented here and its agreement with the experimental observations would argue for the constancy of the Loitsiansky integral for at least the initial conditions of the experiment cited, independent of assumptions about the negligibility of the inertial terms. What is needed is an explanation as to how viscosity modifies the tails of the correlation function as the Reynolds number is reduced, without requiring that the inertial terms vanish. It might be noted that this same behavior also appears in the eddy-damped quasinormal Markovian (EDQNM) turbulence model, which yields a constant Loitsiansky integral for any initial spectrum rising from zero wave number at least as fast as k^4 (see Lesieur[18]). It would thus appear that it is impossible to have a turbulence that decays faster than $n = -5/2$.

From the above, it appears that the fact that a turbulence decays as $t^{-5/2}$ by no means uniquely characterizes it as the final period of decay, but may instead be only a function of the manner in which the turbulence is generated. The real final period of decay is reached only when the Reynolds number becomes sufficiently low (of order unity) that the inertial terms are truly negligible. It is interesting to note in this context that, if the arguments about turbulence at infinite Reynolds number being characterized by the von Kármán–Howarth similarity theory are correct, then this final period of decay can never be reached because R_λ is constant throughout the decay.

X. THE RELATION OF THE EXPERIMENTS TO DECAYING ISOTROPIC TURBULENCE

Until recently, almost all of the attempts to simulate the turbulence described by Eq. (1) have (following the example of Taylor[29]) utilized a grid in a wind tunnel. In such experiments, the wakes generated by obstructions (most often a biplane grid of round or square bars) in a uniform airstream merge to form a turbulent flow that is approximately homogeneous in planes perpendicular to the direction of the mean flow. Since there are no mean velocity gradients, the turbulence can only decay and the spatial variation of its kinetic energy is described by

$$U\frac{\partial\langle q^2\rangle/2}{\partial x}=-\frac{\partial}{\partial x}\left[\langle pu\rangle+\frac{1}{2}\langle q^2u\rangle-2\nu\langle u_i e_{i1}\rangle\right]-\epsilon, \quad (63)$$

where e_{ij} is the fluctuating strain rate. The terms in brackets on the right- hand side are of third order in u/U. Since $u/U \sim 10^{-2}$ typically, they can be neglected with respect to the others. If we move with the flow, $U\partial/\partial x=\partial/\partial t$, and Eq. (63) can be seen to be the equivalent of Eq. (8) for temporally decaying turbulence. Thus the distance behind the grid x is the equivalent of the time t, i.e., $t=x/U$.

Generally, the turbulence behind a grid tends to be anisotropic with the streamwise component slightly larger than the cross-stream components. Comte-Bellot and Corrsin[30] were able to generate turbulence very close to an isotropic state by introducing a slight contraction after the grid. However, in most of the investigations, all of the experimentally determined second-order quantities were in reasonable agreement with the isotropic relations even with the slight anisotropy, as long as the distance behind the grid was sufficient (typically $x/M > 40$, where M is the grid mesh length).

The preceding considerations alone do not ensure that the spectral equations for grid-generated wind-tunnel turbulence are the same as for temporally decaying turbulence. However, since the flow is homogeneous in planes perpendicular to the mean flow direction and varies only slowly in the streamwise direction, it is reasonable to suppose the turbulence to behave as homogeneous turbulence in an infinite environment, *at least for scales of motion much smaller than the size of the tunnel dimensions.* Thus spectral and correlation measurements in such flows should be expected to satisfy those for homogeneous (and in most cases isotropic) turbulence for all but the largest scales of motion. Because of the difficulty in making sufficient measurements in space to compute wave-number spectra, Taylor[29] introduced the idea of a frozen field for which time variations at a fixed point could be treated as spatial variations moving by the probe. For such a frozen field the temporal correlations measured at a fixed point $C_{i,j}(\tau)$ are related to the spatial correlations $B_{i,j}(\mathbf{r})$ by

$$B_{i,j}(U\tau,0,0)=C_{i,j}(\tau), \tag{64}$$

where τ is the time delay and $\mathbf{r}=(r,0,0)$ is the streamwise separation and they are related by $r=U\tau$. The one-dimensional wave-number spectrum $F_{i,j}(k_1)$ and the frequency spectrum $S_{i,j}(f)$ measured by a fixed probe can be similarly related, i.e.,

$$F_{i,j}(k_1)=(U/2\pi)S_{i,j}(Uk_1/2\pi), \tag{65}$$

where f is the circular frequency and k_1 is given by $k_1=2\pi f/U$.

In the following sections, the wealth of experimental data on grid-generated turbulence acquired over the past 50 years will be examined. The major difficulty that will be encountered is the limited range of variation of R_λ *for a particular set of initial conditions.* (Note that this is quite different from the usual lament of turbulence theoreticians that the Reynolds number is not high enough.) Previous investigators appear to have believed that the character of turbulence behind a grid should be independent of its initial conditions. As a consequence, most of the Reynolds number variation reported in the literature has been obtained by changing the initial conditions either by varying the velocity or changing the grid mesh length. These experiments (which unfortunately constitute the bulk of those performed) are of little use in evaluating the theory proposed herein.

Only a few of the experiments in the literature report detailed measurements over a wide enough range of decay times (distances downstream) for R_λ to have varied significantly. The problem is that R_λ depends only weakly on distance from the grid. From Eq. (39), it follows that

$$R_\lambda \sim t^{(n+1)/2} \sim x^{(n+1)/2}. \tag{66}$$

For the experiments considered below, $n \sim -1.2$ so that $R_\lambda \sim x^{-0.1}$. Thus a factor of 10 variation in x/M is necessary for even a 20% variation in R_λ! Most of the early experiments (Batchelor and Townsend[31] and Mills *et al.*,[32] for example) only report data for $20<x/M<70$. A few (Uberoi,[33] Frenkiel and Klebanoff,[34] and Wahrhaft and Lumley[35]) report data to about $x/M=110$–170. Only Comte-Bellot and Corrsin[30,36] report streamwise variations even approaching an order of magnitude $45<x/M<385$. Particular attention will be paid to these experiments for which the spectral and decay data have been conveniently tabulated, and the data can be shown to be internally consistent.

A related problem with the bulk of the experimental data is that few of the experiments report measurements of either the velocity derivative skewness or the two-point triple-velocity correlation. Thus there is no basis for deciding whether these terms are still increasing, or are remaining nearly constant. Since the product of derivative skewness and R_λ can be expected to be constant only while the former has not reached a maximum value, there is no basis for deciding whether the theory should be expected to describe the data or not if the derivative skewness measurements are absent. The experiments discussed below all either present this information, or provide the means to obtain it.

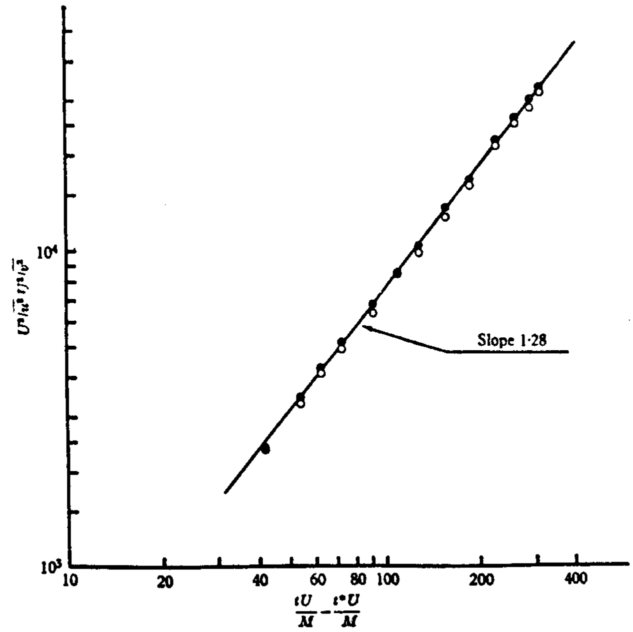

FIG. 1. Turbulence intensity decay in a typical case after a contraction at 18 M; square-rod grid with $M=2.54$ cm and $U=20$ m/sec. O: $U^2\sqrt{\overline{u^2}}$ with $t_*U/M=4$; +: $U^2\sqrt{\overline{v^2}}$ with $t_*U/M=4$ (from Comte-Bellot and Corrsin[17]).

XI. THE TURBULENCE INTENSITY VARIATION BEHIND THE GRID

The variation of the turbulence intensity with distance downstream of a grid has been investigated many times since the experiments reported by Simmons and Salter[37] and Dryden *et al.*[38] These early experiments seemed consistent with the von Kármán–Howarth predictions that $u^2 \sim t^{-1}$ or $U^2/u^2 \sim (x-x_0)/M$, where x_0 represents a virtual origin. However, subsequent studies by Corrsin and co-workers indicated that a better fit to the experimental data could be obtained by a relation of the form

$$\frac{U^2}{\overline{u^2}}=A_1\left[\frac{x-x_0}{M}\right]^{-n}, \tag{67}$$

where the coefficient A_1 and the exponent n appeared to depend on the particular geometry and Reynolds number of the grid. The most comprehensive of these studies is due to Comte-Bellot and Corrsin[30] from which Fig. 1 is taken.

So well established is Eq. (67) on empirical grounds that few would doubt its validity, nor that the constants depend on the initial conditions. Thus the turbulence intensity measurements would appear to strongly support this prediction of the proposed theory. (The recent contributions of Mohammed and LaRue,[39] which might appear to dispute this, are discussed later.) It should be noted that it has been previously recognized [Eq. (40)] that there must be a relation between the spectral shape (especially near $k=0$) and the decay exponent (see Lesieur[18]).

A second aspect of the turbulence intensity variation concerns the possible dependence of the exponent n on *grid* Reynolds number $R_M = UM/\nu$ and whether or not $n \to -1$ in the limit as $R_M \to \infty$. From the definitions of R_M and R_λ and Eq. (67), the coefficient A_1 can be obtained as

$$A_1 = -\frac{10}{n}\frac{R_M}{R_\lambda^2}\left[\frac{Ut}{M}\right]^{n+1}. \tag{68}$$

(Note that the explicit time dependence is canceled by the time variation of R_λ.) If $n \to -1$ in the limit as R_M and $R_\lambda \to \infty$, then the limiting value of A_1 is given by

$$A_{1\infty} = 10\left[\frac{R_M}{R_\lambda^2}\right]. \tag{69}$$

Thus, in this limit, R_M/R_λ^2 is a constant and uniquely determines the decay, and can, at most, be a function of the grid geometry. This relation is generally consistent with the experimental data of Comte-Bellot and Corrsin.[30] It was introduced by Batchelor and Townsend[25] on empirical grounds, but follows directly from the theoretical arguments presented herein.

From the discussions in Sec. VII, it is clear that whether $n \to -1$ as $R_M \to \infty$ is closely related to the question of the applicability of Kolmogorov's theory to grid turbulence. However, it has long been recognized that the Kolmogorov theory applies only beyond the Reynolds number range of most grid turbulence experiments (cf. Stewart and Townsend[40]). In fact, only two experiments (Kistler and Vrebalovich[41] and Schedvin *et al.*[42]) were carried out at high-enough grid Reynolds numbers to even begin to observe the $k^{-5/3}$ range expected for high Reynolds number turbulence. Both of these experiments reported best-fit decay laws for which $n=-1$. Consistent with this is that there is some evidence (most notably the square bar data of Comte-Bellot and Corrsin[30] and the experiments of Ling and Wan[10]) that the decay exponent increases from $n<-1$ toward $n=-1$ with increasing grid Reynolds number. Figure 2 shows the variation of exponent with grid Reynolds number for the square bar grid data of Comte-Bellot and Corrsin.[30] Also plotted is the exponent determined by Kistler and Vrebalovich[41] at very high Reynolds numbers using a round bar grid with the same solidity ($\sigma=0.34$). Even without this latter point, the trend is clearly downward, and is especially apparent when each of the three grids used is considered independently.

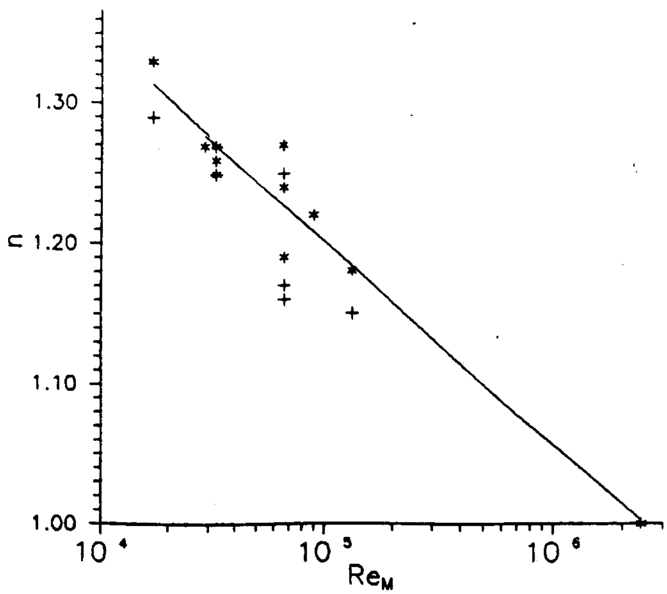

FIG. 2. Variation of n with R_M, square-bar grid for $U^2\sqrt{u^2}$ data of Comte-Bellot and Corrsin[17] (✱: no contraction; +: with contraction).

XII. THE LENGTH SCALES

Von Kármán and Howarth[3] showed for isotropic turbulence that, if the turbulence decayed as a power law in time, then the Taylor microscale would increase as the square root of time. In view of the consensus reported above for the turbulence intensities, it is no surprise that all of the experiments beginning with that of Dryden[38] are consistent with Eq. (37).

Figure 3 shows the variation of the square of the Taylor microscale with Ut/M for the two grids of Comte-Bellot and Corrsin.[36] Both show the expected dependence of λ^2 as confirmed by the linear regression curve fits. From the tabulated data, the ratio $\lambda^2/\nu t$ can be computed and found to be 8.24 and 8.20 for the 25.4 mm and 50.8 mm grids, respectively. Since, from Eq. (37), the coefficient should be $-10/n$, where n is the exponent for the energy decay, $-n$ can be computed as 1.20 and 1.21 for the two grids, respectively. These values are very close to the value of 1.25 suggested by the authors from direct plots of the intensity data, the difference corresponding, in part, to the choices of virtual origin: $Ut_0/M = 3.5$ for both grids by the authors from the turbulence intensity measurements, $Ut_0/M = 8.95$ and 4.54 for the 25.4 mm and 50.8 mm grids, respectively, here. Note that the theory requires that the virtual origin for all statistical quantities from a single

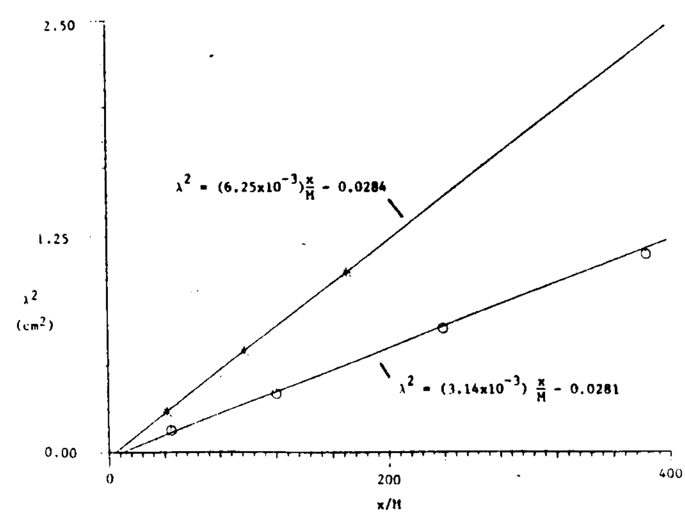

FIG. 3. Taylor microscale versus downstream distance (data of Comte-Bellot and Corrsin[23]). (O: 25.4 mm grid; ✱: 50.8 mm grid.)

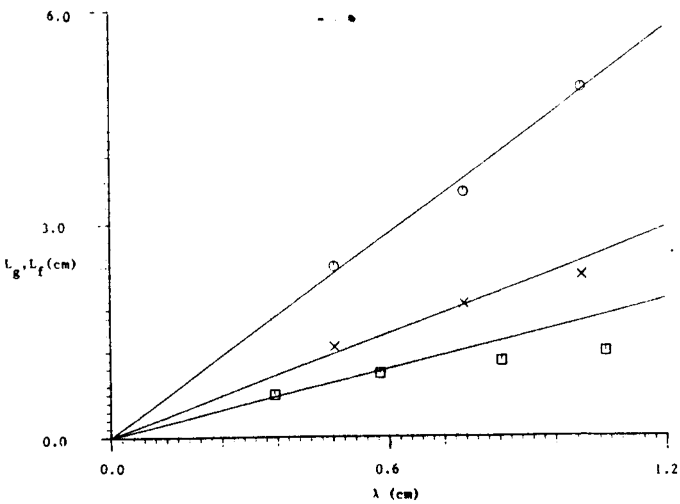

FIG. 4. Integral scale versus Taylor microscale (data of Comte-Bellot and Corrsin[23]) (50.8 mm grid, ×: lateral, ○: longitudinal; 25.4 mm grid, □: lateral.

experiment be the same, which has not always been the practice of experimenters.

Another test of the theory here is whether or not the integral scales are directly proportional to the Taylor microscale. In the experiments of Comte-Bellot and Corrsin,[36] the transverse integral scale was computed by integrating the transverse velocity correlation to the point where it crossed the r axis. The longitudinal integral scale was obtained from the measured one-dimensional spectrum in the limit of zero wave number. Neither of these procedures is particularly satisfactory, as pointed out by the authors, because of the high-pass filtering of the data by ac coupling, and the difficulties in carrying out the extrapolations. In addition, there must always be some concern as to whether the largest scales of motion reasonably approximate an isotropic turbulence in an infinite environment. The results are, however, in general agreement with prediction as illustrated in Fig. 4, which plots the measured integral scales versus the measured Taylor microscale for the two grids at various x/M. It is suggested that the values at larger x/M are most susceptible to the problems mentioned above (since the spectrum progressively shifts to lower frequencies and larger scales), and are therefore the least reliable.

XIII. THE SPECTRAL SCALING

The principal difficulty in verifying the spectral predictions of the theory is that, despite the numerous experiments in grid turbulence over the past 50 years, there exist only a few sets of experimental data that are both well tabulated and over a wide range of downstream positions *for fixed initial conditions*. The exceptions to this are the experiments of Comte-Bellot and Corrsin[36] for which the spectral and decay data have been conveniently tabulated. Therefore these experiments will be examined in detail first, then other experimental evidence will be considered.

Before carrying out this examination of the spectral data, however, it is useful to review what comparisons will

TABLE I. Dependence of spectral scaling on x/M for $n=-1.2$.

Scaling	Wave number	Spectrum
Taylor	$k_1\lambda \sim k_1[x/M]^{0.5}$	$E_{11}/u^2\lambda \sim [x/M]^{0.7}E_{11}$
von Kármán/ Howarth	$k_1 l \sim k_1[x/M]^{0.4}$	$E_{11}/u^2 l \sim [x/M]^{0.8}E_{11}$
Kolmogorov	$k_1\eta_K \sim k_1[x/M]^{0.55}$	$E_{11}/\nu^{5/4}\epsilon^{1/4} \sim [x/M]^{0.55}E_{11}$

be made and what kind of agreement (or disagreement) with other theories might be expected. Since the data are usually presented as a function of position behind the grid x/M, it is interesting to note how the various scaling parameters vary with x/M. Three types of scaling are of interest: the Taylor scaling proposed here, the von Kármán–Howarth scaling, and the Kolmogorov scaling. The appropriate spectral scalings are summarized in Table I.

Of particular interest will be the degree to which the spectral data at different x/M can be collapsed, and especially the range of wave numbers for which the scaling appears to be valid. Since the von Kármán–Howarth scaling is proposed for only the energy-containing range, it should not successfully collapse the high-wave-number region (since the Reynolds number also varies with distance from the grid). The Kolmogorov scaling, on the other hand, should collapse only the high-wave-number region and not the low since it is based on the local similarity of the smallest scales of motion. The Taylor scaling proposed here must be expected to collapse the entire spectral range from lowest to highest wave numbers if the theory is correct.

This uniformity of collapse over all scales for the proposed theory will be seen to be the most useful test of the proposed theory's validity. The reasons for this can be seen by examining how the various scaling parameters vary with distance from the grid. All measurements can be shown to be consistent with a power-law decay for the energy and a square-root growth of the Taylor microscale, and to be adequately described by the frozen field hypothesis. The rate of dissipation can thus be shown to be proportional to

$$\epsilon \sim \left[\frac{x-x_0}{M}\right]^{n-1}. \tag{70}$$

It follows immediately that the length scale of the von Kármán–Howarth theory is given by

$$l \equiv \frac{u^3}{\epsilon} \sim \left[\frac{x-x_0}{M}\right]^{n/2} \tag{71}$$

and the Kolmogorov microscale is given by

$$\eta_K \equiv \left[\frac{\nu}{\epsilon}\right]^{1/4} \sim \left[\frac{x-x_0}{M}\right]^{(1-n)/4}. \tag{72}$$

The magnitude of the differences that can be expected can be estimated by taking a nominal value for the energy decay exponent as $n=-1.2$ for which the three length scales vary as the 0.4, 0.55, and 0.50 power of

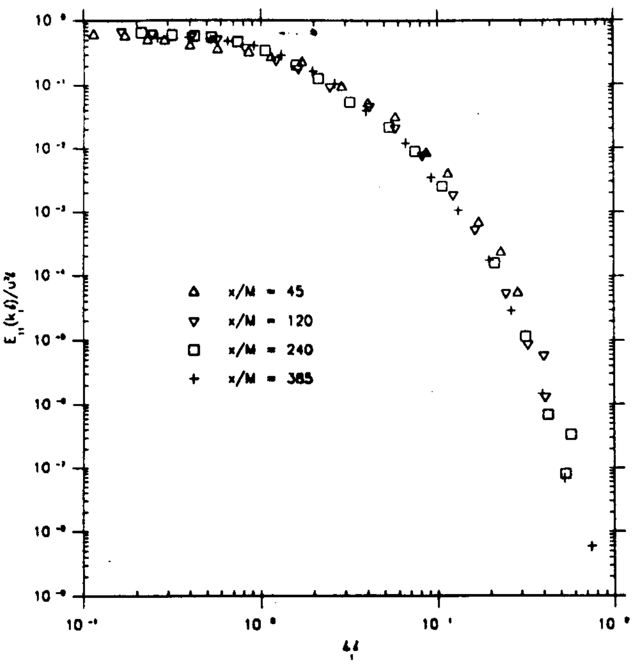

FIG. 5. Spectra from 25.4 mm grid normalized in von Kármán–Howarth (energy) variables (data of Comte-Bellot and Corrsin[36]). (Δ: $x/m=45$; ∇: 120; —: 240; +: 385.)

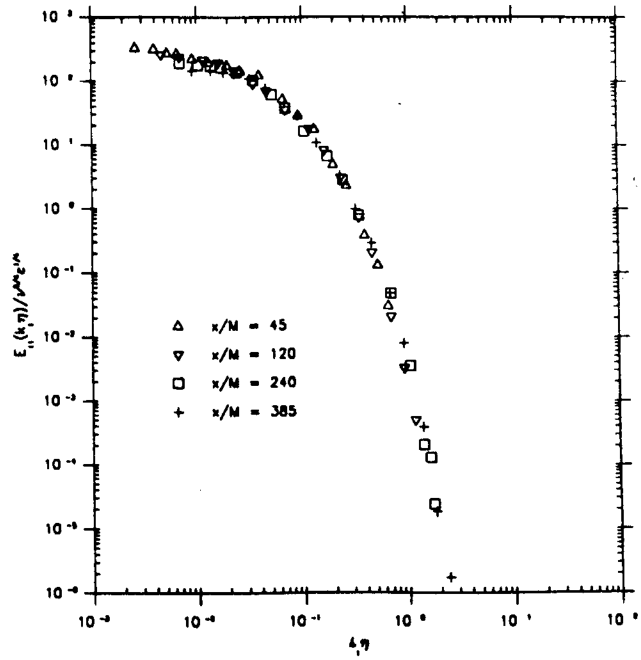

FIG. 6. Spectra from 25.4 mm grid normalized in Kolmogorov variables (data of Comte-Bellot and Corrsin[23]). (Δ: $x/m=45$; ∇: 120; —: 240; +: 385.)

$(x-x_0)/M$, respectively. The implications of these slight differences in the length scale variation on the spectral scaling are summarized in Table I. Since x/M varies by less than a factor of 10 in all the experiments, the difference between the Taylor scaling and the others will be less than 25%. Thus it will be most difficult to sort out these differences from just the quality of the collapse. The uniformity of the collapse required by only the theory proposed herein is, therefore, an important and distinguishing feature.

The two sets of experimental data to be considered from Comte-Bellot and Corrsin[36] were obtained in a 1.0 ×1.3 m wind tunnel at an airspeed of 10 m/sec. Two grids of 25.4 and 50.8 mm mesh size were used corresponding to grid Reynolds numbers of 17 000 and 34 000, respectively. The one-dimensional spectrum was measured for the 25.4 mm grid at downstream positions corresponding to Ut/M of 45, 120, 240, and 385, and for the 50.8 mm grid measurements were reported at 42, 98, and 171. The spectral data are tabulated by the authors as $E_{11}(k_1)$ vs k_1 in Table 2 and the relevant data on scales in Table 4 of their paper. [Note that the authors also computed the spectrum $E(k)$ by graphically differentiating $E_{11}(k_1)$ and using the isotropic relationship relating them. Thus these data contain no more information than already present in the one-dimensional spectra, and, in fact, are not as reliable as them because of the procedures employed to derive them. Hence the comparisons below utilize only E_{11}.]

Figures 5–7 show plots of the logarithm of the spectrum versus the logarithm of wave number using the three nondimensionalizations for the 25.4 mm grid (for which x/M variation is the greatest). The spectra plotted in von Kármán–Howarth variables (Fig. 5) collapse equally well over the entire range of wave numbers (especially if the position closest to the grid is ignored), and, in fact, better at the high wave numbers than the Kolmogorov scaling (Fig. 6). This is particularly curious since the Kolmogorov scaling collapses the data also almost as well at the lowest wave numbers as at the highest. Both of these observations are contrary to previous expectations (see, for example, Batchelor[12]).

The Taylor variable scaling (Fig. 7) works well over

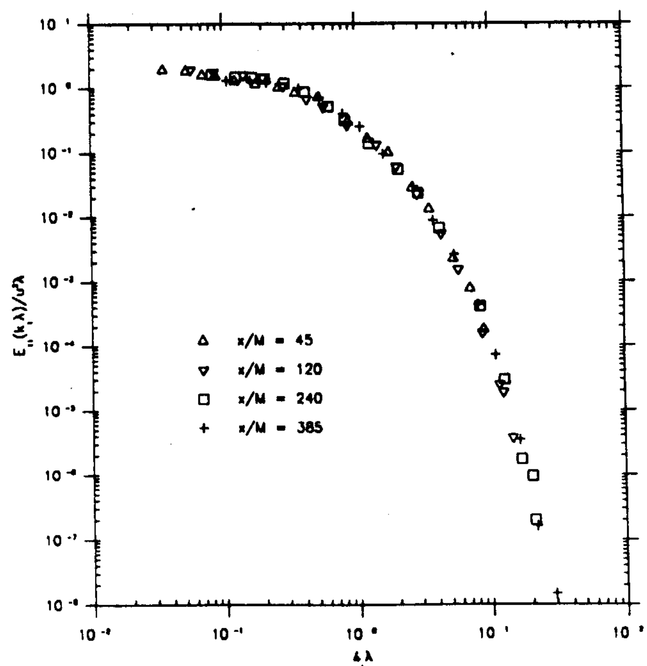

FIG. 7. Spectra from 25.4 mm grid normalized in Taylor variables (data of Comte-Bellot and Corrsin[23]). (Δ: $x/m=45$; ∇: 120; —: 240; +: 385.)

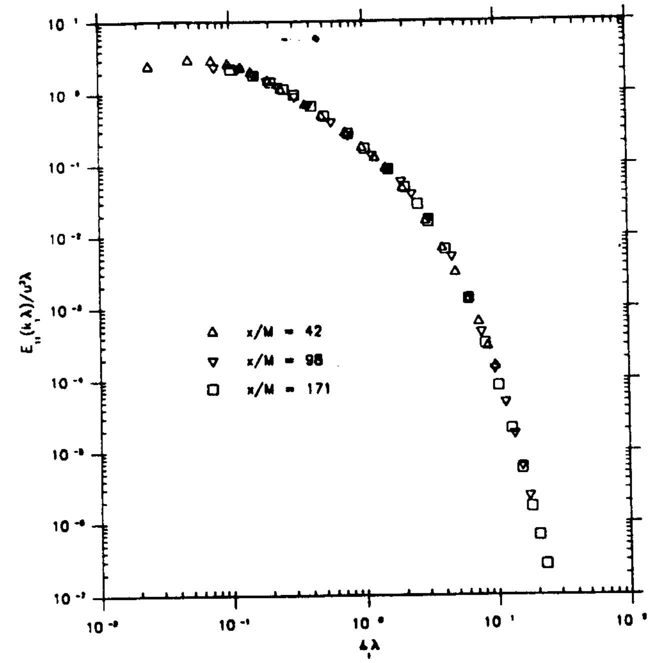

FIG. 8. Spectra from 50.8 mm grid normalized in Taylor variables (data of Comte-Bellot and Corrsin[23]). (Δ: $x/m=42$; ∇: 98; —: 171.)

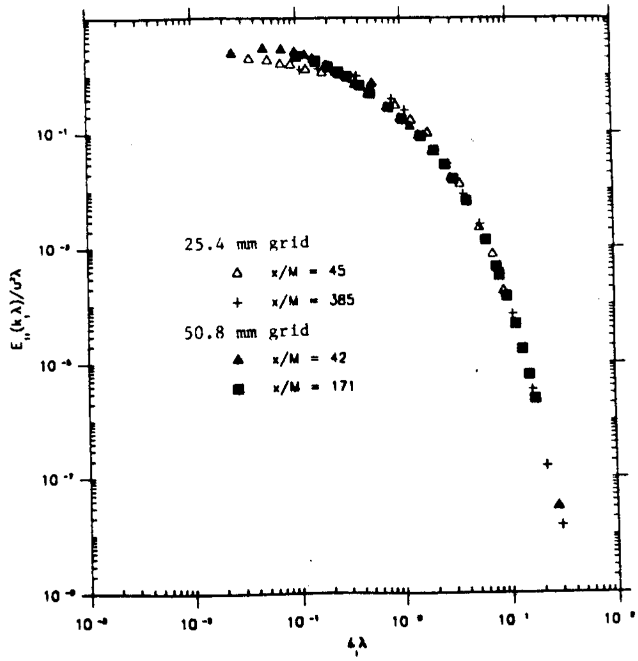

FIG. 9. Spectra from both grids plotted in Taylor variables showing different spectral shape for each grid (data of Comte-Bellot and Corrsin[23]). (25.4 mm grid, Δ: $x/m=45$, ∇: 385; 50.8 mm grid, —: 42, +: 171.)

the entire range of wave numbers from well below l^{-1} to above $3\eta_K^{-1}$. This is as predicted from the theory presented herein. Figure 8 shows the spectral data from the 50.8 mm grid plotted in Taylor variables, and the same collapse over all wave numbers is indicated. Even without the relative comparisons among the three scalings, the collapse in Taylor variables over the entire spectral range of nearly nine decades and three decades in wave number is quite spectacular.

Figure 9 shows the spectra from both grids plotted together. It is clear that each experiment has its unique spectral shape. This is also consistent with the theoretical interpretation provided earlier, which emphasized the dependence on initial conditions.

Prior to the experiments of Comte-Bellot and Corrsin, there were numerous efforts over the years to establish experimentally the self-preserving character of turbulence decay behind a grid. Particularly noteworthy were the results of Stewart and Townsend[40] and Uberoi.[33] Monin and Yaglom[16] provide an excellent review of these efforts. Despite the lack of a theory of self-preservation that utilized the turbulence intensity and Taylor microscale (except for the final period of decay), these investigators did plot some of their spectral and two-point correlation results using u^2 and λ. Unfortunately, often the data from experiments at different initial conditions (especially grid mesh Reynolds number) were plotted on the same piot. As remarked earlier and demonstrated above, turbulence behind different grids or even similar grids at different Reynolds numbers is not universal. Therefore some of the scatter in these early experiments can be attributed to the differing shapes associated with different initial conditions.

Stewart and Townsend[40] show plots of both the two-point velocity correlation and the one-dimensional spectrum normalized by u^2 and λ. A cursory study of their results would appear to indicate the scaling to be less successful than for the later Comte-Bellot and Corrsin experiments. The reasons for this can largely be attributed to three causes. First, the plots include data from as close to the grid as $x/M=10$ whereas there is now a general consensus that a self-preserving flow cannot be achieved before $x/M=40$. Second, the low turbulence intensities behind the grid and the broadband nature of the signals seriously stretched the limits of 1940–50s electronic technology, especially when the multiplication of signals was involved. When these factors are taken into account, the Stewart and Townsend measurements are not inconsistent with the proposed scaling, and had the present analysis been available, might have been offered in support of it. Uberoi[33] presents measurements at three positions ($x/M=48$, 72, and 110) behind a grid at a single-grid Reynolds number. The decay measurements are in relatively close agreement to those of Comte-Bellot and Corrsin. However, Uberoi provides extensive plots of the one-dimensional spectrum normalized by the turbulence intensity and the Taylor microscale calculated from the measured decay rate, and by the turbulence intensity and the integral scale (determined from the spectral intercept). Both of these scalings (and especially the former) are successful over the entire spectral range. This is, of course, the expected result since the integral scale and Taylor microscale should be (from the analysis presented herein), and are, proportional.

In summary, the proposed Taylor variable scaling is consistent with the measured spectral data. Aside from the inferences that can be made regarding the quality of the collapse at high and low wave numbers, it will not be pos-

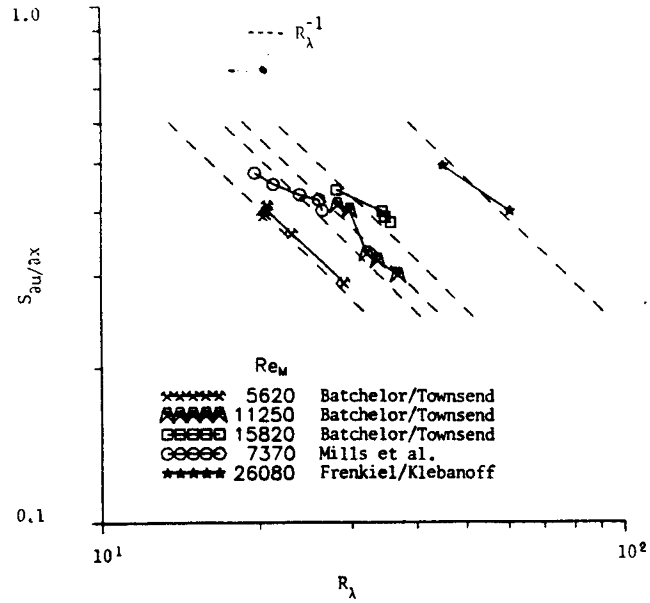

FIG. 10. Velocity derivative skewness versus Reynolds number.

sible to further distinguish between the Kolmogorov and Taylor scalings until experiments are carried out over a wider range of decay parameters.

XIV. THE VELOCITY DERIVATIVE SKEWNESS, THE SPECTRAL TRANSFER, AND THE TRIPLE-VELOCITY CORRELATION

The first measurements of the velocity derivative skewness were due to Batchelor and Townsend.[31] Subsequently, there have been only a few attempts to document its variation with increasing distance behind a grid. There has been no shortage of theories, however.

The Kolmogorov[5] theory for the universality of the small-scale motions implies that the velocity derivative skewness in high Reynolds number turbulent flows should be constant. The modified Kolmogorov theory (Kolmogorov[14]), which attempts to account for the internal intermittency of turbulent flows, and the phenomenological theories of Corrsin[43] and Tennekes[44] imply that the skewness should increase with Reynolds number. While there is evidence that this is the case for turbulent shear flows, particularly in the atmosphere, there is no support for these theories from the grid turbulence experiments for fixed initial conditions. For example, Mills *et al.*,[32] Frenkiel and Klebanoff,[34] and Bennett and Corrsin[26] all state *explicitly* that, in their experiments, the velocity-derivative skewness *increases with increasing distance from the grid* (or with decreasing R_λ). This is consistent with Eq. (46) since $n < -1$ for all these experiments.

Figure 10 summarizes the velocity-derivative skewness data as a plot of S vs R_λ. (Note that this plot is an expanded version of that provided by Tavoularis *et al.*[13] but with the data for fixed initial conditions identified as such.) The data of Bennett and Corrsin[26] at very low-grid Reynolds numbers are not included, but show the same trend as the other experiments. While the scatter is significant and the Reynolds number variation is limited (because of the limited distances from the grid as discussed earlier), the data are in reasonable agreement with the prediction of Eq. (44) that $S \sim R_\lambda^{-1}$. Also, clearly evident is the dependence of the constant $SR_\lambda = \gamma(*)$ on the initial conditions. This was first noticed by Batchelor and Townsend,[31] but its significance was apparently not understood, perhaps because of the small variation of both R_λ and S in their experiments.

Other features of the derivative skewness can also be seen in Fig. 10. The first is that the derivative skewness has an apparent maximum value, consistent with the arguments of Secs. V and VII that it should. Whether this is a consequence of the limited extent of the tunnel, or actually reflects a real limiting value needs further investigation. The second feature is the diminishing range of variation of both R_λ and S for the higher Reynolds number experiments. This is consistent with the postulated increase in the decay exponent toward $n = -1$ for which all of the data at the highest Reynolds numbers would be expected to converge to a single line given by the limiting value of derivative skewness of Eq. (56).

The relation between the velocity derivative skewness and the spectral transfer has already been noted in Eq. (42). By substituting for $T(k,t)$ from Eq. (1), we can relate the velocity derivative skewness directly to the moments of the one-dimensional energy spectrum by

$$S = S_1 + S_2, \tag{73}$$

where S_1 is defined by

$$S_1 = \frac{-\frac{3}{7}(d/dt)\int_0^\infty k_1^2 E_{11}(k_1,t)dk_1}{[\int_0^\infty k_1^2 E_{11}(k_1,t)dk_1]^{3/2}} \tag{74}$$

and S_2 by

$$S_2 = \frac{-2\nu\int_0^\infty k_1^4 E_{11}(k_1,t)dk_1}{[\int_0^\infty k_1^2 E_{11}(k_1,t)dk_1]^{3/2}}. \tag{75}$$

It is easy to see from Eq. (74) that S_1 is simply related to the time rate of change of the dissipation, i.e.,

$$S_1 = \frac{-(3/7)(d/dt)[\epsilon/15\nu]}{[\epsilon/15\nu]^{3/2}}. \tag{76}$$

But for the power-law decay of Eq. (67),

$$\epsilon = -\frac{3}{2}\frac{U^3}{M}\frac{d}{dx/M}\left[\frac{u^2}{U^2}\right], \tag{77}$$

since for the wind tunnel experiments $d/dt = Ud/dx$. It follows after some algebra that

$$S_1 = \tfrac{30}{7}[(n-1)/n]R_\lambda^{-1}, \tag{78}$$

which is exactly the first term of Eq. (47). Thus the fact that the energy decay is described experimentally by a power law dictates that at least a part of the velocity derivative skewness scales in a manner consistent with the proposed self-preservation.

It is interesting to note that Eqs. (73)–(75) can be written as

TABLE II. Variation of G and $R_\lambda S$ with R_M (data of Batchelor and Townsend[31]).

R_M	G	$R_\lambda S$	$SR_\lambda/R_M^{1/2}$
5444	8.0	7.4	0.10
10 888	10.0	11.4	0.11
15 392	11.5	14.4	0.12
30 784	14.0	19.4	0.11

$$SR_\lambda=\tfrac{30}{7}[(n-1)/n]-2G, \tag{79}$$

where

$$G=\lambda^4\frac{\partial^4\rho}{\partial r^4}\bigg|_{r=0} \tag{80}$$

and ρ is the longitudinal correlation coefficient. For $n=-1$, Eq. (79) reduces to the result of Batchelor and Townsend.[31] Figure 10 of their paper shows both the constancy of G and its dependence on the initial conditions. These experimental results could be previously understood only by assuming that the decay exponent was given by $n=-1$, but are clearly consistent with the more general theory proposed here. Another immediate consequence of Eqs. (69) and (56) is that SR_λ varies asymptotically as $R_M^{1/2}$, as suggested on empirical grounds by Batchelor and Townsend.[31] Table II summarizes their results and demonstrates that $SR_\lambda/R_M^{1/2}$ is constant at about 0.11 for their grid.

Table III shows the results of an attempt to use the grid data of Comte-Bellot and Corrsin[36] to calculate S_2 from Eq. (75). While there is considerable scatter in the computed values of SR_λ, this is largely due to systematic errors in the spectral data at the very highest wave number which are perhaps attributable to the method for thermally compensating the constant current hot wires used. Thus even these data are consistent with the proposed scaling. It should, however, be noted that the values of the velocity derivative skewness computed using the spectra are considerably higher than those reported using direct measurements. Unfortunately, there appear to be no results where both spectral and direct derivative measurements can be used to check the internal consistency of the data.

There have been only a few attempts to measure the spectral transfer (Van Atta and Chen,[45] Helland *et al.*,[46] and Uberoi[33]). Only Uberoi presents data at several downstream distances so that scaling arguments can be evaluated. The spectral transfer function collapses reasonably well when normalized by u^3 and plotted as a function of

TABLE III. Calculated velocity derivative skewness (1 in. grid, Comte-Bellot and Corrsin[36]).

x/M	R_λ	S_1R_λ	S_2R_λ	SR_λ	S
45	48.6	7.9	−36.2	−28.3	−0.58
120	41.1	7.9	−32.8	−24.9	−0.61
240	38.1	7.9	−43.6	−35.7	−0.93
385	36.6	7.9	−42.6	−34.7	−0.96

TABLE IV. Maximum of triple-velocity correlation (Mills *et al.*[32]).

x/M	R_λ	k_{max}	$k_{max}R_\lambda$
17	26.5	0.056	1.48
32	26.1	0.062	1.63
69	20.9	0.067	1.41

wave number normalized by integral scale. This is not in accord with the theory presented here, which requires normalization by $R_\lambda^{-1}u^3$ instead of u^3. However, for this experiment, the ratio of u^3 and $R_\lambda^{-1}u^3$ is proportional to $(x/M)^{0.1}$, and therefore the ratio (and R_λ itself) varies by only 9% over the entire range of the data. Thus it is not possible to distinguish between the two scalings, both collapsing the data to within the experimental error. It is interesting to note that Helland *et al.*,[46] using the data of Schedvin *et al.*,[42] introduced on empirical grounds a self-preserving form of the spectrum to compensate for a very limited range of downstream positions ($x/M=37-41$). Their empirical scaling can be shown to be indistinguishable from that proposed here over the range of data considered.

The measurements of Van Atta and Chen[44] of the one-dimensional spectra and of the one-dimensional triple spectra were made at only a single position ($x/M=40$) downstream of two geometrically similar grids operated at identical Reynolds numbers. Since only a single downstream location was used and since the Reynolds numbers (all of them) were the same for both grids, the results are not useful for verifying the self-preservation theory presented here. However, unlike the experiments of Comte-Bellot and Corrsin, where identical grids at different Reynolds numbers gave spectra that were distinctly different, the spectra of Van Atta and Chen collapsed perfectly. (Note that the choice of L_f, λ, or η is irrelevant since the Reynolds number was the same.) This contrast highlights the fact that the decay is determined by the conditions as measured by grid geometry and grid Reynolds number.

Finally, there have been several attempts to measure the triple-correlation functions directly (Stewart and Townsend,[40] Mills *et al.*,[32] Van Atta and Chen,[45] and Frenkiel and Klebanoff[34]). The earlier measurements appeared to suffer from high-pass filtering problems at the largest wave numbers, while the latter report insufficient downstream positions for a definitive test of the various scaling laws. Nonetheless, all of the data show an *increase* in the triple correlation with *decreasing* R_λ as predicted. Stewart and Townsend even show that appropriate length scale is proportional to $x^{1/2}$, although they did not relate it directly to λ. Table IV summarizes the values of the maximum from the data of Mills *et al.*, which is consistent (to within experimental error) with the proposed scaling which requires $k_{max}R_\lambda=$ const.

In summary, despite the very limited range of R_λ for a given set of initial conditions, there is evidence that the proposed scaling of the nonlinear terms may be correct, at least until a maximum value of these terms is achieved. While there has been a tendency to dismiss the variations in the measured values of the velocity derivative skewness as scatter in the data, there have been clear and persistent warnings from the experimental community for the past 30 years that the trends in the data were not in accord with previous theories. Had the present theory been available when those measurements were made, it is possible that all of the results would have been interpreted as confirming it.

XV. OVERALL ASSESSMENT OF EXPERIMENTAL EVIDENCE

In addition to the body of data discussed above, there exist the extensive experiments of decaying turbulence behind grids by Ling and co-workers (Ling and Wan[10] and Ling and Huang[47]). (These experiments appear to have been the motivation for the theoretical contributions of Barenblatt and Gavriolov,[9] which as noted, somewhat parallel the arguments presented herein.) These authors found fully self-preserving decay in which the intensity decayed as a power law with exponent dependent on the grid shape and Reynolds number. Moreover, they found the second-order correlation functions to collapse with a single length scale that increased as the square root of the decay time (in one case for nearly a decade in R_λ).

Finally, they noted that the decay exponent increased toward -1 as the grid Reynolds number increased. In the absence of a theory to explain the results, they identified their turbulence as *weak turbulence*. These results are, in fact, exactly consistent with the theory presented here, and in the light of this new understanding, might more properly be labeled *early turbulence* since the nonlinear terms (as measured in this case by the peak in the triple correlation) never reached saturation and continued to increase during decay.

Finally, the very recent results of Mohammed and LaRue[47] must be considered in which they both presented their own measurements behind a grid, and reanalyzed those of others. By focusing their attention away from the initial period of decay in which the derivative skewness was increasing they argued that all of the experiments can be described by a single decay law with exponent equal to about -1.3. Whether or not this is an asymptotic state is irrelevant to the present theory, since the present theory can make no statement as to what happens once the nonlinear terms can no longer increase to ensure self-preservation. What is relevant is the fact that, by limiting their attention to the region where saturation occurred, the authors excluded from consideration the very region where the theory presented here applies. It is interesting to contrast the relatively rapid rise of the derivative skewness in these measurements with the apparently slow rise in the experiments of Ling and Huang cited above. Taken together they emphasize the role of the initial conditions, which, in turn, probably account for much of the confusion over the years in trying to make definitive statements about the decay of grid turbulence.

It has long been recognized that experiments are most useful when designed to test between conflicting theories and hypotheses. At the very least the theory proposed herein would seem to provide a strong rationale for a new generation of experiments (or numerical simulations). Such experiments should be especially designed to monitor the evolution of the turbulence at great distance from the grid, and should include measurements of the velocity derivative skewness and spectral transfer with careful attention to ensuring consistency between the two. Only when these second- (or third-) generation experiments are performed will a more definitive verification be possible.

XVI. SUMMARY AND CONCLUSIONS

A new theory of self-preservation for decaying isotropic turbulence has been proposed in which the kinetic energy and the Taylor microscale are the appropriate scaling parameters for all scales of motion. The kinetic energy decays as a power law in time, the Taylor microscale grows as the square root of time, and the nonlinear terms (as measured by the spectral transfer or the velocity derivative skewness) vary *during decay* as R_λ^{-1}. The coefficients and decay exponent are determined by the initial conditions. The theory is valid at both finite and infinite Reynolds numbers. By insisting that Kolmogorov's theory be valid in the limit of infinite (grid or initial) Reynolds number, the von Kármán–Howarth theory is recovered as the limiting case. Because self-preservation is accomplished by the nonlinear spectral transfer terms, it is argued that these will eventually saturate and thereby limit the range of application of the theory to the initial period of decay in which these terms are still increasing. After this point is reached, there is the possibility that another self-preserving regime is entered in which the turbulence decays as t^{-1}, but with constants that may still depend on the initial conditions (Speziale/Bernard suggestion[15]). The theory appears to be consistent with the wealth of experimental data.

The theory is used to reexamine the nature of integral invariants in decaying isotropic turbulence, with the result that the appropriate invariant is seen to be governed by the decay exponent, and therefore by the initial conditions. Also, the $t^{-5/2}$ energy decay usually associated with the *final period of decay* is shown to result directly from self-preservation and the assumed constancy of the Loitsiansky integral, with no assumptions required about the negligibility of the inertial terms. From this, two types of $t^{-5/2}$ behavior were identified: one where the inertial terms were important, and a second true final period where they were negligible. It seems likely from experiments that the former is only realized when the initial conditions permit by virtue of the initial spectrum.

It has been the custom since the introduction of Kolmogorov's ideas on the statistical independence of the small-scale motions to view turbulence as being made up of large and small scales of motion that only weakly interact through an energy cascade. The results of this paper would certainly appear to call this view into question, at least at finite Reynolds numbers. For isotropic turbulence (at least), the local similarity theory of small-scale turbulence (which may still describe many shear flows) gives way to a higher principle—that of self-preservation at all scales—at least when certain conditions are satisfied.

It is also clear from the arguments presented here that the large-scale structures must play a crucial role since they determine the integral invariants that appear to control (or perhaps reflect) how the decaying turbulence moves from one self-preserving regime to another. Kerr[48] has suggested that the evolution of the large scale structures is, itself, the energy cascade. This is consistent with the concept of a self-preserving flow where large eddies evolve by vortex concentration and breakdown. Such a process has recently been demonstrated by Glauser and George[49] for an axisymmetric jet mixing layer, and may manifest itself in different ways for different flows. The initial conditions come into play by dictating how this evolution begins, and the structures are then locked into this evolution until the physics dictates otherwise.

In fact, if one considers the lifetime of a coherent structure in decaying turbulence to be characterized by the time scale l/u, then the distance behind the grid for this life cycle to be completed is proportional to Ul/u. Since l is typically of order M and $u/U \sim 10^{-2}$, this distance can be measured in hundreds of mesh lengths from the grid. Thus the decay of turbulence behind a grid may, in fact, represent just a random collection of single-vortical structures (perhaps the generalized "vortrons" suggested by Moffatt[50]), which simply evolve until diffused by viscosity, or until their vorticity is so concentrated that they break down. Such an evolution would appear to be quite consistent with the rather elegant idea of a flow in which all terms in the averaged equations remain in relative balance. If so, perhaps the first real links between coherent structures and the dynamics of turbulence will be found.

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