Influence of Wire Spacing on Velocity Derivative Measurement with Parallel Hot-Wire Probes

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I. Abstract

The spatial resolution of multiple wire probes is studied in order to evaluate the effect of wire spacing on the measurements of small scale turbulent fluctuations. Pao's model for the high wavenumber velocity spectrum is used to investigate the effect of the wire spacings on the measured difference spectra. These results are used to evaluate the effect of wire spacing on the direct measurements of spatial derivatives using parallel wire probes.

II. Introduction

Direct measurements of the dissipation rate and mean-square vorticity in turbulent shear flows require the determination of a number of the spatial derivatives of the velocity components. In the direct measurement of spatial derivatives it is common to use a difference method utilizing multiple wire probes. This difference method involves approximating the velocity gradients by taking the difference of the measured velocity at two points that are in close proximity to each other. However, the spatial filtering imposed on multiple wire probes by the wire separation has not previously been investigated.

For measurements of the dissipation of energy as well as those of the mean-square vorticity, Hussein and George[3], Champagne[4], the dissipative scales of turbulence are the determining factor in choosing the size of the measuring probe since the velocity gradients are determined by the smallest scales of motion. It is well-known that the smallest scales of the turbulence are strongly affected by the line averaging due to the finite wire lengths (Uberoi and Kovasnay[1], Wyngaard[2]). The effect of the wire spacing on direct derivative estimation by differencing has not, however , been previously investigated, at least theoretically. The objective of this paper is to analyze the spatial filtering arising from the separation of the individual wires, and to assess the effect of separation on estimation of derivative moments.

III. Transfer Function for the parallel wire probe

For a homogeneous random field, the velocity can be represented as (Lumley[6]):

$$u(\bar{x}) = \int e^{i\bar{k}\cdot\bar{x}} \Omega(\bar{k})\, d\bar{k} \tag{1}$$

where $\bar{k}$ represents the wave number vector and Ω is the Fourier transform (in the sense of generalized functions) of the velocity vector.

We consider only the streamwise velocity component, $\bar{U}(x$, and evaluate it at two different positions $\bar{x}_1$ and $\bar{x}_2$ which differ only in the y-coordinate, i.e.

$$\bar{x}_1 = (x, y_1, z)$$

$$\bar{x}_2 = (x, y_2, z) \quad .$$

The separation between the wires is defined to be

$$\Delta = y_1 - y_2$$

Suppressing for now the dependence on x and z we can write

$$u(y_1) = \int e^{-i\vec{k}.\vec{x}_1}\, \hat{u}(\vec{k})\, d\vec{k} \tag{2}$$

$$u(y_2) = \int e^{-i\vec{k}.\vec{x}_2}\, \hat{u}(\vec{k})\, d\vec{k} \tag{3}$$

from which it follows that

$$[u(y_1) - u(y_2)] = \int d\vec{k}\, \hat{u}(\vec{k})\, [e^{-i\vec{k}.\vec{x}_1} - e^{-i\vec{k}.\vec{x}_2}] \ . \tag{4}$$

The mean squared velocity difference is now given by,

$$\overline{[u(y_1) - u(y_2)]^2} = \int d\vec{k}\, d\vec{k}'\, \overline{\hat{u}(\vec{k})\, \hat{u}^*(\vec{k}')}\, [e^{-i\vec{k}.\vec{x}_1} - e^{-i\vec{k}.\vec{x}_2}]\, [e^{+i\vec{k}'.\vec{x}_1} - e^{+i\vec{k}'.\vec{x}_2}]$$

(5

For a homogeneous random field (Monin and Yaglom[7])

$$\overline{a_i(\vec{k})\, a_j^*(\vec{k}')}\, d\vec{k}\, d\vec{k}' = F_{ij}(\vec{k})\, \delta(\vec{k}'-\vec{k})\, d\vec{k}\, d\vec{k}' \tag{6}$$

where F_{ij} is the three-dimensional velocity spectrum. Therefore

$$\overline{[u(y_1)-u(y_2)]^2} = \int d\vec{k} F_{11}(\vec{k})\, [2-e^{i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}-e^{i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}] \tag{7}$$

$$= \int d\vec{k} F_{11}(\vec{k})\, 2\,[1-\cos k_2(\Delta)] \tag{8}$$

The measured one-dimensional difference spectrum can be obtained in terms of the one-dimensional spectra of the turbulence as

$$F_\Delta^1(k_1) = \int_{-\infty}^{+\infty}\!\!\int 2\,[1-\cos k_2\Delta]\, F_{11}(\vec{k})\, dk_2 dk_3 \tag{9}$$

Note that this is the spectrum which could be inferred by differencing the frequency spectra measured by each probe in a connected field (using Taylor's hypothesis); or alternately by measuring the frequency spectrum and the difference signal.

The cosine term in the above equation is a transfer function due to the separation between the two wires. Thus the separation acts as a high pass spatial filter removing the contribution of the low wavenumbers in the 2-direction. For small values of $K_3\Delta$, the cosine term can be expanded in a Taylor series to obtain

$$F_{\Delta}^{2}(k_1) = \Delta^2\left[\int_{-\infty}^{+\infty}\!\!\int k_2^2 F_{11}(\bar{k})\, dk_2 dk_3\right] - \frac{1}{12}\Delta^4\left[\int_{-\infty}^{+\infty}\!\!\int k_2^4 F_{11}(\bar{k})\, dk_2 dk_3\right] \qquad (10)$$

and

$$\overline{[u(y_1)-u(y_2)]^2} = \Delta^2\left[\int d\bar{k}\, k_2^2 F_{11}(\bar{k})\right] - \frac{1}{12}\Delta^4\left[\int d\bar{k}\, k_2^4 F_{11}(\bar{k})\right] \qquad (11)$$

The first terms on the right hand sides in the above two equations correspond to the first x_2-derivative of the velocity and it's one-dimensional spectrum respectively; while the second terms correspond to the second x_2-derivative and it's spectrum. If the second term in equation 10 is negligible compared to the first term, then the measured difference provides an estimate of the mean square derivative in the 2-direction.

IV. The Effect of the Higher Order Terms on the Effective Transfer Function

Calculations were carried out to evaluate the effect of the transfer function on the measured difference spectra assuming isotropic turbulence and using Pao's' spectrum. Figure 1 shows the measured difference spectra compared with the one-dimensional spectra of the turbulence for three cases: Δ/η = 1,2 and 5 where η is the Kolmogorov length. The one-dimensional spectrum, the first and second terms in equation (10) and the difference of these two terms for several different spacings are shown in Figure 1. The difference spectra progressively deviates from the derivative spectra as the spacing between the wires is increased. For $\Delta=\eta$ the resulting measured spectrum shows little effect from the filtering for $k<\eta^{-1}$. However a separation of 5η shows a large effect at wave numbers above $k\eta$ of 0.2. It is interesting to note that unlike the spectrum of $\frac{\partial u}{\partial x_1}$ which peaks off-axis and rises as $k_1^{1/3}$ for low wavenumbers, the spectrum of $\frac{\partial u}{\partial x_2}$ decreases monotonically.

Figure 2 shows the difference spectra for parallel wires with spacing from one to nine times the Kolmogorov length. It is clear that the effect of the spatial filtering due to wire spacing is negligible for wire spacing close to the Kolmogorov scale. Figure 3 shows the effective transfer function that is obtained by dividing both sides of equation 10 by the first term on the right hand side of the equation (i.e., the desired result). This effective transfer function would be unity for wire spacing that is equal to zero and should decrease with increasing wire spacing for a given wavenumber. The figure shows the calculations for 5 different wire spacings from η to 10η. A wire spacing of 1η is seen to be necessary to resolve the dissipation range.

V. Effect of Wire Spacing on Derivative Estimate

To show the effect of this spectral contamination on the measured derivative, the mean square derivative was calculated by integrating the difference spectrum, i.e.,

$$\overline{\left[\frac{\partial u}{\partial y}\right]_m^2} = \int_{-\infty}^{+\infty} F_\Delta^1 (k_1)\, dk_1 \qquad (12)$$

The subscript m denotes the calculation for the measured quantity. Figure 4 shows results for the calculations comparing the measured cross-stream derivative normalized by the actual derivative (i.e., that computed with the transfer function equal to unity). As shown on the plot, a wire spacing of $\Delta = \eta$ is necessary to achieve an accuracy of about 95%.

VI. Conclusions and Discussions

Calculations of the response of parallel wires in homogeneous turbulence show the effect of wire spacing on the difference spectra. This contamination of the difference spectra depends on wire spacing and the dissipative scales of turbulence. It was shown that at wire spacings close to the Kolmogorov microscale, the spatial filtering of the cross-stream derivative is negligible.

References

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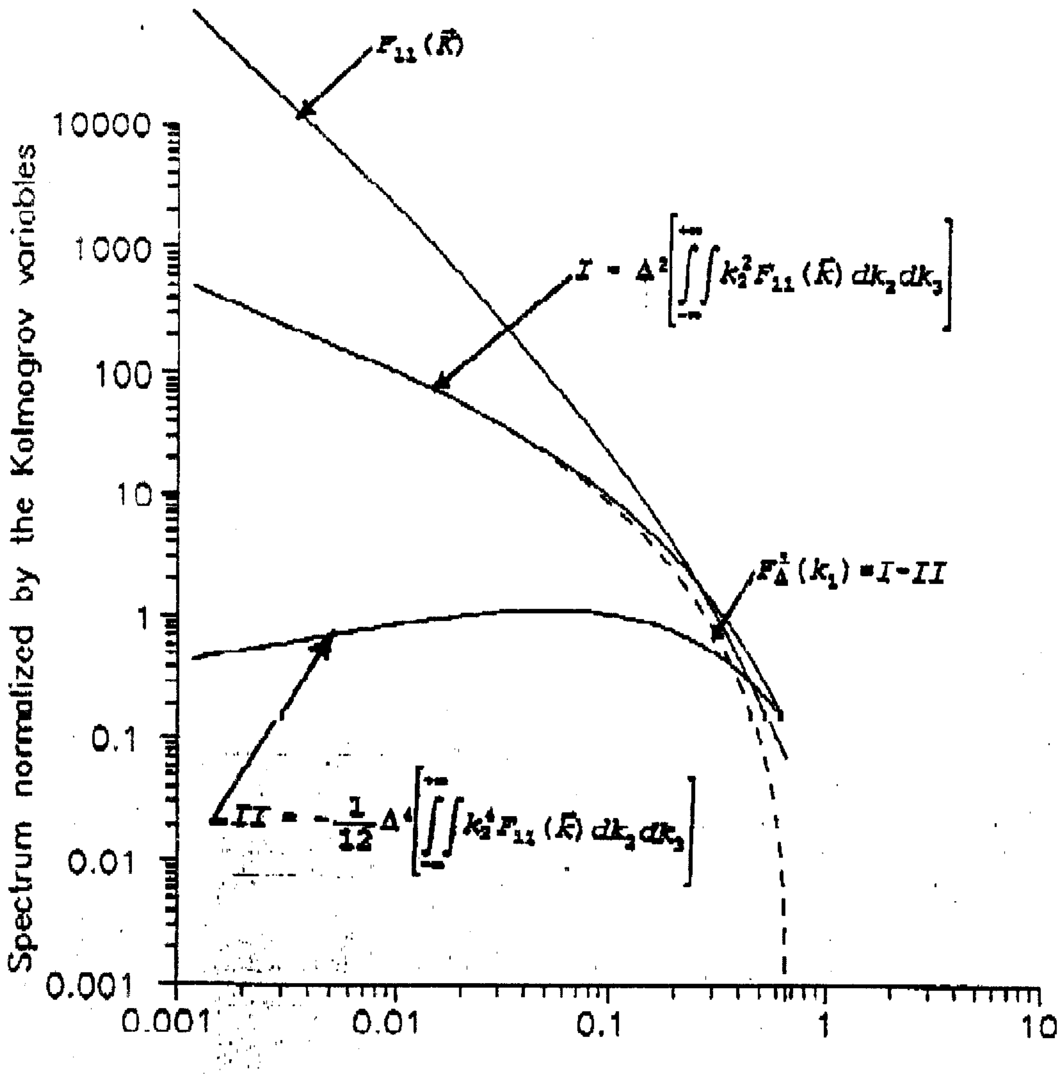

Figure 1c. Comparison of leading terms for the difference spectra. Wire spacing of 5η

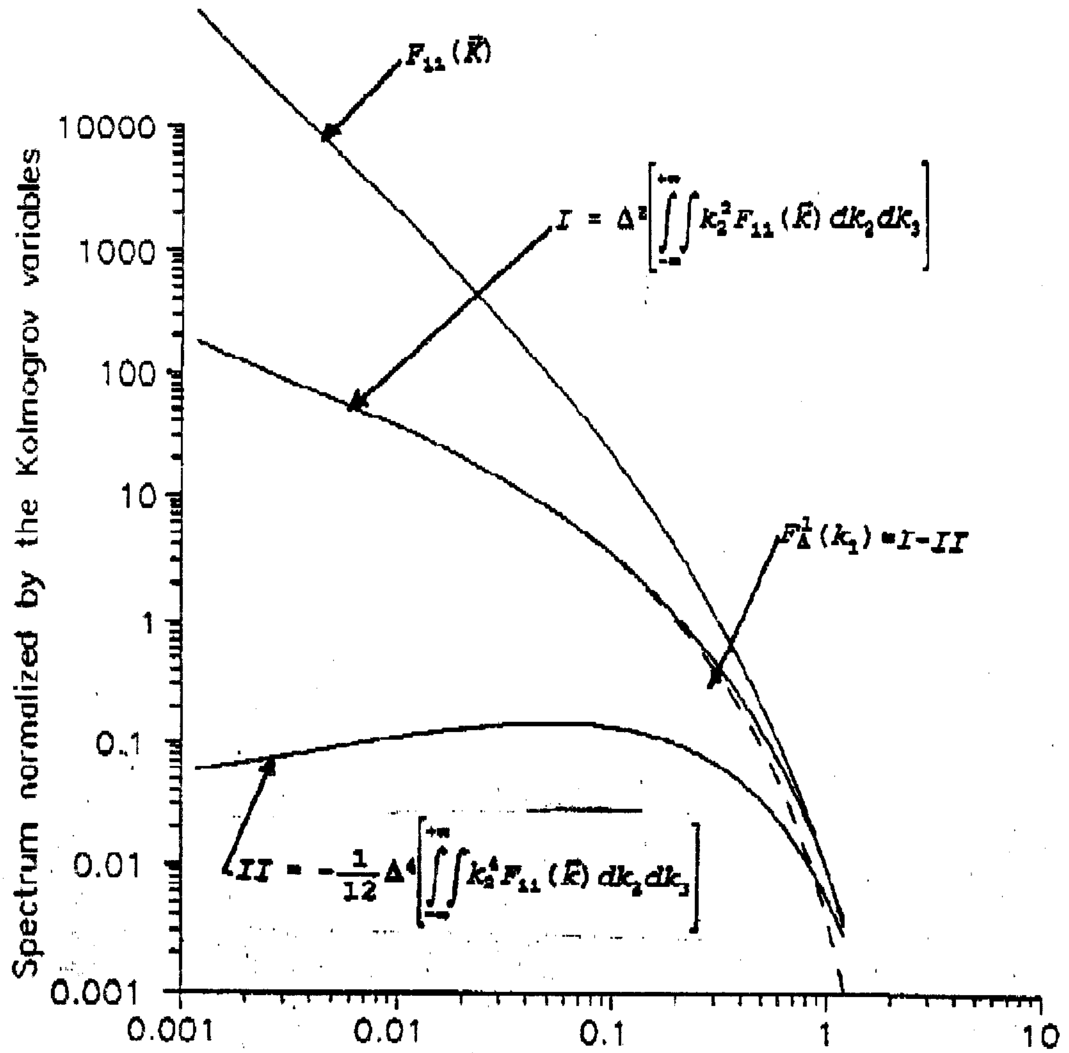

Figure 1b. Comparison of leading terms for the difference spectra. Wire spacing of 2η

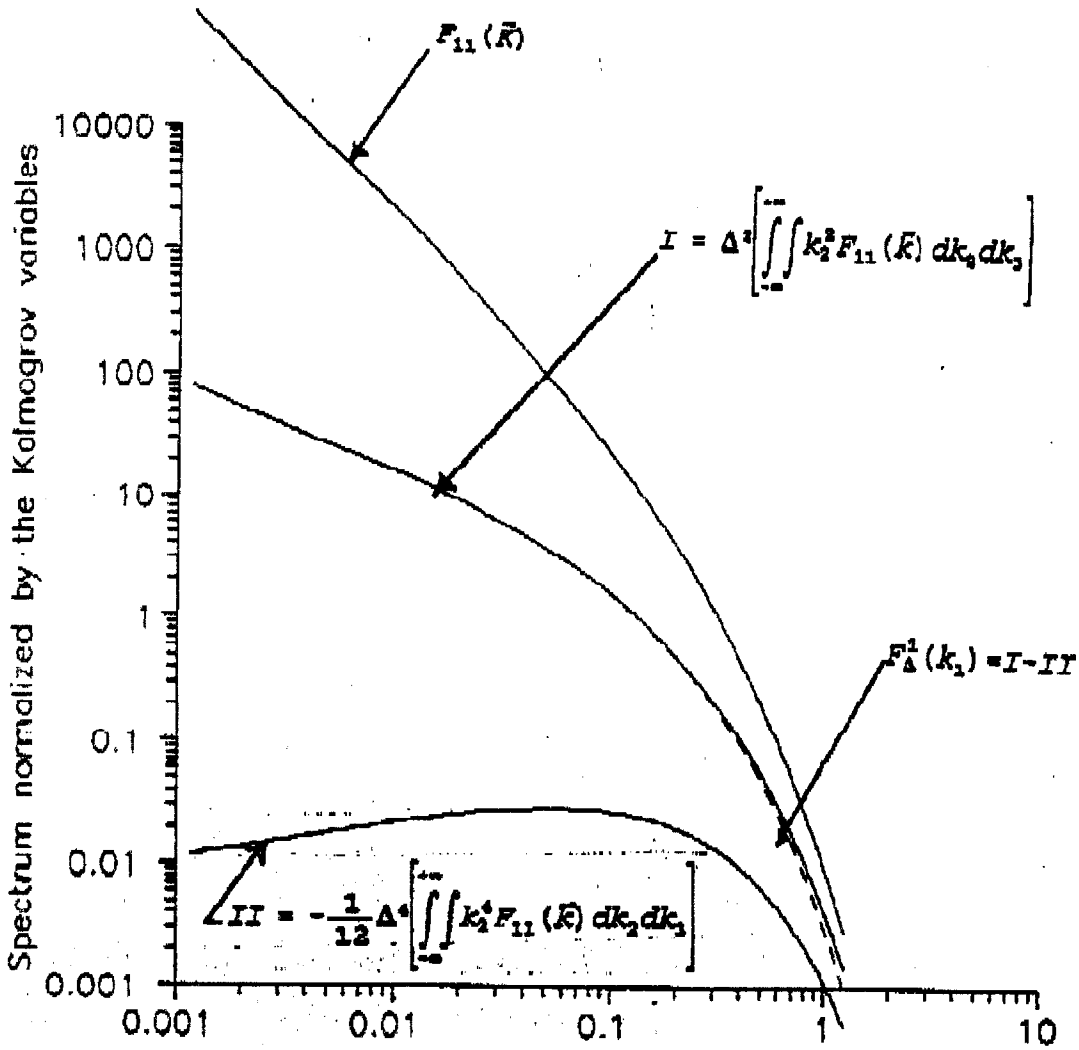

Figure 1a. Comparison of leading terms for the difference spectra. Wire spacing of 1η

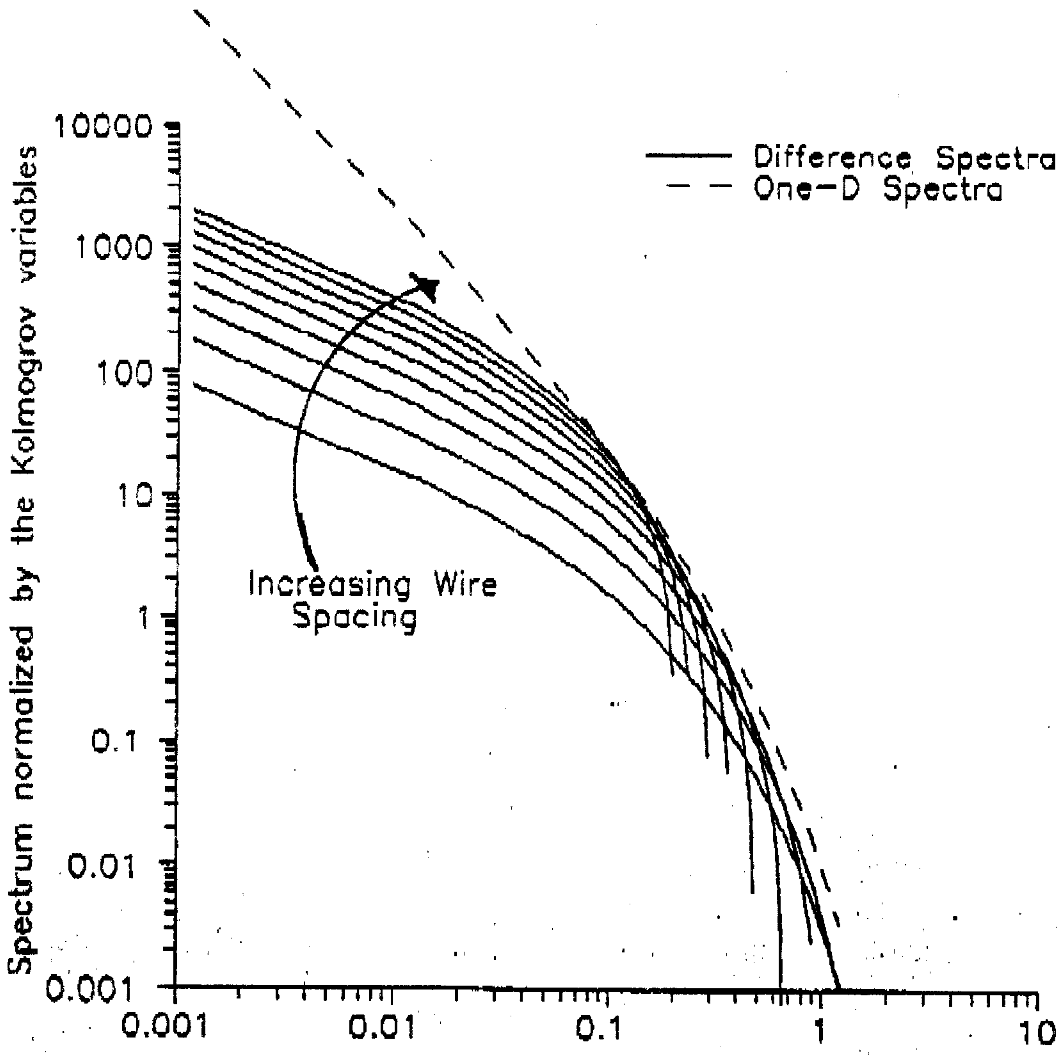

Figure 2. Calculated difference spectra

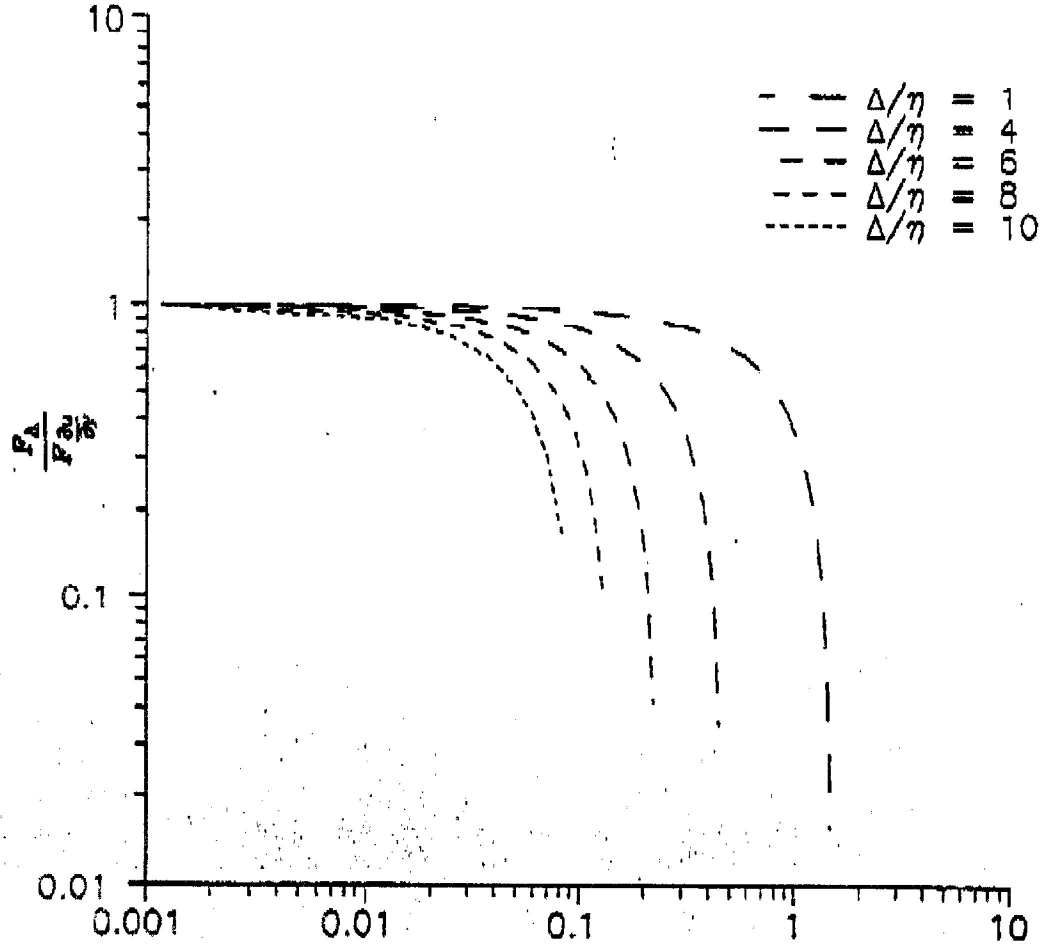

Figure 3. Effective transfer function

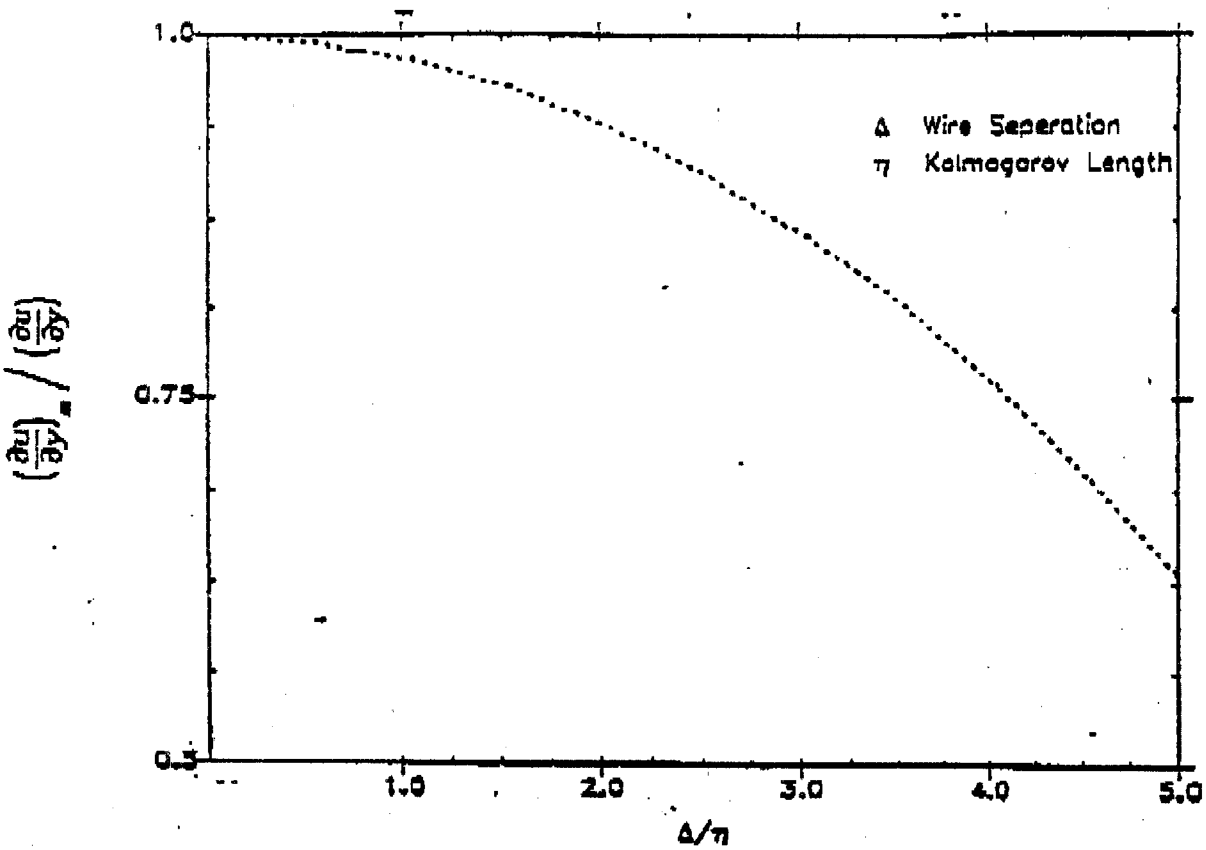

Figure 4. Effect of wire spacing on measured derivative