Proceedings of FEDSM'02
2002 ASME Fluid Engineering Division Summer Meeting
July 14-18, 2002, Montreal, Quebec, Canada

FEDSM2002-31411

FURTHER STUDIES OF THE AXISYMMETRIC DISK WAKE USING THE 'SLICE POD'

Peter B.V. Johansson
Turbulence Research Laboratory
Department of Thermo and Fluid Dynamics
Chalmers University of Technology
Gothenburg, SE-41296, SWEDEN
Email: jope@tfd.chalmers.se

William K. George
Turbulence Research Laboratory
Department of Thermo and Fluid Dynamics
Chalmers University of Technology
Gothenburg, SE-41296, SWEDEN
Email: wkgeorge@tfd.chalmers.se

ABSTRACT

This paper presents the findings of three experiments using multi-point hot-wire arrays in the high Reynolds number axisymmetric turbulent wake behind a disk. The purpose of the multiple experiments was to validate earlier and less extensive experiments. The 'slice POD' was applied to all sets to examine the effects of array coverage and the disk support system. The Reynolds number based on the free stream velocity and disk diameter was kept constant at 28,000. The investigated region spanned from 10 to 60 disk diameters downstream.

These results confirm the earlier findings. In particular, the eigenvalues integrated over frequency show a azimuthal mode-1 dominance at $x/D = 10$ which evolves to a mode-2 dominance by $x/D = 50$. For all downstream positions, two distinct peaks were found in the first eigenspectrum: one for azimuthal mode-2 at near zero frequency, and another for azimuthal mode-1 at a Strouhal number (fd/U_∞) of 0.126. Both peaks decrease in magnitude as the flow evolves downstream, but the peak at the Strouhal number 0.126 decrease more rapidly then the one at near-zero frequency, leaving the latter to eventually dominate.

INTRODUCTION

This is the second in a series of papers on the application of 'slice' POD techniques to the axisymmetric wake behind a circular disk. In the first paper (Johansson et al., 2002) the axisymmetric turbulent wake behind a disk was studied using a 'slice POD' for three fixed downstream cross-sections of the flow ($x/D = 10$, 30, and 50). For all downstream positions, two distinct peaks were found in the eigenspectrum of the lowest (and most dominant) radial POD mode: one for azimuthal mode-2 at near zero frequency, and another for azimuthal mode-1 at a fixed Strouhal number ($St = fd/U_\infty$) of 0.126. (Note that measured frequency should be interpreted as a streamwise wavenumber using $k_1 = 2\pi f/U_\infty$ for all frequencies except the very low ones to properly apply Taylor's hypothesis according to the criteria of Lumley, 1965.) Both peaks decreased in magnitude as the flow evolved downstream, but the peak at the Strouhal number 0.126 decreased more rapidly so the latter eventually dominated. The authors did deliberately not associate a Strouhal number with the 'near-zero' frequency, since it was found to be the lowest resolved non-zero frequency (0.73 Hz). When integrated over frequency and normalized by the total kinetic energy, it was seen that the first eigenspectrum accounted for more than 60% of the energy.

These were surprising findings, especially the evolution to azimuthal mode-2 far downstream which had not been previously observed. Fuchs et al. (1979) computed azimuthal cross-spectra using two hot-wires at a fixed radius in the wake, and found an azimuthal mode-2 peak at low frequency (St = 0.005). The investigation was limited to two positions downstream, x/D = 3 and 9, and mode-1 was found to be dominant at these positions. Brücker (2001) investigated the wakes behind a sphere and an axially oriented cylinder with an elliptic nose and a blunt base at low Reynolds numbers (up to Re = 1000), and stated that the results indicated a simultaneous existence of the primary instability causing the vortex shedding together with a long-wave instability (St = 0.05 for the sphere, and St = 0.03 for the cylin-

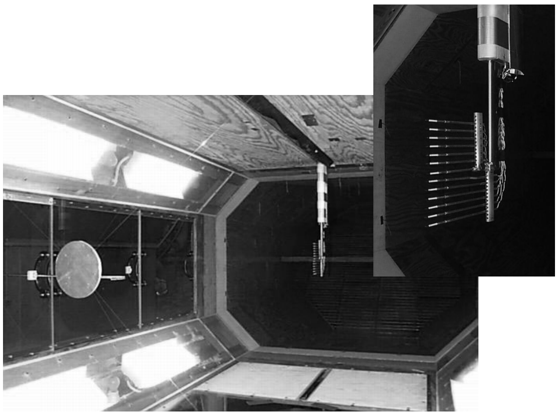

Figure 1. The probe array and disk in the Chalmers wind tunnel

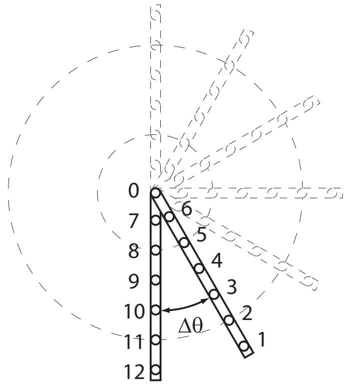

Figure 2. The traversing scheme, shown in 30° increments in $\Delta\theta$ for simplicity

der). He assumed that both of these were associated with an azimuthal mode-1 type of motion. It has long been suspected that there might be a connection between the theory for instability of laminar wakes (see e.g., Monkewitz, 1988) and the behavior of fully turbulent high Reynolds number wakes. The stability results are usually based on the Orr-Sommerfeld equations which are in turn derived from the linearized Navier-Stokes equations with parallel flow approximations. The results of such efforts suggest the emergence of azimuthal mode-1 as dominant, since it is the fastest growing disturbance. Therefore the emergence of mode-2 was quite unexpected. If true, it would seem to imply either that non-linearities must dominate, or non-parallel effects, or simply that such theories may not be relevant to turbulence at all. Regardless, the implications of the experimental POD results are so important that it is of paramount importance that they be confirmed to be true beyond all reasonable doubt, before significant effort is expended trying to explain them. This paper attempts to do that by:

1. Increasing the number of measuring locations.
2. Examining the influence of measurement locations with respect to the wake width.
3. Changing the support structure of the disk to alter any modal excitation by them.

EXPERIMENTAL SETUP

The experiments were performed in the low-turbulence wind tunnel at Chalmers University of Technology, Gothenburg, Sweden. The free-stream streamwise turbulence intensity over the span of velocities related to this work was less than 0.03%. The measuring cross-section in the tunnel is 1.80×1.25 m^2 and the downstream length is 3.00 m. The tunnel velocity was kept constant at 15 m/s during the experiment. The disk was a Swedish five kronor coin with a diameter of 28 mm. The total area ratio between disk and tunnel cross-section was less then 0.03%. The Reynolds number based on the free stream velocity and disk diameter was 28,000.

For the earlier experiments and most of the present work the disk was suspended with four pairs of wires, each with the diameter 0.2 mm and placed at 90°. The photograph shown in Fig. 1 shows the experimental setup. Results are also reported for an alternative suspension arrangement that uses only three pairs of support wires positioned at 120°.

Two different rakes were used for the present experiments: the original 13 hot-wire rake used in Johansson et al. (2002), and a 15 hot-wire rake obtained by extending the same rake to include two additional wires. The two arrays hot-wires were used in the manner of Glauser and George (1987) as shown in Fig. 1. The arrays were used to obtain the two-point velocity cross-spectra for all combinations of locations shown in Fig. 2. The measurement grid was chosen following the guidelines of Glauser and George (1992) to avoid as much spatial aliasing as possible when making the azimuthal Fourier decompositions to obtain the cross-spectra. Of primary concern in this investigation was to determine whether the previous results were influenced by the grid, hence the extra probes.

Half the array of probes was movable, and traversed from a 15° separation up to 180° with 15° increments in $\Delta\theta$, see Fig. 2. Each hot-wire probe is numbered and marked by a circle. Using this scheme, half the cross-section of the wake at a fixed downstream position was scanned, and pairs of instantaneous velocity correlations for a fixed angle separation computed. Note that the cross-spectra corresponding to the remaining half-plane were available from the azimuthal symmetry of the flow. This was justified by an initial test where cross-spectra were obtained on both sides of the wake center plane ($\Delta\theta = 180°$ in Fig. 2). These cross-spectra were impossible to distinguish from each other when the probe rake was properly centered behind the disk. For the particular angle for which the movable probe rake caught the wake of the suspending wires, the measurements on either side were used to estimate this position.

Each single hot-wire, 3 mm long and made of unplated 5μm tungsten wire, was oriented to measure the downstream component of the velocity. The probes were connected to an AN2000 Constant Temperature Anemometer (CTA) system, and sampled with an IO Tech Wavebook 516 16 bit sample and hold A/D converter. The data was low-pass filtered at 1 kHz and sampled at 4 kHz for all configurations, substantially higher than the temporal Nyquist criterion. Measurements were made simultaneously at all positions. Each block had 4096 samples, and a total of 360 blocks of data was taken per probe for each angular probe location, ensuring a variability of less than 4% for the cross-spectra used in the POD.

PROPER ORTHOGONAL DECOMPOSITION

The POD and the manner in which it is used in this investigation was described in detail in Johansson et al. (2002), so only the most essential features will be reviewed here. Note that the experimental setup is only capable of providing the necessary information to obtain the POD modes, but not the information to project them back on the instantaneous flow. This is because the latter requires measurement at all positions simultaneously in the manner of Citriniti and George (2000).

The POD results from a projection of the velocity field, u_i, into a coordinate system, ϕ_i, optimal in terms of kinetic energy. If the field has finite total energy, Hilbert-Schmidt theory assures that the solution exists and consists of a denumerable, infinite, set of eigenvalues, $\lambda^{(n)}$, and corresponding eigenfunctions, $\phi_i^{(n)}$. For an axisymmetric shear flow such as a jet or a wake, this is true in the radial direction at a single downstream location, hence the term ‘slice POD’.

For the axisymmetric wake considered here, the turbulent velocity field is *stationary* in time and *periodic* in the azimuthal direction. Therefore, the Hilbert-Schmidt theory does not apply to them, but instead Fourier modes are appropriate. If only the streamwise velocity component is considered (i. e., $i = j = 1$), the following integral equation(s) must be solved:

$$\int_0^\infty B_{1,1}(m,f,r,r';x)\psi_1^{(n)}(m,f,r';x)r'\,dr' = \lambda^{(n)}(m,f;x)\psi_1(m,f,r;x) \quad (1)$$

where $B_{1,1}(m,f,r,r';x)$ is the two-point velocity correlation Fourier transformed in time and expanded in Fourier series in the azimuthal direction and ψ is the corresponding eigenfunction. Note that ψ and λ are now functions of frequency, f, and azimuthal mode number, m,

In practice, the following steps are taken (following Glauser and George (1987)):

1. Measurement of the instantaneous velocity at two points.
2. Fourier transformation in time and computation of the cross-spectrum.
3. Repetition of 1. and 2. for many pairs of points.
4. Expansion of the cross-spectra obtained in 2. in Fourier series in the azimuthal direction
5. Solution of the remaining eigenvalue problem in the radial direction, Eq. 1, for each frequency and azimuthal mode number.

STATISTICAL RESULTS

The earlier paper of Johansson et al. (2002) details the first order statistics for this flow, and the difficulties in obtaining them. The primary problem is that the velocity deficit is very small (less than 2% of free stream velocity by the last measuring position), and thus extremely sensitive to hot-wire drift and calibration errors. Unlike many other free shear flows, the turbulence intensities, u_{RMS}/U_∞, are very small, from 9% at $x/D = 10$ to below 3% at $x/D = 50$. As a consequence, the hot-wire anemometer is clearly the instrument of choice, since virtually no other technique can resolve these very weak fluctuations.

Power spectral densities were computed for all downstream positions as well as rotations of the movable rake. These are not shown, but are identical to those in Johansson et al. (2002). All PSD’s off the center of the wake show a prominent peak at 67 Hz, corresponding to a Strouhal number ($St = fd/U_\infty$) of 0.126. This is consistent with Strouhal number measured by Miau et al. (1997) in the near wake. The magnitude of this peak decreases with increased downstream position, but is still clearly visible at $x/D = 60$. From the absence of this peak in the center of the wake, one can immediately infer that this dominant feature of the PSD can not be related to an azimuthal mode-0 motion.

POD RESULTS

Eigenspectra were computed for the three downstream locations by solving Eq. 1 to obtain the distribution $\lambda(m,f;x)$. Even this simple result from the POD provides a large amount of information regarding the energy distribution in the flow.

The eigenspectra, $\lambda(m,f;x)$, are representations of how the energy is distributed as function of azimuthal mode number, m, and frequency, f, at a given downstream position, x. Therefore their evolution show how the main characteristics of the flow evolve. Three-dimensional plots of the eigenspectra for the first POD mode for the four wire supported wake are presented in Figs. 3 and 4. Here, also the spectral content in each azimuthal mode is shown for the first three azimuthal modes (m = 0, 1, and 2) for all the downstream distances together with the full spec-

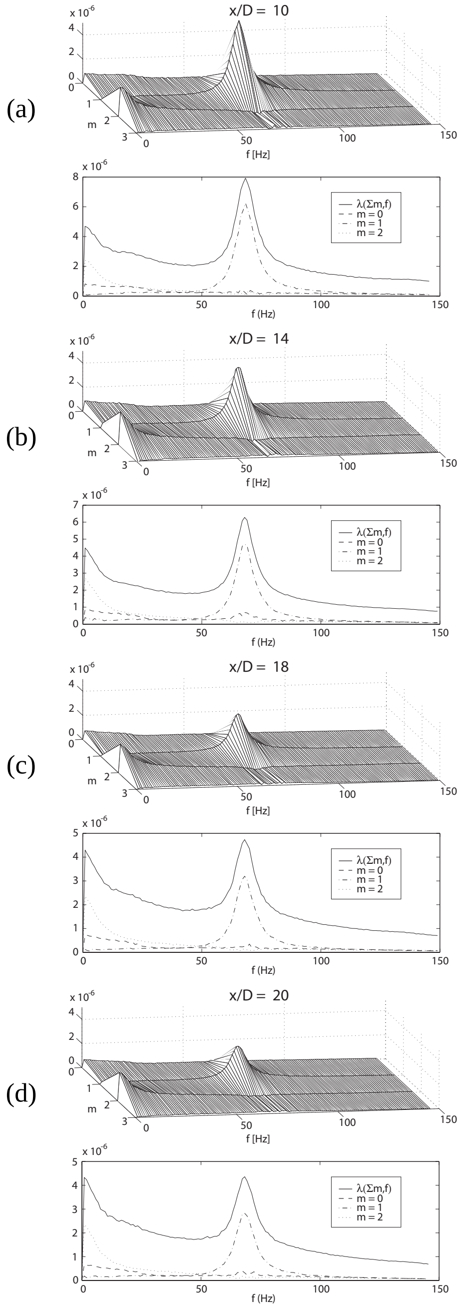

Figure 3. Eigenspectrum function of azimuthal mode number (m) and frequency (f) at different positions: (a) $x/D = 10$, (b) 14, (c) 18, and (d) 20.

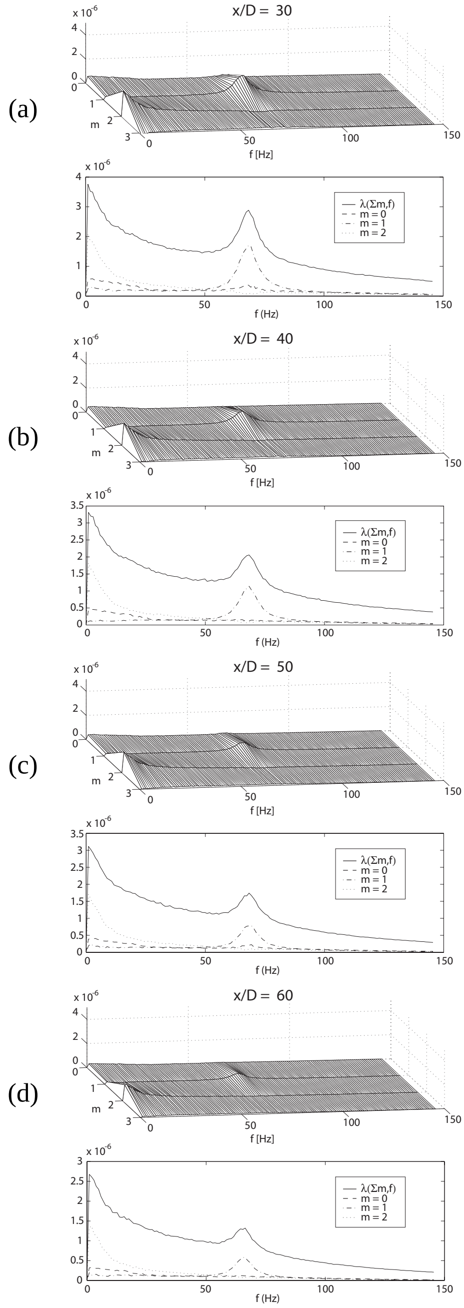

Figure 4. Eigenspectrum as function of azimuthal mode number (m) and frequency (f) at different positions: (a) x/D = 30, (b) 40, (c) 50, and (d) 60.

trum to illustrate the portion of the total energy that is captured by each mode.

As for the earlier results 13-wire results, the pictures are strikingly similar, showing two dominating events. The energy is concentrated around two separate peaks in the f-m plane. One is at near-zero frequency for azimuthal mode-2 and the other for mode-1 at a higher frequency, 67 Hz. This second peak for mode-1 corresponds to a Strouhal number of 0.126. This Strouhal number does not change with downstream position, and is exactly the same as the one detected in the PSD's off the wake

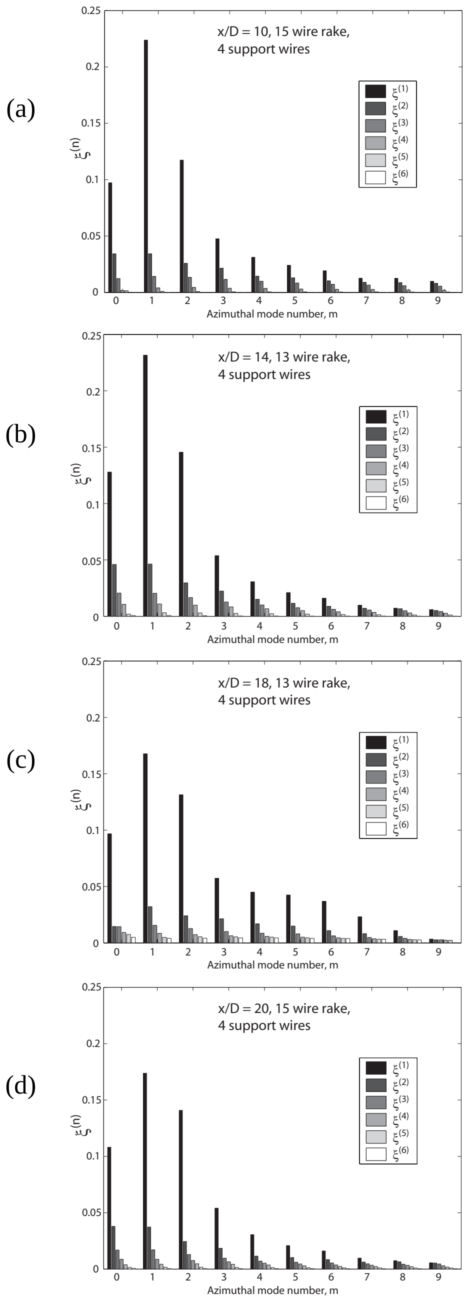

Figure 5. Eigenspectrum integrated over frequency (as defined in equation 2) as function of azimuthal mode number (m) at different positions: (a) x/D = 10, (b) 14, (c) 18, and (d) 20.

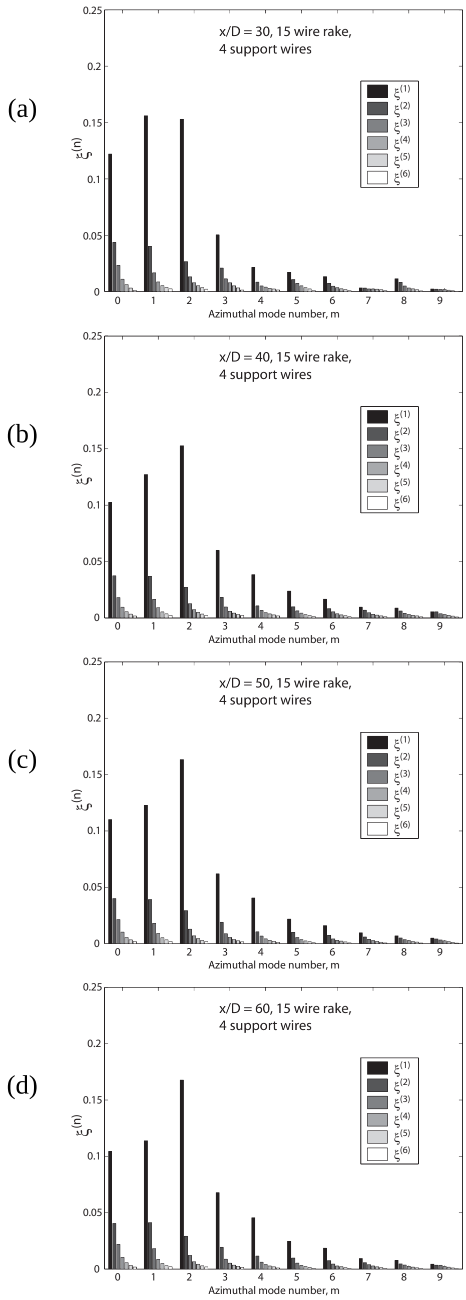

Figure 6. Eigenspectrum integrated over frequency (as defined in equation 2) as function of azimuthal mode number (m) at different positions: (a) x/D = 30, (b) 40, (c) 50, and (d) 60.

center described in the previous section. The peak at 'near-zero' frequency does not either seem to change with downstream distance, even though this has to be investigated further. This is because the 'near-zero' peak lies at the lowest measurable frequency in this experiment, 0.98 Hz.

The eigenspectra can be integrated over frequency to illustrate another key property of the POD, its ability to show how the kinetic energy of the flow is distributed among the various azimuthal modes. To visualize the energy distribution per azimuthal mode number, m, we computed for each downstream

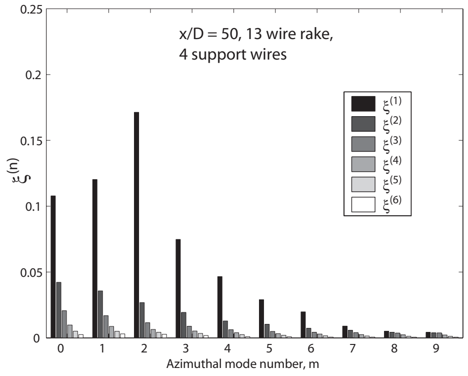

Figure 7.

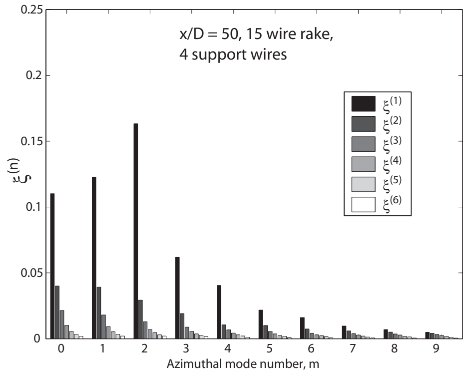

Figure 8.

position the quantity $\xi(m;x)$ where:

$$\xi(m;x) = \frac{\int_f \lambda(m,f;x)\,df}{\sum_m \int_f \lambda(m,f;x)\,df} \qquad (2)$$

Here, the denominator is the total kinetic energy in the field. The resulting normalized eigenspectra, $\xi(m;x)$ for the four-wire supported wake using the 15 hot-wire rake are plotted in Fig. 5 and 6. It is clear that at $x/D = 10$, most of the energy lies in the azimuthal mode-1, while by $x/D = 50$, the most energetic azimuthal mode is number 2.

Figure 7 shows same wake at $x/D = 50$, but using the data obtained with the 13-wire rake presented in Johansson et al. (2002). There are only very small differences, one being that mode 0 is slightly larger for the 15-wire rake. This can be explained by the fact that this rake covers a larger portion of the wake. Certainly, this effect is very small.

Figures 8 and 9 shows the same plots for the four and three wire supported rakes, also obtained using the 15 hot-wire rake at $x/D = 50$. The results are virtually indistinguishable, suggesting strongly that the whatever the physical cause of the observations, it is not a consequence of how the disk is supported in the wind tunnel.

Figure 10 shows plots of ru^2 versus r for all downstream positions. The total energy in the POD is the integral under these curves. Clearly as the rake is traversed downstream, progressively more and more of the total energy is not included in the decomposition (since the hot-wire rake is fixed). (This was one of the primary reasons for expanding from 13 to 15 wires.) The lost energy is less than 1% at $x/D = 10$ but perhaps as much as 20% at $x/D = 60$. As Fig. 6 makes clear, the evolution from azimuthal mode-1 peak to a peak at mode-2 takes place between $x/D = 30$ and 40. Beyond $x/D = 40$ there is virtually no change in the eigenspectra, even though progressively more of the energy is lost. This suggests strongly that the outside energy does not affect the eigenspectra (at least in the lower modes). This is consistent with the lack of observed differences between the 13 and 15-wire arrays.

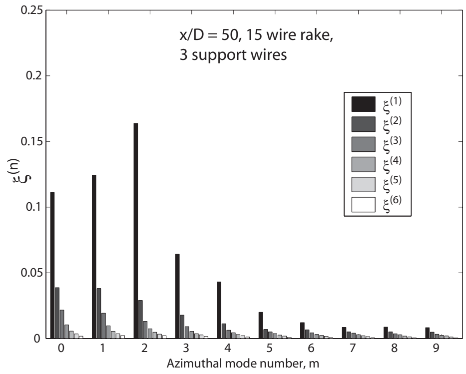

Figure 9.

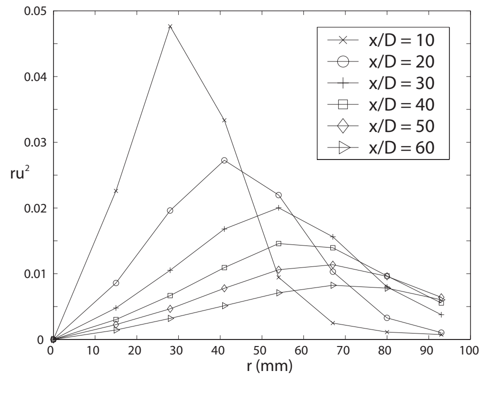

Figure 10.

The investigation was very recently expanded to further downstream distances and is presented in Johansson and George (2002). In this paper, experiments in a different wind tunnel using different anemometers, with the coin replaced with a disk machined in acrylic with the diameter 20 mm is presented. To maintain a constant Reynolds number with this smaller disk, the velocity was increased. The eigenspectra integrated over frequency, $\xi^{(n)}$, are in perfect agreement to those presented in this paper, ensuring that an eventual asymmetry of the coin is not the cause of the mode-2 dominance.

CONCLUDING REMARKS

Three different 'slice POD' investigations are reported of the axisymmetric turbulent wake behind a disk. Two different hot-wire rake configurations were used: a 13 hot-wire rake and a 15 hot-wire rake covering a larger area of the flow. Also two different supporting methods were used: one a four wire support system, the other a three wire support system. The results were essentially independent of rake, flow coverage or wake support system. The results confirm the earlier observations of Johansson et al. (2001), but with a much more extensive data base.

For all downstream positions, two distinct peaks were found in the eigenspectrum of the lowest (and most dominant) radial POD mode: one for azimuthal mode-2 at near zero frequency, and another for azimuthal mode-1 at a fixed Strouhal number (fd/U_∞) of 0.126. Both peaks decrease in magnitude as the flow evolves downstream, but the peak at the Strouhal number 0.126 decreased more rapidly so the latter eventually dominated. When integrated over frequency and normalized by the total kinetic energy, it was seen that the first eigenspectrum accounts for more than 60% of the energy.

As noted by Johansson et al. (2002), the results are strikingly similar to recent POD results for the far axisymmetric jet obtained with 138 hot-wires presented in Gamard et al. (2002). In particular, azimuthal mode-2 dominates the far downstream development. The main difference from the jet results are that peak at the non-zero frequency scales with the local Strouhal number for the jet, but remains fixed in frequency for the wake.

ACKNOWLEDGMENT

The authors gratefully acknowledge the assistance of Scott Woodward throughout this investigation. This work was initially supported by the US Air Force Office of Scientific Research, grant number F49620-98-1-0143 and Chalmers University of Technology. It continues with the support of the Swedish Research Council, grant number 2641.

REFERENCES

C. Brücker. Spatio-temporal reconstruction of vortex dynamics in axisymmetric wakes. *Journal of Fluids and Structures*, 15: 543–554, 2001.

J. H. Citriniti and W. K. George. Reconstruction of the global velocity field in the axisymmetric mixing layer utilizing the proper orthogonal decomposition. *Journal of Fluid Mechanics*, 418:137–166, 2000.

H. V. Fuchs, E. Mercker, and U. Michel. Large scale coherent structures in the wake of axisymmetric bodies. *Journal of Fluid Mechanics*, 93:189–211, 1979.

S. Gamard, D. Jung, S. Woodward, and W. K. George. Application of a 'slice' POD to the far field of an axisymmetric turbulent jet. *Accepted for publication in Physics of Fluids*, June 2002.

M. N. Glauser and W. K. George. Application of multipoint measurements for flow characterization. *Experimental Thermal and Fluid Science*, 5:617–632, 1992.

M.N. Glauser and W.K. George. Orthogonal decomposition of the axisymmetric jet mixing layer including azimuthal dependence. In G. Comte-Bellot and J. Mathieu, editors, *Advances in Turbulence*, pages 357–366. Spring-Verlag, 1987.

P. B. V. Johansson and W. K. George. Far downstream development of POD modes in a turbulent disk wake. In *Ninth European Turbulence Conference*, Southampton, July 2-5, 2002.

P. B. V. Johansson, W. K. George, and S. H. Woodward. Proper orthogonal decomposition of an axisymmetric turbulent wake behind a disk. *Accepted for publication in Physics of Fluids*, June 2002.

J. L. Lumley. On the interpretation of time spectra in high intensity shear flows. *Physics of Fluids*, 8:1056–1062, 1965.

J. J. Miau, T. S. Leu, T. W. Liu, and J. H. Chou. On vortex shedding behind a circular disk. *Experiments in Fluids*, 23: 225–233, 1997.

P. A. Monkewitz. A note on vortex shedding from axisymmetric bluff bodies. *Journal of Fluid Mechanics*, 192:561–575, 1988.

