AN LDA BASED METHOD FOR ACCURATE MEASUREMENTS OF THE DISSIPATION RATE TENSOR WITH APPLICATION TO A CIRCULAR JET

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ABSTRACT

We report on the development of an LDA based method by which all the different terms in the dissipation rate tensor can be measured. It is shown that the homogenous part of this tensor can be expressed as a sum of two terms, the second derivative of the two point correlation tensor with respect to the separation between the points and the second derivative with respect to position of the Reynolds stress tensor. The inhomogenous part contains mixed derivatives of the same quantities. An error analysis is performed which shows that the dominating errors are due to the uncertainty in the determination of the distance between the measuring points and to misalignment of the laser beams. The method is applied to the study of a low Reynolds number circular air jet with Kolmogorov microscale of the order of 100 µm and, except for a region close to the outlet nozzle, the error is shown to be of order 5 - 10%. Some results from the measurements are also presented.

1. INTRODUCTION

Reynolds stress modelling of turbulent flow fields requires that an accurate and general model of the dissipation rate tensor is available. A number of such models have been proposed, but direct experimental verification of them is very difficult. For a discussion of these and related matters see e.g. George & Taulbee (1990). In the Reynolds stress equations the terms associated with dissipation usually appears as

$$D_{ij} = D_{ij}^{h} + D_{ij}^{n} \tag{1}$$

$$D_{ij}^{h} = 2\,\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} \tag{2}$$

$$D_{ij}^{n} = \nu \frac{\partial}{\partial x_k}\left(\overline{u_i \frac{\partial u_k}{\partial x_j}}\right) + \nu \frac{\partial}{\partial x_k}\left(\overline{u_j \frac{\partial u_k}{\partial x_i}}\right) \tag{3}$$

where ν is the kinematic viscosity of the fluid, u_i is the velocity vector and x_i is co-ordinate direction. The overbar denotes an unbiased ensemble average. The terms appearing in equation (3) are sometimes written in slightly modified forms, but the way of expression used in equation (3) is best adopted to our way of measurement.

It must be noted that the upper index, h or n appearing in equations (1)-(3), are not tensor indices. h is used to indicate the homogenous part of the tensor and n the non-homogenous part. This convention is also followed for other quantities appearing as superscripts in this paper.

One of the main difficulties in experimental studies of dissipation is that they require that measurements are obtained in two nearby points, without the measurement in one of the points disturbs the measurement in the other, and that the small difference in velocity between the two points is well resolved. In addition to this the effective measuring control volume must be very small, of the order of the Kolmogorov microscale, to avoid that the velocity may vary noticeably across the measuring control volume or that the distance between the measuring points may not be accurately known.

The laser Doppler anemometer has for a long time been considered attractive for this kind of measurements, since it automatically solves one of the most severe of these problems; it measures without disturbing the flow. The size of the measuring control volume can also be made very small by using expanded laser beams and side scattering. It has however some severe drawbacks, in particular the influence of a rather high noise level, usually of the order of 1 percent, and influence of imperfect beam alignment.

We report here on the development of an LDA technique which solves these problems, thus making accurate direct measurements of all the elements in the dissipation rate tensor possible. Some results on a circular jet are included.

2. THE DISSIPATION RATE TENSOR

2.1 Homogeneous part of the dissipation rate tensor

To try to measure the various terms appearing in the homogenous dissipation rate tensor by direct measurements of the velocity in two points simultaneously and forming the average of the product of their difference would lead to unacceptable error levels. This is due to the fact that the calibration constants of the two systems can not be determined accurately enough to permit the small difference in velocity between the two measuring points to be measured accurately. We therefore have to transform the expression (2) to a form that will permit more accurate measurements to be performed. We write

$$\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k} = \lim_{\Delta x_k \to 0} \frac{u_i(\mathbf{x}_a) - u_i(\mathbf{x}_b)}{\Delta x_k} \times \frac{u_j(\mathbf{x}_a) - u_j(\mathbf{x}_b)}{\Delta x_k}$$

$$= \lim_{\Delta x_k \to 0} \left(\frac{u_i u_j(\mathbf{x}_a) + u_i u_j(\mathbf{x}_b)}{\Delta x_k^2} - \frac{u_i(\mathbf{x}_a)u_j(\mathbf{x}_b) + u_i(\mathbf{x}_b)u_j(\mathbf{x}_a)}{\Delta x_k^2} \right) \tag{4}$$

where we have written

$$u_i(\mathbf{x}_a) = u_i\left(\mathbf{x}_0 + \frac{\Delta x_k}{2}\right) \tag{5}$$

$$u_i(\mathbf{x}_b) = u_i\left(\mathbf{x}_0 - \frac{\Delta x_k}{2}\right) \tag{6}$$

and we, thus, have made a symmetric separation of the two points around the centre point x_0.

The first term on the right hand side of equation (4) contains only one-point correlations. By expanding them in a Taylor series around x_0 we obtain after some algebra

$$u_i(x_a)u_j(x_a) + u_i(x_b)u_j(x_b) = 2R_{ij} + \frac{\Delta x_k^2}{4}\frac{\partial^2 R_{ij}}{\partial x^2} + O\left(\Delta x_k^4\right) \tag{7}$$

where

$$R_{ij} = u_i u_j \tag{8}$$

and it is understood that all terms on the right hand side of (7) are to be evaluated at the point x_0.

The second term on the right hand side of equation (4) is of a different nature. It consists of products of velocity components obtained in two different points. If we write it in the form

$$u_i(x_a)u_j(x_b) + u_i(x_b)u_j(x_a) = u_i\left(x_0 + \frac{\Delta x_k}{2}\right)u_j\left(x_0 - \frac{\Delta x_k}{2}\right) + u_i\left(x_0 - \frac{\Delta x_k}{2}\right)u_j\left(x_0 + \frac{\Delta x_k}{2}\right) \tag{9}$$

we see that it is symmetric in Δx. We will now consider each term in (9) as a two-point function, denoted by Q_{ij}, of the separation Δx_k. Due to the symmetry we thus have

$$u_i(x_a)u_j(x_b) + u_i(x_b)u_j(x_a) = Q_{ij}(x_0, \Delta x_k) + Q_{ij}(x_0, -\Delta x_k) \tag{10}$$

where

$$Q_{ij}(x_0, \Delta x_k) = u_i\left(x_0 + \frac{\Delta x_k}{2}\right)u_j\left(x_0 - \frac{\Delta x_k}{2}\right) \tag{11}$$

Expanding this in a Taylor series around $\Delta x_k=0$ gives

$$u_i(x_a)u_j(x_b) + u_i(x_b)u_j(x_a) = 2R_{ij}(x_0) + \Delta x_k^2\frac{\partial^2 Q_{ij}}{\partial \Delta x_k^2} + O\left(\Delta x_k^4\right) \tag{12}$$

If equations (7) and (12) now are entered into equation (4) and an ensemble average is taken of the resulting expression we obtain

$$D_{ij}^h = 2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} = 2\nu\left(\frac{1}{4}\frac{\partial^2 \overline{R_{ij}}}{\partial x_k^2} - \frac{\partial^2 \overline{Q_{ij}}}{\partial \Delta x_k^2}\right) \tag{13}$$

It must be noted that the first term on the right hand side of equation (13) is the second derivative of a one-point correlation, $\overline{R_{ij}}$, with respect to the co-ordinate x_k, while the second term is the derivative of a two-point correlation, $\overline{Q_{ij}}$, with respect to the separation between the two points.

2.2 Inhomogenous part of the dissipation rate tensor

The primary expression to be considered when evaluating the inhomogenous dissipation rate tensor is according to equation (3)

$$u_i\frac{\partial u_k}{\partial x_j}$$

This can be expressed as

$$u_i\frac{\partial u_k}{\partial x_j} = \lim_{\Delta x_j \to 0} \frac{u_i\left(x_0 + \frac{\Delta x_j}{2}\right) + u_i\left(x_0 - \frac{\Delta x_j}{2}\right)}{2} \times \frac{u_k\left(x_0 + \frac{\Delta x_j}{2}\right) - u_k\left(x_0 - \frac{\Delta x_j}{2}\right)}{\Delta x_j}$$

$$= \lim_{\Delta x_j \to 0} \frac{R_{ik}\left(x_0 + \frac{\Delta x_j}{2}\right) - R_{ik}\left(x_0 - \frac{\Delta x_j}{2}\right)}{2\Delta x_j} - \lim_{\Delta x_j \to 0} \frac{Q_{ik}(x_0, \Delta x_j) - Q_{ik}(x_0, -\Delta x_j)}{2\Delta x_j} \tag{14}$$

where R_{ik} and Q_{ik} is defined by equations (8) and (11) respectively. Note again that R_{ik} is a quantity defined in one point, while Q_{ik} is a two-point quantity.

If we expand the terms in equation (14) which contain R_{ik} in a Taylor series around x_0 we obtain

$$R_{ik}\left(x_0 + \frac{\Delta x_j}{2}\right) - R_{ik}\left(x_0 - \frac{\Delta x_j}{2}\right) = \Delta x_j\frac{\partial R_{ik}}{\partial x_j} + O\left(\Delta x_j^2\right) \tag{15}$$

Similarly, if we expand the terms containing Q_{ik} in equation (14) in a Taylor series around $\Delta x_j=0$, keeping x_0 constant, we obtain

$$-Q_{ik}\left(x_0,\Delta x_j\right)+Q_{ik}\left(x_0,-\Delta x_j\right)=-2\Delta x_j\frac{\partial Q_{ik}}{\partial \Delta x_j}+O\left(\Delta x_j^2\right) \quad (16)$$

If the last two expressions are entered into equation (14) and an ensemble average is formed on the resulting expression we obtain

$$\overline{u_i\frac{\partial u_k}{\partial x_j}}=\frac{\partial \overline{R_{ik}}}{\partial x_j}-2\frac{\partial \overline{Q_{ik}}}{\partial \Delta x_j} \quad (17)$$

The non-homogenous dissipation rate tensor can thus be expressed as

$$\begin{aligned}D_{ij}^n &= \nu\frac{\partial}{\partial x_k}\left(\overline{u_i\frac{\partial u_k}{\partial x_j}}+\overline{u_j\frac{\partial u_k}{\partial x_i}}\right)\\ &= \nu\frac{\partial}{\partial x_k}\left(\frac{\partial \overline{R_{ik}}}{\partial x_j}+\frac{\partial \overline{R_{jk}}}{\partial x_i}-2\left(\frac{\partial \overline{Q_{ik}}}{\partial \Delta x_j}+\frac{\partial \overline{Q_{jk}}}{\partial \Delta x_i}\right)\right)\end{aligned} \quad (18)$$

It is thus clear that the primary quantities to measure in order to obtain a measurement of the dissipation rate tensor are the Reynolds stress tensor, $\overline{R_{ik}}$, and the two-point correlation tensor, $\overline{Q_{ik}}$.

3. EXPERIMENT

3.1 Measurement technique

The basic requirement of an instrument used to measure the dissipation rate tensor is an ability to measure in two different points simultaneously. We accomplish this by using four fibre optic probes operating in pairs, see figure 1. One probe in each pair is used to emit the laser beams and the other to collect the scattered light at 90° collection angle. With this arrangement it is possible to reduce the measuring control volume considerably. Its diameter decreases in proportion to

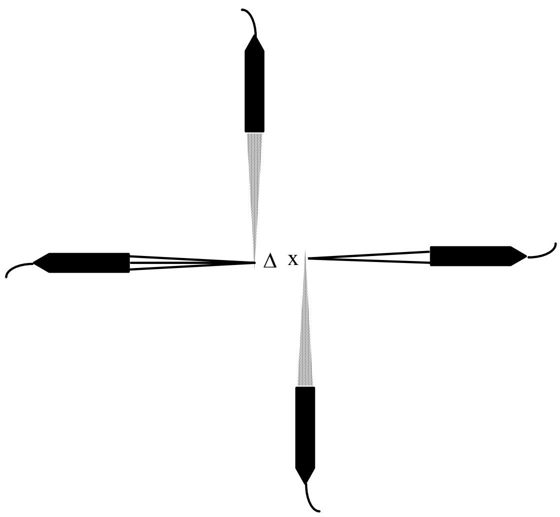

Figure 1. Probe configuration.

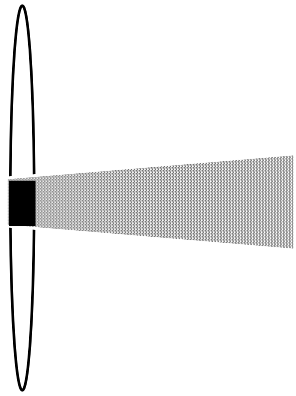

Figure 2. The measuring control volume.

the reciprocal of the F-number of the emitted beams, which is given by the optics used and can be reduced by beam expansion modules. The length of the measuring control volume, see figure 2, is determined by the receiving optics. In our case, we use Dantec probes and the length of the measuring control volume is then approximately 80% larger than the diameter of the beam waists. We obtain a diameter of 45 µm and a length of 80 µm.

Each pair of the probes are traversed symmetrically about a centre point. In this way a number of error terms directly proportional to the distance between the measuring control volumes vanish identically (as opposed to a case when only one of the probe volumes would be traversed). The probes are brought to measure the velocities in the two points at five different separation distances in turn and the correlation coefficient and the rms values of the measured velocity components are computed. These data are then fitted to a fourth order polynomial

$$\overline{C_{ij}}=p_{0,ij}+p_{2,ij}(\Delta x)^2+p_{3,ij}(\Delta x)^3+p_{4,ij}(\Delta x)^4 \quad (19)$$

(if i = j, $p_{3,ij} = 0$) and the parameter $p_{2,ij}$ is evaluated. The second derivative of $\overline{C_{ij}}$ with respect to Δx as Δx goes to zero is equal to twice the value of this parameter. In this way the second derivative of the correlation function at zero separation is properly obtained. The procedure is then repeated for separation in the other two co-ordinate directions.

One of the probe pairs is always used to measure the velocity in the main flow direction, even in cases when we do not need to measure this component to obtain a particular element of the dissipation rate tensor In this way we are always able to perform a speed weighting of our data, thus permitting us to compensate for particle statistical bias.

The Reynolds stresses were measured using one of the probe pairs in a conventional set up to measure all three velocity components simultaneously.

3.2 The jet flow rig

A simple experiment has been carried out. An air jet with a speed of 10 m/s at the outlet of a nozzle with a diameter d=8 mm entered into a chamber 70∞70 cm in cross section and 1.2 m high, see figure 3. The nozzle was made as a part of the bottom wall of the chamber and was turned in the shape of a

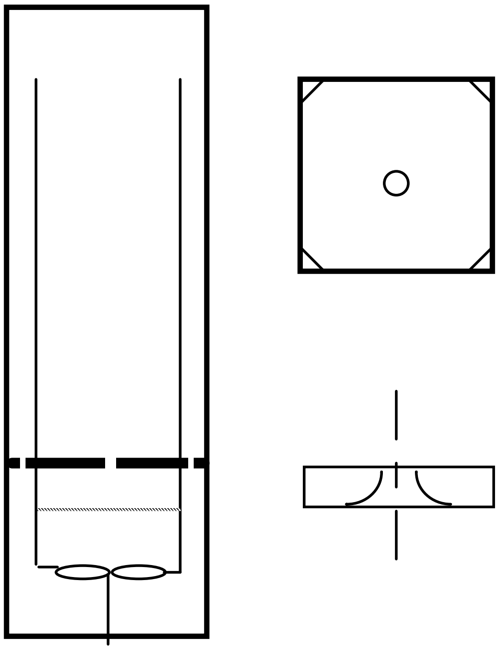

Figure 3. The jet flow rig.

circular arc, see inset in figure 3. The walls of the chamber were made of glass to permit the laser beams to enter into the chamber from the probes which were placed outside the chamber. The flow returned through triangular channels at the corners of the chamber. The flow was driven by a carefully regulated fan. Between the fan and the nozzle a number of screens were placed to reduce the disturbance level of the air stream entering the nozzle. The outlet velocity profile had a top hat shape with very constant velocity across the nozzle and a low turbulence level.

4. ERROR ESTIMATES

Our method to measure the different terms in the dissipation rate tensor involves the use of two separate LDA systems. The Doppler frequency measured by one of the systems (I) in point x_a at the particular instant of time x^I is given by

$$F_D^I\left(x_a, t^I\right) = \overline{F_D^I}\left(x_a, t^I\right) + f_D^I\left(x_a, t^I\right) + f_n^I\left(x_a, t^I\right) \tag{20}$$

$$\overline{F_D^I}\left(x_a, t^I\right) = c^I \mathbf{s}^I \cdot \overline{\mathbf{V}}\left(x_a, t^I\right) \tag{21}$$

$$f_D^I\left(x_a, t^I\right) = c^I \mathbf{s}^I \cdot \mathbf{v}\left(x_a, t^I\right) \tag{22}$$

where we have split the velocity vector $\mathbf{V}$ into its average $\overline{\mathbf{V}}$, and its fluctuation $\mathbf{v}$. Note that the overbar denotes a non-biased average. Further, c^I is the "calibration" constant of the system I, giving the ratio of the detected Doppler frequency of the projection of the velocity vector $\mathbf{V}$ in the direction $\mathbf{s}^I$ (unit vector), which points in the direction of the bisector to the two emitted beams of system I. f_n^I is the noise contribution to the detected Doppler frequency. Analogous formulas are of course available for the measurement in the point x_b using system II.

To carry the analysis further using general expressions would make our formulas very cluttered. We therefore choose to proceed by developing the expressions only for a certain specific choice of combination of velocity components and to imply the others by way of analogy. We will also confine ourselves to an analysis of the $\overline{Q_{ij}}$ - terms and leave the $\overline{R_{ij}}$ - terms since these are much smaller than the two point correlations and varies with position at a much smaller rate than $\overline{Q_{ij}}$ and thus contribute very little to the final error estimates.

Each system is, at a certain phase of the experiment, used to measure the velocity component in the direction of one of the co-ordinate axis. The unit vectors in which we measure are therefore always one of the following

$$\mathbf{s}_\mathbf{x}^\mathbf{I} = \left(\sqrt{1-\left(\varepsilon_y^I\right)^2-\left(\varepsilon_z^I\right)^2},\ \varepsilon_y^I,\ \varepsilon_z^I\right) \tag{23}$$

$$\mathbf{s}_y^\mathbf{I} = \left(\varepsilon_x^I,\ \sqrt{1-\left(\varepsilon_x^I\right)^2-\left(\varepsilon_z^I\right)^2},\ \varepsilon_z^I\right) \tag{24}$$

$$\mathbf{s}_z^\mathbf{I} = \left(\varepsilon_x^I,\ \varepsilon_y^I,\ \sqrt{1-\left(\varepsilon_x^I\right)^2-\left(\varepsilon_y^I\right)^2}\right) \tag{25}$$

with analogous expressions for system II. ε_x^I, ε_y^I and ε_z^I are small components of the unit vectors due to some inescapable misalignment of the laser beams.

We now consider the specific example of obtaining Q_{12}. With the aid of the above expressions and equation (10) we obtain

$$\begin{aligned} Q_{12} &= u(x_a)v(x_b) \\ &= \frac{\dfrac{f_D^I(x_a)-f_n^I(x_a)}{c^I} - \varepsilon_y^I v(x_a) - \varepsilon_z^I w(x_a)}{\sqrt{1-\left(\varepsilon_y^I\right)^2-\left(\varepsilon_z^I\right)^2}} \\ &\times \frac{\dfrac{f_D^{II}(x_b)-f_n^{II}(x_b)}{c^{II}} - \varepsilon_x^{II} u(x_b) - \varepsilon_z^{II} w(x_b)}{\sqrt{1-\left(\varepsilon_x^{II}\right)^2-\left(\varepsilon_z^{II}\right)^2}} \end{aligned} \tag{26}$$

where we have implicitly assumed that system I is measuring in position x_a and system II in position x_b. In addition to this we have also here written the velocity vector as (u, v, w).

The directional errors, ε, are often of the order of~0.01. This implies that they can safely be neglected compared to 1 in the square roots in the denominator of equation (24). We further assume that the noise terms, f_n^I and f_n^{II}, are uncorrelated with all other terms except when correlated with themselves. Forming a non-biased average of Q_{12} based on a large number of samples we then get

$$\overline{Q_{12}} = \frac{\overline{f_D^I(x_a) f_D^{II}(x_b)}}{c^I c^{II}}$$
$$- \frac{\varepsilon_x^{II}}{c^I} \overline{f_D^I(x_a) u(x_b)} - \frac{\varepsilon_z^{II}}{c^I} \overline{f_D^I(x_a) w(x_b)} \quad (27)$$
$$- \frac{\varepsilon_y^I}{c^{II}} \overline{f_D^{II}(x_b) v(x_a)} - \frac{\varepsilon_z^I}{c^{II}} \overline{f_D^{II}(x_b) w(x_a)}$$

When deriving this expression we have also neglected term which contain products of directional errors. These term are of order 10^{-4} times smaller than the dominating terms. In this specific example (Q_{12}) we note that all noise terms have been cancelled since no terms where they were correlated with themselves were present.

If we further note that $f_D^I / c^I \approx u(x_a)$ and $f_D^{II} / c^{II} \approx v(x_b)$ we find that the last three terms in equation (27) representing the error in the measurement of $\overline{Q_{12}}$ leads to relative errors of order 0.01 since all correlations $\overline{u_i u_j}$ are of the same order of magnitude and the ε:s are of order 0.01. This error is thus of the order of a few percent. Note that signal noise does not contribute to this error.

It turns out that the scatter in the $\overline{Q_{12}}$ data is larger than the scatter in the corresponding correlation coefficient, which is defined as

$$\overline{C_{12}} = \frac{\overline{Q_{12}}}{\sqrt{\overline{u^2(x_a)}}\sqrt{\overline{v^2(x_b)}}} \quad (28)$$

When taking the second derivative of $\overline{Q_{12}}$, as is required to obtain the homogenous dissipation rate tensor according to equation (13), this is of importance, and we thus prefer to work with the correlation coefficient.

The rms values $\sqrt{\overline{u^2(x_a)}}$ and $\sqrt{\overline{v^2(x_b)}}$ can be computed from equations (20) - (22) in a way analogous to the derivation of equation (27). We find

$$\sqrt{\overline{u^2(x_a)}} = \frac{\sqrt{\overline{\left(f_D^I(x_a)\right)^2}}}{c^I} \left(1 + \alpha_u\right) \quad (29)$$

$$\sqrt{\overline{v^2(x_b)}} = \frac{\sqrt{\overline{\left(f_D^{II}(x_b)\right)^2}}}{c^{II}} \left(1 + \alpha_v\right) \quad (30)$$

where

$$\alpha_u = \frac{1}{2} \frac{\overline{\left(f_n^I(x_a)\right)^2}}{\overline{\left(f_D^I(x_a)\right)^2}} - \varepsilon_y^I \frac{\overline{u(x_a) v(x_a)}}{\overline{\left(u(x_a)\right)^2}} - \varepsilon_z^I \frac{\overline{u(x_a) w(x_a)}}{\overline{\left(u(x_a)\right)^2}} \quad (31)$$

$$\alpha_v = \frac{1}{2} \frac{\overline{\left(f_n^{II}(x_b)\right)^2}}{\overline{\left(f_D^{II}(x_b)\right)^2}} - \varepsilon_x^{II} \frac{\overline{u(x_b) v(x_b)}}{\overline{\left(v(x_b)\right)^2}} - \varepsilon_z^{II} \frac{\overline{v(x_b) w(x_b)}}{\overline{\left(v(x_b)\right)^2}} \quad (32)$$

If these expressions are entered into equation (28) we find after considerable algebra

$$\overline{C_{12}} = \frac{\overline{f_D^I(x_a)}\,\overline{f_D^{II}(x_b)}}{\sqrt{\overline{\left(f_D^I(x_a)\right)^2}}\sqrt{\overline{\left(f_D^{II}(x_b)\right)^2}}} \left(1 - \alpha_C(x_k)\right) - \beta_C(\Delta x_k) \quad (33)$$

where

$$\alpha_C = \frac{1}{2} \frac{\overline{\left(f_n^I(x_a)\right)^2}}{\overline{\left(f_D^I(x_a)\right)^2}} + \frac{1}{2} \frac{\overline{\left(f_n^{II}(x_b)\right)^2}}{\overline{\left(f_D^{II}(x_b)\right)^2}} - \varepsilon_x^{II} \frac{\overline{u(x_b) v(x_b)}}{\overline{v(x_b)^2}}$$
$$- \varepsilon_z^{II} \frac{\overline{v(x_b) w(x_b)}}{\overline{v(x_b)^2}} - \varepsilon_y^I \frac{\overline{u(x_a) v(x_a)}}{\overline{u(x_a)^2}} - \varepsilon_z^I \frac{\overline{u(x_a) w(x_a)}}{\overline{u(x_a)^2}} \quad (34)$$

$$\beta_C = \varepsilon_x^{II} \frac{\overline{u(x_a) u(x_b)}}{\sqrt{\overline{u(x_a)^2}}\sqrt{\overline{v(x_b)^2}}} + \varepsilon_z^{II} \frac{\overline{u(x_a) w(x_b)}}{\sqrt{\overline{u(x_a)^2}}\sqrt{\overline{v(x_b)^2}}}$$
$$+ \varepsilon_y^I \frac{\overline{v(x_b) v(x_a)}}{\sqrt{\overline{u(x_a)^2}}\sqrt{\overline{v(x_b)^2}}} + \varepsilon_z^I \frac{\overline{v(x_b) w(x_a)}}{\sqrt{\overline{u(x_a)^2}}\sqrt{\overline{v(x_b)^2}}} \quad (35)$$

When deriving equations (29) - (35) we have as before neglected terms which contain products of directional errors, correlations of noise terms with anything except themselves and products of noise and directional errors

The α- terms contain only one-point data, which changes very slowly compared to two-point terms. The terms are small, of the order of 0.01, and the second derivative of these terms can safely be neglected. A word of warning is however in place, since the noise terms may, due to spurious effects, vary also over short distances.

The second term, β_C, in equation (33), contain terms which are also small, of the order of 0.01. However, they, contain two-point data, which are expected to vary in space at approximately the same speed as the dominating term in equation (33). This implies that they will contribute an error term to the homogenous dissipation rate, which is of the same order as their contribution to the correlation term.

The homogenous dissipation rate is finally computed as

$$D_{12}^h = 2\nu \overline{\frac{\partial u}{\partial x_k} \frac{\partial v}{\partial x_k}} = 2\nu \left(\frac{1}{4} \frac{\partial^2 \overline{R_{12}}}{\partial x_k^2} - \frac{\partial^2 \overline{Q_{12}}}{\partial \Delta x_k^2} \right)$$
$$= 2\nu \left(\frac{1}{4} \frac{\partial^2 \overline{R_{12}}}{\partial x_k^2} - \frac{\partial^2}{\partial \Delta x_k^2} \left(\sqrt{\overline{u^2(x_a)}} \sqrt{\overline{v^2(x_b)}}\; \overline{C_{12}} \right) \right) \quad (36)$$

We have demonstrated that the relative errors due to noise and misalignment in $\overline{Q_{12}}$, $\sqrt{\overline{\left(u(x_a)\right)^2}}$ and $\sqrt{\overline{\left(v(x_b)\right)^2}}$ are all of the order of a few percent. The errors involved in the measurement of $\overline{R_{12}}$ is of the same nature and thus of the same order of magnitude. Moreover, the second derivative of $\overline{R_{12}}$ is to be obtained with respect to x_k, while the derivative of $\overline{Q_{12}}$ is to be evaluated with respect to Δx_k. The variation in space of any quantity with respect to x_k is an order of magnitude slower than the variation with respect to Δx_k, and, thus, the error in D_{12}^h is dominated by the error in $\overline{Q_{12}}$. In the following we will therefore neglect all errors except those in $\overline{Q_{12}}$.

We have so far considered errors due to signal noise and misalignment of the laser beams. There are however some additional errors that must be considered. These are

a) Statistical uncertainty in the measurement of $\overline{Q_{12}}$.
b) Errors in the determination of c^I, C^{II}.
c) Error due to the finite size of the measuring control volumes.
d) Errors in the determination of the effective distance between the two measuring points.
e) Particle statistical bias.
f) Particle density bias.

The statistical uncertainty, in the measurement of $\overline{Q_{12}}$, has been investigated experimentally. In a certain point and for a certain separation we measured $\overline{Q_{12}}$ 20 times. From this ensemble of $\overline{Q_{12}}$-values, the mean and rms values could be calculated. The central limit theorem then tells us that this ensemble can be modelled as having a Gaussian distribution, and thus the uncertainty in the determination of the mean value of $\overline{Q_{12}}$ can be estimated as $\frac{1.96\sigma}{\sqrt{20}}$ at 95% confidence, where σ is the rms value of the ensemble. We found that if each measurement of $\overline{Q_{12}}$ was based on 2.5 minutes integration time, the error was less than 1%. We thus concluded that if we based our estimates of $\overline{Q_{12}}$ on 50 minutes records the corresponding error should be of this order or less.

Errors in c^I and C^{II} enters in the computation of the rms-values multiplying the two-point correlation coefficient in equation (36). These quantities can usually be determined within 1%, and thus contributes to the relative error in the homogenous dissipation tensor with the same value.

The measurement of dissipation is essentially a measurement of the variances of spatial derivatives of the velocity components. It is thus clear that the measuring control volumes must have a linear dimension significantly smaller than a distance over which the instantaneous velocity changes noticeably. This distance is the Kolmogorov microscale. In our experiment this scale varied from about 70 to 180 µm. Our measuring control volumes had a shape like a circular cylinder with diameter 45 µm and length 80 µm. This is considered to be a sufficient spatial resolution to make the error due to the finite size of the measuring control volume negligible.

Errors due to the uncertainty in the determination of the effective distance between the two measuring points can be evaluated as follows. There are three sources of this error, an imperfection in the determination of the position where the two measuring points coincide, Δ_0, an error due to the fact that scattering particles cross the measuring control volume at varying positions, Δ_p, and an error in the traversing of the two measuring control volumes, Δ_t.

The first error, Δ_0, was evaluated during the adjustment of the measuring control volumes. We found that the coincident data rate changed noticeably when we moved one of the measuring control volumes a distance of 5 - 10 µm. This turns out to be the dominating error in the determination of the effective distance between the measuring control volumes. This is thus our estimate of the zero position error.

The particles cross the measuring control volumes at arbitrary positions. The separations were such that the main flow direction had one of the relations to the measuring control volume depicted in figure 4.

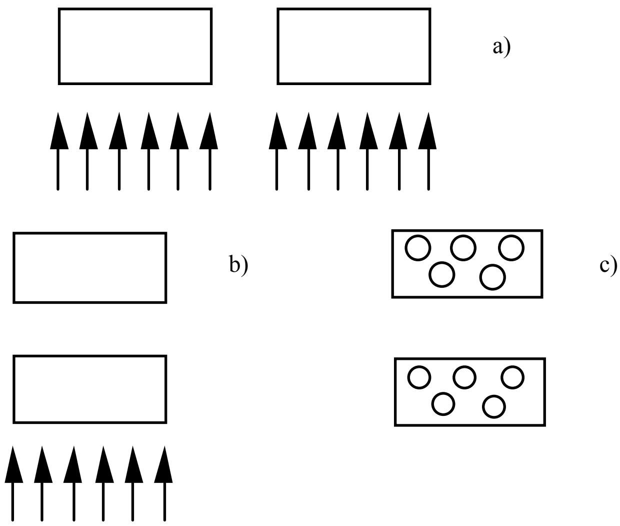

Figure .4 The possible relations between the inflow of particles to the measuring control volume and their separation.

In figure 4a) the measuring control volumes are separated along their longest dimension and the main flow direction is from below. In b) the measuring control volumes have been separated in the main flow direction and in c) they are separated sideways such that the main flow is coming out of the paper. It is immediately realised that in cases a) and c) the influence of the randomness in position of the particles do not change the effective distance. In case b) we examine the cases that the particles give samples of equal probability either anywhere in the two measuring control volumes or anywhere in the lower halves of them. In either case we obtain the result that the effective distance between the two measuring control volumes is unaffected by the randomness of the particle position.

The third position error, Δ_t, due to imperfect translation of the probes is believed to be very small. The mechanical devices used for the translation was very stable and the scales used to read the translations were accurate to within 2 µm. A check on the translation error was performed by returning the probes to zero distance every time the measurements at the longest translation was finished. We were in all cases able to recover the original correlation coefficient very closely

The errors in the primarily measured quantities $\overline{Q_{ij}}$ and $\overline{R_{ij}}$ due to the uncertainty in the determination of the effective distance between the two measuring volumes is dominated by the zero position error Δ_0. In order to estimate the error in the homogenous dissipation rate tensor, D_{ij}^h, due to this error a numerical simulation was performed. The parameter $p_{2,ij}$ in equation (19) was evaluated for three sets of separation distances, corresponding to Δ_0 =-10μm, Δ_0 =0 and Δ_0 =+10 μm. In our particular case we found that $p_{2,ij}$ varied between 2 - 6% at different position at the centreline in our jet flow.

Two possible sources of bias errors are at hand, particle density bias and particle statistical bias. The first of these is due to the fact that only the flow coming out of the jet was seeded. The flow was however recirculated and the flow was seeded at short time intervals, which indicates that the seeding level should be fairly uniform. Although we cannot give a quantitative estimate of the error due to this effect we believe that it is smaller than or at the most of the same order of magnitude as the averaging error, i.e. of the order of 1%.

It is well established that the particle statistical bias is caused by the increased probability to obtain a sample when the volume flow rate through the measuring control volume is high. One can correct for this effect in single point measurements by weighting each sample with the reciprocal of the volume flow rate, see e.g. McLaughlin & Tiederman (1973) or Buchhave et al (1979). In two point measurements the situation is slightly more complicated, but in our case it is simplified by the fact that, due to the short distance between the measuring control volumes, the volume flow rate is very nearly the same in both points. The proper weighting factor is then again the reciprocal of the volume flow rate through either of the measuring control volumes. In all our measurements we have measured the longitudinal velocity component in at least one of the measuring points, i.e. even when the primary interest was to measure the two radial or the two tangential velocity components. In this way we were able to correct properly for the particle statistical bias effect in all measurements.

We have thus found that two sources of error dominates, the error due to misalignment of the laser beams and the error due to the zero position offset. Our analysis shows that the total error in the homogenous dissipation rate tensor thus can be estimated to be of the order of 5 - 10%.

5. SOME RESULTS AND DISCUSSION

We shall show some examples of our results obtained along the centreline of the jet and limit ourselves to the diagonal elements of the homogenous dissipation rate tensor. At the centre line all the off-diagonal elements are zero. Figure 5 shows the two-point correlation coefficient C_{22} at x_1/d=60, for separations in the three co-ordinate directions. It can be seen that C_{22} decreases roughly twice as fast in the x_1-direction as in the x_2-direction. In isotropic turbulence this is expected, but we would also have expected that $C_{22}(\Delta x_1)=C_{22}(\Delta x_3)$ which clearly is not the case in our measurements.

A clearer impression of the second derivatives of the correlation coefficients is obtained by plotting it versus the square of the separation distance. This is done for i=j=1 in figure 6. The open symbols show C_{11} versus $(\Delta x_2/x_1)^2$ at x_1/d=20, 40 and 60. It can be seen that C_{11} drops with a constant slope down to approximately 0.95. The dominating terms in the series expansion, equation (19), are thus $p_{0,11}$ and $p_{2,11}$, the latter being half the second derivative of C_{11} with respect to Δx_2. The estimated values of $p_{2,11}$ shown in the figure are obtained by fitting the data to the polynomial (19) including the $(\Delta x_2)^4$-term, but no higher order terms. Included in the plot is a measurement of C_{11} versus $(\Delta x_3/x_1)^2$

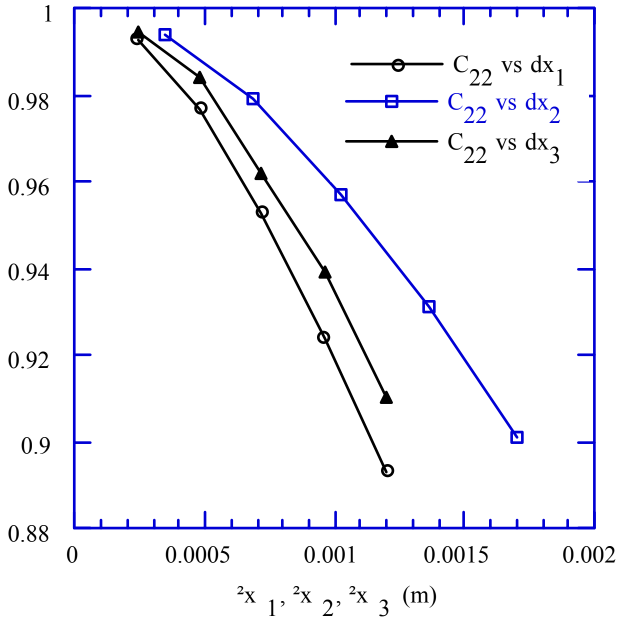

Figure 5. The two point correlation coefficient $\overline{C_{22}}$ for separation of the measuring points in the three co-ordinate directions at the centre-line, 60 diameters downstream of the nozzle.

at x_1/d=40 (filled squares), which should be identical to the measurements for separation in the x_2-direction. Although the measurements differ slightly at large separations the estimated values of $p_{2,11}$ are very close.

A sensitivity analysis of the influence of the zero position error was made using these data. It was found that a position error of 10 μm changed the estimated value of p_2

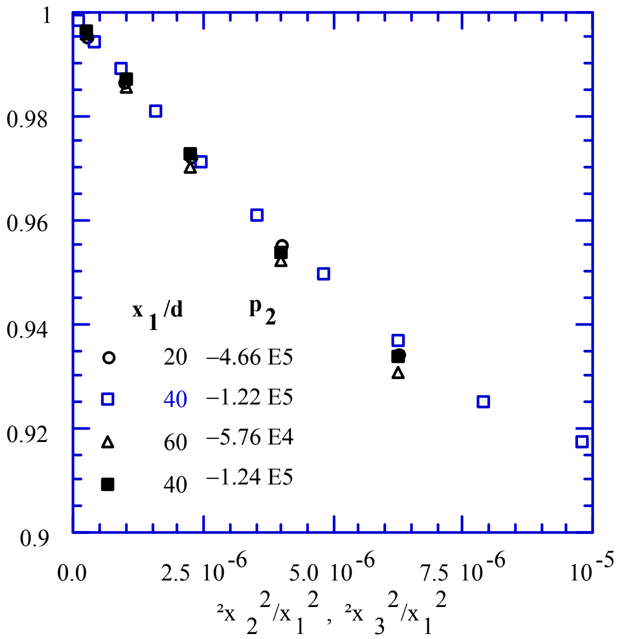

Figure 6. $\overline{C_{11}}$ on the centre-line, 20, 40 and 60 diameters downstream of the nozzle.

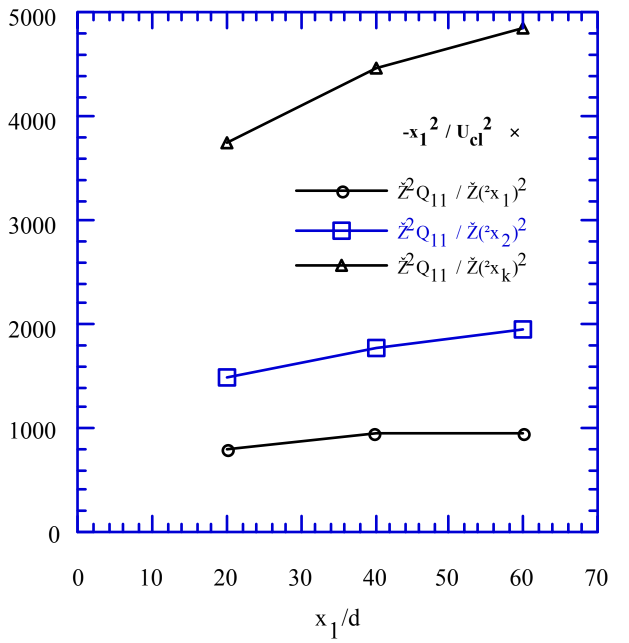

Figure 7. Normalised second derivatives of $\overline{Q_{11}}$.

with about 6% at x_1/d=20, decreasing to about 2% at x_1/d=60.

At the centre-line $\sqrt{\overline{u_i^2(x)}}$ was found to be very close to a constant, why $\partial^2 \overline{Q_{ij}} / \partial(\Delta x_k)^2$ becomes $\overline{R}_{ij} \partial^2 \overline{C_{ij}} / \partial(\Delta x_k)^2$, for i=j. This sum over k, and its elements are shown in figure 7, normalised with $-x_1^2 / U_{cl}^2$. Note that $\partial^2 \overline{Q_{11}} / \partial(\Delta x_3)^2$ is equal to $\partial^2 \overline{Q_{11}} / \partial(\Delta x_2)^2$ due to symmetry.

Figure 8 shows $\partial^2 \overline{Q_{ii}} / \partial(\Delta x_k)^2$ and its components

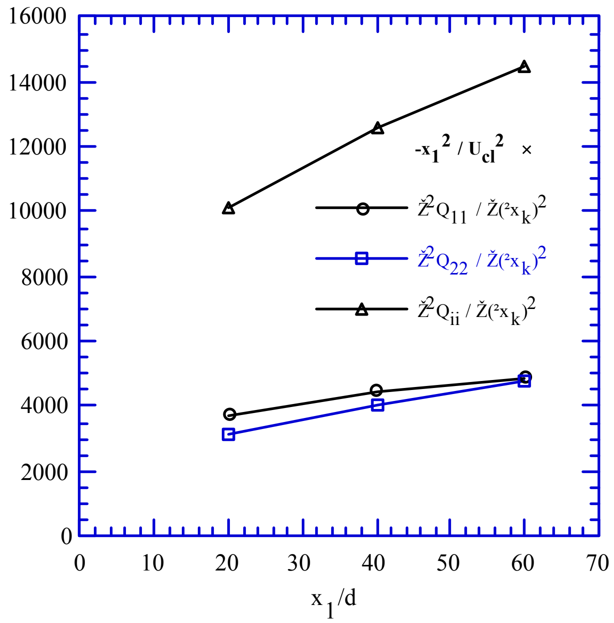

Figure 8. $\partial^2 \overline{Q_{ii}} / \partial(\Delta x_k)^2$ and its elements normalised with $-x_1^2/U_{cl}^2$.

from the different two-point correlations, once again normalised by $-x_1^2/U_{cl}^2$. If the first term on the right hand side of equation (36) is neglected, this represents $D_{ii}^h / 2\nu$, and its components.

All quantities shown in figures 7 and 8 should go to zero near the nozzle and become independent of x_1/d if the jet was self similar.

The normalised value of the scalar dissipation, $D_{ii}/2$, can be obtained from figure 8, if the inhomogenous part is neglected. Normalising the kinematic viscosity with z_1 U_{cl} yields $z_1 D_{ii} / 2U_{cl}^3$=0.255 at x_1/d=60. A number of estimates of this value have been published, preferably in the self similar region and at higher Reynolds numbers. Maybe the best comparison is made with Panchapakesan & Lumley (1993), who obtained the value of 0.174 (32% lower) from an energy balance in an air-jet at Re = 11 000.

6. CONCLUSIONS

We have here reported on a method by which all the elements in the dissipation rate tensor can be measured directly. This is accomplished by putting the expressions for the dissipation rate tensor into a form which contains second derivatives of the two point correlation with respect to the separation between the points and second derivatives for the Reynolds stresses with respect to the position for the homogenous part of the dissipation rate tensor and mixed second order derivatives of the same quantities for the inhomogenous part.

The method works for flows with Kolmogorov microscale of the order of 100 µm or larger. It should be possible to extend this to higher Reynolds number flows by using more expanded beams and very stable high precision mechanical traversing.

It has been shown that error bounds can be computed for the different elements of the dissipation rate tensor. It turns out that an accuracy of the order of 5-10% is attainable.

The method has one severe drawback; it is extremely time consuming. To obtain the complete dissipation rate tensor in one point will typically require several days of measurements.

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