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X-Wire Response in Turbulent Flows of High-Intensity Turbulence and Low Mean Velocities

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■ The angular response of an x-wire is studied at low velocities (0.25–1.4 m/s). It is found that the k-factor in the modified cosine law is strongly velocity-dependent. The implications of this for multicomponent measurement are explored, and a practical scheme for incorporating it is proposed. At lower velocities a total loss of direction sensitivity is observed that leads to additional errors in x-wire measurements. Expressions are derived for evaluating when cross-flow errors begin to affect x-wires. Also, a "dropout" phenomenon is observed in which certain voltage pairs from an x-wire cannot be converted into the velocity components. The implication of this dropout on the turbulence measurements is also discussed.

Keywords: *hot-wire anemometer, x-wire, turbulence measurements*

INTRODUCTION

It can be shown [1] that for laminar flow past an infinitely long and uniformly heated cylinder placed obliquely to a uniform undisturbed velocity field, the heat transfer is proportional only to the normal velocity component. This result is known as the cosine law because it implies

$$U_{\text{eff}}^2 = U_0^2 \cos^2 \phi, \tag{1}$$

where ϕ is the angle between the flow velocity vector and the plane normal to the wire as shown in Fig. 1.

In most hot-wire applications, however, the flow is not laminar and the temperature distribution is not uniform. Nonetheless, the cosine law is still the basic relationship on which empirical relations are based. There are situations in which the mean velocities are high and the turbulent intensities are small and the cosine law may work satisfactorily. There have been, however, few proponents of the cosine law for high-intensity turbulent flows since the work of Champagne and Sleicher [2], which showed significant deviations from this simple response.

Real hot wires depart from the cosine law because of their finite length and because of end losses to the prongs. The effect of both of these is to make the temperature distribution across the wire nonuniform. Hinze [3] and Webster [4] suggested the relation

$$\left(\frac{U_{\text{eff}}}{U_0}\right)^2 = \cos^2 \phi + k^2 \sin^2 \phi, \tag{2}$$

where the sum on the right-hand side accounts for the cooling caused by the velocity component along the wire. Hinze found the value of k to be between 0.1 and 0.3, whereas Webster found it to be 0.2. Champagne and Sleicher [2], in a systematic study on heat transfer of inclined wires, found k to be a function of the wire length-to-diameter ratio, l/d. They found $k = 0.2$ for $l/d = 250$. Other researchers have reported different values of k; for example Kawall et al. [5] report $k = 0.1$ for $l/d = 250$.

In addition to its angular sensitivity and the problems it creates at high turbulence intensity, there are additional problems that arise under these conditions. Hot wires, when used in the conventional way, respond only to the magnitude of velocity and are insensitive to its direction. As a consequence, the wire cannot distinguish when the sign of the velocity vector changes. This phenomenon is known as rectification and can be a major source of error in situations where flow reversals occur on the wire. The errors associated with rectification and cross flow can be greatly reduced by using the techniques of Kawall et al. [5], Legg et al. [6], and Lekakis et al. [7], all of whom employ a three-wire probe instead of the conventional x-wire. The problems associated with the conventional x-wires are discussed in detail in these references and also by Tutu and Chevray [8].

This paper attempts to summarize experience at our laboratory in using x-wires in high-intensity turbulent flows and in flows with low mean velocity, specifically, the effect of the latter on the velocity dependence of the angular response, the loss of directional sensitivity, and "dropout."

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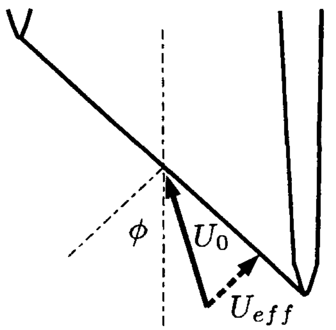

Figure 1. Geometry for inclined wire showing relation of effective velocity, U_{eff}, to flow velocity U_0. For the cosine law, $U_{\text{eff}} = U_0 \cos \phi$.

EXPERIMENTAL RESULTS AND DISCUSSION

The tungsten wires used in this study were 2.5 μm in diameter and were gold-plated at the ends to give an effective l/d of 250. These were operated in the constant-temperature mode using a Dantec 55M10 system. The overheat ratio was 0.4. This low overheat was used to optimize the wire response for its use in a non-isothermal flow and to reduce the velocity at which natural convection effects dominate. An additional hot wire was operated in the constant-current mode to obtain the temperature that was used to extract velocity information from the x-wire probe.

Figure 2 illustrates the angular dependence of a hotwire at velocities from 0.25 to 1.4 m/s. These low velocities occur in buoyancy-dominated flows such as plumes and at the outer edges of many other free shear flows. It is clear that the angular response is strongly dependent on the velocity U_0. Also note that a higher inclinations the wire's angular response is significantly reduced. Other investigators have noted this velocity dependence, although the effects were weaker because the velocities were not as

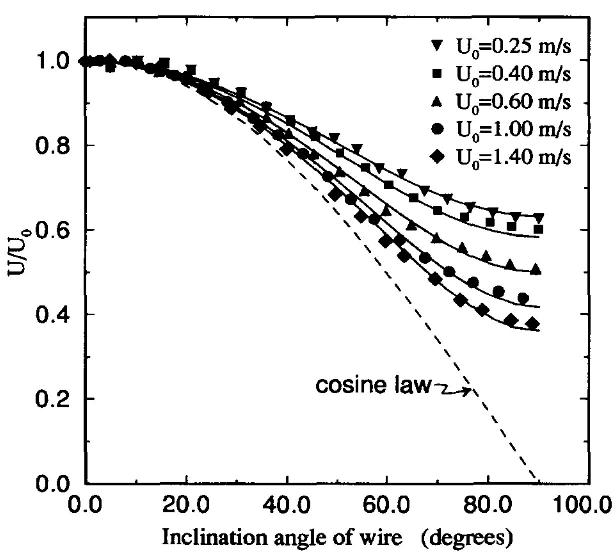

Figure 2. Angular response of an inclined hot wire for different flow velocities. For comparison purposes, the cosine law response is also shown.

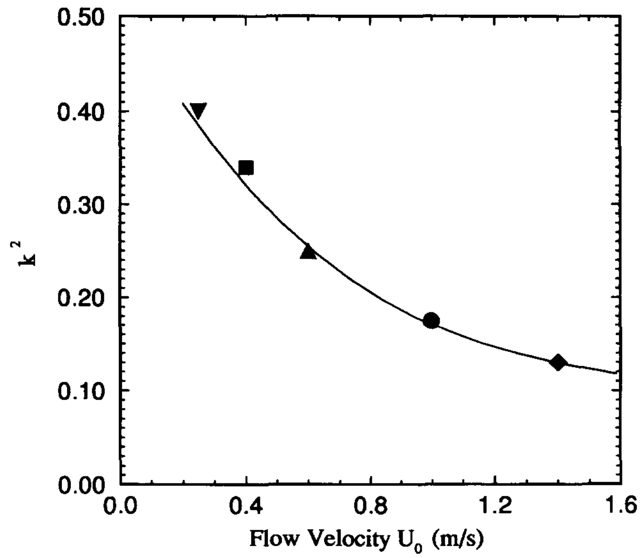

Figure 3. Variation of the factor k^2 with the flow velocity U_0 for the data shown in Fig. 2. The solid line is the curve fit to the data.

low. No experiments were conducted to study the effect of different length-to-diameter ratios of hot wires on the results shown in Fig. 2.

The phenomenon illustrated in Fig. 2 can be attributed to the increased ratio between the conduction to the end supports and the forced convection. This implies that the tangential cooling increases in comparison with the forced convection cooling caused by the flow and thus causes larger deviations from the Cosine Law.

Based on the results of Fig. 2 an alternative to Eq. (1) can be taken as

$$\left(\frac{U_{\text{eff}}}{U_0}\right)^2 = \cos^2 \phi + k^2(U_0) \sin^2 \phi, \tag{3}$$

where k is dependent on the total velocity U_0. This dependence can be expressed by a polynomial

$$k^2(U_0) = B_0 + B_1 U_0 + B_2 U_0^2 + \cdots. \tag{4}$$

Figure 3 shows the values of k^2 with U_0 along with the polynomial fit. Due to the velocity dependence of k, the solution of (3) requires an iterative procedure. First a guess of U_0 is made and the corresponding k is calculated from (4). Then (3) is solved to get a new value of U_0. The procedure is repeated until two consecutive values differ by a specified tolerance. The simple cosine law can be used to make the initial guess of U_0.

CROSS-FLOW ERRORS

The most basic limitation on the hot wire is due to the so-called cross-flow errors, which arise from the fact that a vector field (the velocity) is being mapped into a scalar (the cooling velocity) by the wire. It is possible to evaluate when these cross-flow errors become significant by expanding the cooling velocity about the state where the fluctuating velocities are zero. This has been carried out in detail in [3] for a hot wire placed normal to the flow,

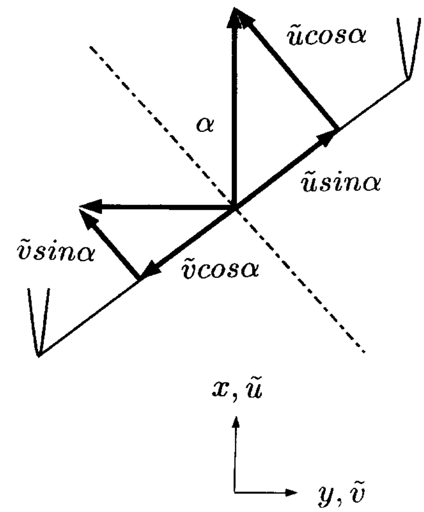

Figure 4. Nomenclature for the velocity components for an inclined hot wire.

and here we will extend these analyses to the x-wire. Consider Fig. 4, which shows a hot wire inclined at an angle α to the vertical direction. The effective cooling velocity for this wire is given by

$$\tilde{u}_{\text{eff}}^2 = (\tilde{u}\cos\alpha + \tilde{v}\sin\alpha)^2 + k^2(\tilde{u}\sin\alpha - \tilde{v}\cos\alpha)^2 + \tilde{w}^2, \tag{5}$$

where $\tilde{w}$ is the velocity component normal to the uv plane, that is, the cross-flow component. For simplification it will be assumed that α is constant and is equal to 45°. Therefore,

$$\tilde{u}_{\text{eff}}^2 = \tfrac{1}{2}(\tilde{u} + \tilde{v})^2 + \tfrac{1}{2}k^2(\tilde{u} - \tilde{v})^2 + \tilde{w}^2. \tag{6}$$

Since when using an x-wire the $\tilde{w}$ component is not known, the effective cooling velocity must be assumed to be

$$\tilde{u}_{\text{eff}}^2 = \tfrac{1}{2}(\tilde{u}_{\text{m}} + \tilde{v}_{\text{m}})^2 + \tfrac{1}{2}k^2(\tilde{u}_{\text{m}} - \tilde{v}_{\text{m}})^2, \tag{7}$$

where we have used the subscript m for the velocities measured using a hot wire in order to distinguish them from the true velocities. It is obvious from Eqs. (6) and (7) that the cross-flow error arises due to the neglect of $\tilde{w}^2$ in Eq. (7).

Tutu and Chevray [8], by assuming a joint Gaussaian probability density function for the two velocity components, have calculated the errors in various turbulence moments due to cross flow and rectification. However, our objective here is to obtain algebraic expressions that an experimentalist can use to estimate when these errors begin to affect his or her measurements in boundary free shear flows. To achieve this we equate the true and assumed forms of the cooling velocity of Eqs. (6) and (7) to obtain

$$(\tilde{u} + \tilde{v})^2 + k^2(\tilde{u} - \tilde{v})^2 + 2\tilde{w}^2 = (\tilde{u}_{\text{m}} + \tilde{v}_{\text{m}})^2 + k^2(\tilde{u}_{\text{m}} - \tilde{v}_{\text{m}})^2. \tag{8}$$

Similarly for the second inclined wire ($\alpha = -45°$) we have

$$(\tilde{u} - \tilde{v})^2 + k^2(\tilde{u} + \tilde{v}^2) + 2\tilde{w}^2 = (\tilde{u}_{\text{m}} - \tilde{v}_{\text{m}})^2 + k^2(\tilde{u}_{\text{m}} + \tilde{v}_{\text{m}})^2. \tag{9}$$

Equations (8) and (9) involve two unknowns, $\tilde{u}_{\text{m}}$ and $\tilde{v}_{\text{m}}$, and therefore these two equations can be manipulated to find expressions for $\tilde{u}_{\text{m}}$ and $\tilde{v}_{\text{m}}$ in terms of the true velocity components $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$. These expressions can then be used to form the statistical moments such as mean velocities and their root mean square values, by carrying out the appropriate decomposition and averaging.

Subtracting (9) from (8) yields

$$\tilde{u}_{\text{m}}\tilde{v}_{\text{m}} = \tilde{u}\tilde{v}. \tag{10}$$

Thus the measured product of the two velocity components is exactly the true product. Substituting $\tilde{u} = U + u$ and $\tilde{v} = V + v$ into Eq. (10) and averaging gives

$$\overline{u_{\text{m}}v_{\text{m}}} = \overline{uv} + (UV - U_{\text{m}}V_{\text{m}}). \tag{11}$$

This equation shows that cross-flow errors in the measured shear stress enter only through the measured *mean* velocities. By using (10) to eliminate V from (8), we can get an expression that is quadratic in the instantaneous measured velocity $\tilde{u}$. The solution of this equation is given by

$$\tilde{u}_{\text{m}}^2 = \frac{1}{2}\left(\tilde{u}^2 + \tilde{v}^2 + \frac{2\tilde{w}^2}{1+k^2}\right) + \frac{1}{2}\left[\left(\tilde{u}^2 + \tilde{v}^2 + \frac{2\tilde{w}^2}{1+k^2}\right) - 4\tilde{u}\tilde{v}\right]^{1/2}, \tag{12}$$

where the positive root is needed to recover the proper expression for the case when $\tilde{w}^2 = 0$. To obtain an equation for the mean measured velocity U_{m}, Eq. (12) has to be simplified using binomial expansions. The final expression for $\tilde{u}$ is then decomposed into a mean and a fluctuating part and then time-averaged. The algebra, although lengthy, is straightforward. In obtaining the following results it was assumed that $W = 0$ (ie., $\tilde{u} = U + \tilde{u}$, $\tilde{v} = V + v$, and $\tilde{w} = w$) and $V \ll U$, a situation that corresponds to the boundary-free turbulent shear flows. The expressions that give the cross-flow errors in the measurements of mean velocities with an x-wire are

$$U_{\text{m}} = U\left[1 + \frac{1}{1+k^2}\left(\frac{\overline{w^2}}{U^2} - \frac{\overline{uw^2}}{U^3} + \frac{\overline{u^2w^2}}{U^4} - 2\frac{\overline{w^4}}{U^4} + \cdots\right)\right], \tag{13}$$

$$V_{\text{m}} = V\left[1 + \frac{1}{1+k^2}\left(\frac{\overline{w^2}}{U^2} - 2\frac{\overline{uw^2}}{U^3} + \frac{\overline{uw^2}}{VU^2} + 3\frac{\overline{u^2w^2}}{U^4} + \cdots\right) + \frac{1}{(1+k^2)^2}\left(2\frac{\overline{w^4}}{U^4} + \cdots\right)\right]. \tag{14}$$

Similar analysis for a hot wire placed normal to the flow (page 106 of Hinze [3]) gives

$$U_{\mathrm{m}} = U\left\{1 + \frac{1}{2}\left[\frac{\overline{w^2}}{U^2} - \frac{\overline{uw^2}}{U^3} + \frac{\overline{u^2w^2}}{U^4} - \frac{1}{4}\left(\frac{\overline{w^4}}{U^4}\right) + \cdots\right]\right\}. \quad (15)$$

It is clear from these expressions that the cross-flow errors for an x-wire are greater than for a hot wire placed normal to the flow by a factor of $2/(1 + k^2)$. Suppose we consider a flow where $\sqrt{\overline{w^2}}/U = 0.25$. Then for an x-wire (with $k^2 = 0.2$) the mean velocity will be overestimated by 5.2% whereas for a hot wire placed normal to the flow it will be overestimated by 3.1%.

Similarly we can obtain the following expressions, which give the errors arising due to the cross flow in the measurement of second moments $\overline{u^2}$ and $\overline{v^2}$ with an x-wire.

$$\overline{u_{\mathrm{m}}^2} = \overline{u^2}\left\{1 + \frac{2}{1 + k^2}\left[\frac{\overline{uw^2}}{\overline{u^2}U} - \frac{\overline{u^2w^2}}{\overline{u^2}U^2} + \frac{1}{2}\left(\frac{\overline{w^4} - \left(\overline{w^2}\right)^2}{\overline{u^2}U^2}\right) + \cdots\right]\right\}, \quad (16)$$

$$\overline{v_{\mathrm{m}}^2} = \overline{v^2}\left\{1 + \frac{2}{1 + k^2}\left[\left(\frac{\overline{vw^2}}{\overline{v^2}U}V + \cdots\right) + \frac{1}{(1 + k^2)^2}\left(\frac{\overline{w^4} - \overline{w^2}^2}{\overline{v^2}U^2}\right) + \cdots\right]\right\}. \quad (17)$$

For a hot wire placed normal to the flow we have

$$\overline{u_{\mathrm{m}}^2} = \overline{u^2}\left[1 + \frac{\overline{uw^2}}{\overline{u^2}U} - \frac{\overline{u^2w^2}}{\overline{u^2}U^2} + \frac{1}{4}\left(\frac{\overline{w^4} - \left(\overline{w^2}\right)^2}{\overline{u^2}U^2}\right) + \cdots\right]. \quad (18)$$

In order to compare the errors in the measurement of $\overline{u^2}$ from the x-wire to those of the hot wire placed normal to the flow, consider a case where $\overline{uw^2}/U^3 = 0.01$ and $\sqrt{\overline{u^2}}/U = 0.40$ (a situation that corresponds to a round jet or a round buoyant plume). Equation (16) shows that, to the leading order, $\overline{u^2}$ is overestimated by 10.4% by an x-wire (with $k^2 = 0.2$). In comparison, the hot wire placed normal to the flow overestimates it by 6.25%. Note that the sign of the error depends on the sign of the third moment $\overline{uw^2}$.

In summary these equations show that for x-wires the cross-flow errors can significantly affect the x-wire measured moments when turbulence intensity is large. Note that because the higher order terms become increasingly important as the turbulence intensity is increased, these expressions can at best serve as indicators of when cross flow is a problem. Furthermore, when using these expressions the assumptions behind them must be kept in mind.

ADDITIONAL EFFECTS DUE TO DROPOUT

From the preceding it is clear that there are three primary sources of error in an x-wire signal at low velocities: rectification, cross flow, and lack of directional sensitivity at higher inclinations. The problem of rectification is obvious for a hot wire placed normal to the flow in which the flow must reverse its direction for rectification to occur. Tutu and Chevray [8] have pointed out that rectification errors are more subtle and serious for x-wires than for hot wires placed normal to the flow.

An additional manifestation of the rectification phenomenon is the occurrence of voltage pairs that could not be resolved into velocity pairs from the angle calibration. In other words, the instantaneous voltage pairs obtained do not lie in the calibrated region of the x-wire and cannot be inverted by Eq. (3). As a consequence these data must be dropped from the statistics. For such data the word "dropout" is probably a more accurate description than "rectification." Dropout is usually caused by a high intensity in the u or v component and is especially troublesome when the mean velocity is low. This is because wires are fairly insensitive to direction at low velocities and any small measurement error (electronic noise, prong support interference, velocity component perpendicular to the x-wire plane, wake of one wire on another, or a velocity or temperature gradient between the wires) can create a large error in the output. The dropout is small in the central regions of flows such as buoyant plumes but has been observed to be as large as 40% at the outer edges.

It is important to note that dropout cannot be detected by common analog signal processing schemes (for example, summing and differencing circuits). As a consequence, the processed data will be incorrect without the experimenter being aware of the problem. The effects will be most noticeable in the higher moments where the scrambled tails of the distribution are most heavily weighted.

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NOMENCLATURE

- d diameter of hot wire, μm
- k coefficient in Eq. (2), dimensionless
- l length of hot wire, mm
- $\tilde{u}$ instantaneous velocity in x direction, m/s
- U mean velocity in x direction, m/s
- U_{eff} effective cooling velocity, m/s
- U_0 flow velocity, m/s
- $\tilde{v}$ instantaneous velocity in y direction, m/s
- V mean velocity in y direction, m/s

Greek Symbols

- α angle between the flow velocity vector and the plane normal to the inclined hot wire, rad

ϕ angle between the flow velocity vector and the plane normal to the hot wire, rad

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