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TURBULENT NATURAL CONVECTION IN A DIFFERENTIALLY HEATED VERTICAL CHANNEL

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ABSTRACT

The turbulence natural convection boundary layer inside a infinite vertical channel with differentially heated walls is analyzed based on a similarity solution methodology. The differences between mean temperature and velocity profiles in a boundary layer along a vertical flat plate and in a channel flow, make it necessary to introduce new sets of scaling parameters. In the limit as $H_* \to \infty$, two distinctive parts are considered: an outer region which dominates the core of the flow and inner constant heat flux region close to the walls. The proper inner scaling velocity is showed to be determined by the outer parameters due to momentum integral. The theory is contrasted with the one suggested by George & Capp (1), the deficiencies of which are identified.

NOMENCLATURE

C_p specific heat at constant pressure.
F_o dimensionless heat transfer rate $\left(\frac{q_w}{\rho C_p}\right)$.
g gravitational acceleration.
h channel half-width and outer length scale.
H H-number based on temperature difference, $\left(\frac{g\beta\Delta T_w h^3}{\alpha^2}\right)$.
H_* H-number for constant heat flux walls, $\left(\frac{g\beta F_o h^4}{\alpha^3}\right)$.
Gr_* Grashof number for constant heat flux walls, $\left(\frac{g\beta F_o h^4}{\nu^3}\right)$.
Nu_h Nusselt number based on h, $\left(\frac{F_o h}{\Delta T_w \alpha}\right)$.
Pr Prandtl number $\frac{\nu}{\alpha}$.
q_w wall heat flux.
T mean temperature.
T_{It} inner temperature scale.
T_o outer temperature scale.
T_w wall temperature.
T_{CL} center-line temperature.
ΔT_w walls temperature difference.
U mean velocity in x direction.
U_I inner velocity scale,constant heat flux wall.
U_o outer velocity scale.
V mean velocity in y direction.
x distance indirection parallel to walls.
y distance indirection perpendicular to walls.
$\tilde{y}$ dimensionless outer variable, $= (y/h)$.
y^+ dimensionless inner variable, $= (y/\eta)$.
α thermal diffusivity.
β thermal expansion coefficient.
η_{It} inner length scale for constant heat flux wall.
ρ density.
ν viscosity.
τ_w wall shear stress.
ξ length scales ratio, $= (\eta/h)$.

INTRODUCTION

Despite its great importance in many industrial and environmental processes and several decades of attention, the basic scaling parameters for turbulence natural convection flow near vertical surfaces are still the subject of some debate. In the modeling of such flows, the near wall behavior of the turbulence quantities is essential in determining the relations among temperature, heat fluxes and velocity. Like other wall-bounded flows, the problem is usually approached by seeking scaling laws for the different regions of the flow, then exploring the consequences of matching the various regions. For the past three decades, the primary model under consideration has been that of George & Capp (1), who noted the absence of a constant stress layer (due to gravity). They then postulated the existence of a buoyant sublayer within the constant heat flux layer in which the buoyancy flux from the wall, $g\beta F_o$, was the crucial parameter, where g is the gravitational acceleration, β is coefficient of thermal expansion and F_o is the wall heat flux divided by density and specific heat. Using this assumption they proposed inner and outer scales for velocity to be $(g\beta F_o \alpha)^{1/4}$ and $(g\beta F_o h)^{1/3}$ respectively, where α is the thermal diffusivity and h is the channel half-width. The theory has generally been found to be in excellent agreement with numerous experimental studies for the temperature and heat transfer relations; in particular the temperature varies inversely with the cube root of distance from wall in the buoyant sublayer, and the Nusselt number is proportional to the cube of the Rayleigh number to within a factor dependent on the Prandtl number. The velocity profile scaling, however, has long been recognized as problematical, especially with more recent experiments and DNS. Although the outer velocity scaling matched the experimental results, the inner scaling and cube root velocity profile in the buoyant sublayer seemed to show systematic departures from the theory which could not be explained. Experimental data (2) and numerical simulation (3; 4; 5) of the natural convection inside a differentially heated channel also shows that the mean profiles of velocity and temperature do not follow the suggested asymptotic behavior in channel flow. Also, the Reynolds shear stresses at the center of the channel do not go to zero, unlike the flat plate with semi-infinite fluid beside it. Therefore the momentum integral equation across the channel will be seen to depend on the values of the shear stress and viscous stress at the center of channel.

This work reconsiders the George & Capp (1) similarity solution methodology and shows their result to be inconsistent with the momentum integral in natural convection channel flow. The proper inner scaling velocity is in fact determined by outer parameters and the momentum integral to be $u_i = (g\beta F_o h)^{1/3}$. Since the inner and outer velocity profiles are now scaled by the same parameter, the velocity profile in buoyant sublayer must be logarithmic. The temperature and heat transfer laws are the same as before.

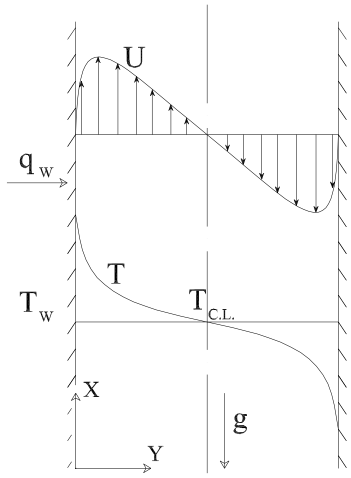

Figure 1. SCHEMATIC OF FULLY DEVELOPED TURBULENT BOUNDARY LAYER INSIDE AN INFINITE VERTICAL CHANNEL WITH DIFFERENTIALLY HEATED WALLS.

GOVERNING EQUATIONS

An infinite vertical channel with differentially heated walls is shown in figure 1. The distance between the two walls is $2h$ and the temperature difference is ΔT_w. The kinematic viscosity ν, the thermal diffusivity α and the thermal expansion coefficient β are the fluid properties and considered to be constant. A uniform and steady heat flux is applied across the walls. Assuming the flow to be homogeneous in the streamwise direction, x, and the channel to be of infinite extent, both the wall temperature and wall heat flux must independent of x . The Boussinesq approximation will be assumed to be valid, so density differences can be neglected except as they affect the gravitational term. Further, the temperature difference between the two sides is considered to be small enough that we can assume (at least for the sake of argument here) that density and temperature differences are proportional; i.e., $\Delta\rho/\rho_o \approx -\beta\Delta T$.

We are herein interested only in fully-developed channel flow, so the streamwise gradient of all properties is assumed to be identically zero, so that all the mean convection terms become identically zero. Also the velocity and temperature profiles are antisymmetric across the channel, which gives the boundary condition of zero values at the centerline of the channel. By contrast, the derivatives of these profiles and Reynolds stresses are symmetrical with respect to the center-line. Also the y-momentum equation of the flow can be used to eliminate the pressure in the x-momentum equation. For the limit of very high Reynolds number, all these assumptions reduce the equations of motion and energy for entire channel to:

$$0 = \frac{\partial}{\partial y}\left(\nu\frac{\partial U}{\partial y} - \overline{uv}\right) + g\beta(T - T_{ref}) \quad (1)$$

and

$$0 = \frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y} - \overline{vt}\right) \quad (2)$$

Capital letters refer to mean values while lower case letters are used for fluctuating quantities. These equations are valid both for near wall and the core region of the flow, but the viscous terms and conduction terms in the core (or outer) region of the flow have less effect (and no effect at all in the limit of infinite Reynolds number).

The energy (or temperature) equation 2 can be integrated across the channel width (from wall to center-line) to yield

$$-\overline{vt} + \alpha\frac{\partial T}{\partial y} = -\frac{q_w}{\rho C_p} \equiv F_0 \quad (3)$$

where q_w is the wall heat flux and C_p is the specific heat at constant pressure. Equation 3 shows that the entire channel boundary layer to be a *constant heat flux layer* and the total kinematic heat flux, F_o across the layer is independent of distance from the wall (c.f., George and Capp (1)).

The momentum equation 1 can be similarly integrated across the channel to give momentum integral equation as:

$$-\overline{uv} + \nu\frac{\partial U}{\partial y} + \int_0^y g\beta(T - T_{ref})dy' = \nu\frac{\partial U}{\partial y}\Big|_{y=0} - \overline{uv}\Big|_{y=0} \quad (4)$$

We can use the wall shear stress, τ_w, to define the friction velocity, u_* by:

$$u_*^2 = \tau_w/\rho = \nu\frac{\partial U}{\partial y}\Big|_{y=0} \quad (5)$$

Also, the kinematic boundary condition at the wall implies:

$$-\overline{uv}\Big|_{y=0} = 0. \quad (6)$$

Using these conditions and choosing the reference point for the temperature to be the centerline, say T_{CL}, equation 4 can be rewritten as:

$$u_*^2 - \left(\nu\frac{\partial U}{\partial y}\Big|_{y=h} - \overline{uv}\Big|_{y=h}\right) = \int_0^h g\beta(T - T_{CL})dy' \quad (7)$$

Clearly as first noted by George and Capp (1), unlike forced flow, natural convection flow does not have a constant stress layer because of the buoyancy term in the momentum equation. Thus unlike forced flow in channels or pipes, it would appear that the wall shear stress cannot be imposed as a constant parameter at all distances from the wall.

Note that in general, all of the values at the left-hand side of the equation are constant (i.e., x-independent and non-zero). Thus we can define:

$$\Phi_{CL}^2 = \nu\frac{\partial U}{\partial y}\Big|_{y=h} - \overline{uv}\Big|_{y=h} \quad (8)$$

Although we might expect the viscous term to be negligible in the limit of infinite Reynolds (or Peclet) number, the Reynolds stress terms is most probably not due to the possible presence of highly correlated motions (or coherent structures) that span the channel. This undoubtedly accounts for the observations of Betts & Bokhari (2) and also numerical simulations of the natural convection channel flow (3) where the centerline Reynolds stress is quite significant. It also represents an important difference from the natural convection boundary layer next to a vertical surface in which the Reynolds stress vanishes at large distances from the wall. Recognition of this difference is part of the purpose of this paper. And it raises interesting questions about why some recent LES simulations of low aspect ratio channels seem to suggest that the Reynolds shear stress vanishes even in channels (e.g., (6)).

INNER AND OUTER SCALING

The Temperature Profile

George and Capp (1) argued that natural convection channel flow at high turbulence Reynolds numbers should in fact be viewed as a classical inner-outer problem, with viscosity and thermal conductivity dominating the inner (or near wall) layer but having no effect on the core region. A direct consequence of this is that the viscous and conduction terms in the equations above are negligible in the core region. By contrast, the combination $g\beta$, and the kinematic heat flux, F_o, were important everywhere. Thus the very near wall region could be completely characterized by $g\beta$, F_o, α and ν. Note that equation of the near wall region do not really know about the channel width, h, nor do they know the net temperature difference $\Delta T_w = T_w - T_{cl}$ (since at least a small part of it occurs across the outer layer)[1]

The core region in the limit of infinite Reynolds number, by contrast, knows only about the channel width, h, $g\beta$, and

[1]The choice of inner scales is actually quite problematical. It can be argued that ΔT_w should be included, since it occurs explicitly in the momentum equation at the wall. Or it might be argued that since the temperature and heat flux determine a length scale, these should be included and $g\beta$ ignored (c.f. Wosnik and George (7), Wosnik et al. (8). Unfortunately this does not seem to lead to the expected heat transfer law, perhaps because ΔT_w and F_o are not independent parameters, one being determined if the other is specified. Hence we have opted herein for the original George & Capp (1) formulation.

F_o, the latter because of the constancy of the kinematic heat flux across the flow. It followed immediately on dimensional grounds that the only choice for an outer temperature scale was $T_o = (g\beta F_o h)^{1/3}$. And on similar grounds, the inner temperature scale was chosen to be $T_i = F_o^{3/4}/(g\beta\alpha)^{1/4}$ (although ν could have been used instead of α, implying a residual Prandtl number dependence of the inner layer). The obvious choice of length scale for the core region was the channel half-width, h, while the inner length scale was chosen on dimensional grounds as $\eta_{It} = [\alpha^3/(g\beta F_o)]^{1/4}$. Note that the latter could have equally been chosen as $\eta_{Iv} = [\nu^3/(g\beta F_o)]^{1/4}$, but α was preferred on experimental grounds since it varies with ν in gases, and has significantly less dependence on temperature in liquids.

A major contribution of George & Capp (1) was the recognition that there must exist a layer in between the inner and outer layers, the so-called *buoyant sublayer*, in which both the thermal diffusivity and kinematic viscosity could be ignored, as well as the outer length scale, h. It follows on dimensional grounds alone that $dT/dy \propto [F_o^{2/3}/(g\beta)^{1/3}]y^{-4/3}$. This layer was presumed to lie between the molecular viscosity/diffusivity dominated region very close to the wall (the viscous and conductive sublayers respectively) and the peak in the mean velocity profile.

Integration across the inner layer leads immediately to two equivalent forms for the temperature in the buoyant sublayer, one in inner variables and one in outer variables, both of which are equally valid if the whole idea of the overlap region is sound. For the inner profile the result is:

$$\frac{T - T_w}{T_{It}} = K_2 \left[\frac{y}{\eta_{It}}\right]^{-1/3} + A(Pr) \tag{9}$$

where the Prandtl number dependent additive constant reflects the Prandtl number dependence of the inner layer. By contrast, integration from the centerline yields the same profile in outer variables as:

$$\frac{T - T_{CL}}{T_o} = K_2 \left[\frac{y}{h}\right]^{-1/3} + A_1 \tag{10}$$

where A_1 is expected to be universal, or at most dependent on the nature of the core flow (i.e., coherent versus incoherent motions).

Such a *buoyant sublayer* region can be expected to exist only when there exists a region for which $y << h$ and simultaneously $y >> \eta_{It}$ and $y >> \eta_{Iv}$. Thus a necessary condition for the existence of a buoyant sublayer is that the ratio of inner to outer length scales be much greater than unity; i.e., $H_*^{1/4} >> 1$ and $Gr_*^{1/4} >> 1$ since:

$$\frac{h}{\eta_{It}} = \left[\frac{g\beta F_o h^4}{\alpha^3}\right]^{1/4} = H_*^{1/4} \tag{11}$$

$$\frac{h}{\eta_{Iv}} = \left[\frac{g\beta F_o h^4}{\nu^3}\right]^{1/4} = Gr_*^{1/4} \tag{12}$$

where H_* and G_* are defined respectively by:

$$H_* = \frac{g\beta F_o h^4}{\alpha^3} \tag{13}$$

$$Gr_* = \frac{g\beta F_o h^4}{\nu^3} \tag{14}$$

Thus unlike forced boundary layers where the local Reynolds number $\delta^+ = u_*\delta/\nu$ is the ratio of length scales, here it is the 1/4-root of H_* (or the corresponding Raleigh, Ra_* or Grashof, Gr_*, numbers). Many have concluded that values of H_* of order 10^6 are sufficient to achieve an asymptotic state, but in fact it is clear from equations 11 and 12 that much higher values of these are necessary to achieve even a modest separation of scales. (E.g., typically h/η_{It} or h/η_{Iv} greater than 10 would be considered to be an absolute minimum for any asymptotic theory to even begin to be valid.) Unfortunately there is very little experimental data and no DNS data satisfying these conditions; in fact, recent attempts have typically been $h/\eta_{It} < 10$ and sometimes even as low as 3 - 4 (e.g., refs. (2; 3)). So in spite of the advances of experimental and computational techniques over the past few decades, there has been (to the best of our knowledge) virtually nothing to contribute to our understanding of a high H-number buoyant sublayer, or the applicability of the theoretical deductions from it.

The Heat Transfer Law

An immediate consequence from matching equations 9 and 10 is that the asymptotic heat transfer law given by:

$$Nu^{-1}H_*^{1/4} = -A(Pr) + A_1 H_*^{-1/12} \tag{15}$$

So the corresponding heat transfer law is given by:

$$Nu = -A(Pr)H_*^{1/4} + A_1 H_*^{1/6} \tag{16}$$

Clearly $A(Pr) < 0$ is the only physically realistic possibility.

In the limit as $H_* \to \infty$, the first term dominates so the asymptotic heat transfer law is given by:

$$Nu = -A(Pr)H_*^{1/4} \tag{17}$$

But this should only be expected for very large H_*; i.e., when $H_*^{1/6} >> 1$ ($H_* > 10^8$ or greater), because of the 1/6th root dependence of the additive term.

Note that the heat transfer 'law' could equally well be rewritten in the more familiar form as:

$$Nu = [-A(Pr)]^{4/3} H^{1/3} \tag{18}$$

where H is defined using the temperature difference ΔT_w as:

$$H = \frac{g\beta\Delta T_w h^3}{\alpha^2} \tag{19}$$

The heat transfer result of George & Capp (1) was particularly significant, since it accorded with the long-standing observation of Arpaci and others that the heat transfer in channels appeared to be independent of the channel width (Note the same factor of h on both sides of equations 18 and 19.) The slightly varying values of the coefficient can probably be attributed to the absence of data at high enough values of H_*, and fitting the heat transfer law without the extra $H_*^{1/6}$ term. Surprisingly there seems to have been little effort over the past few decades to relate the parameters in these equations to the actual temperature profiles as the theory suggests. Perhaps this is because of the concentration of effort on relatively low H-number flows.

THE MEAN VELOCITY PROFILE

The Problematical George & Capp Inner Scaling

The second major point of this paper is that the George & Capp (1) analysis of the mean velocity profile for the inner region is incorrect. While the results for the temperature and heat transfer laws above have been reasonably well-received by the research community (recent low H-number experiments and DNS notwithstanding), the George & Capp (1) results for the mean velocity profile have always been problematical. First the mean velocity near the wall does not seem to scale very universally with their proposed inner velocity scale, $(g\beta F_o\alpha)^{1/4}$. Second the predicted cube root region corresponding to the buoyant sublayer does not seem to be the best fit to the data.

First let us note that almost all recent investigations (including DNS and LES) conclude that it is the friction velocity, u_*, and the corresponding viscous length scale, ν/u_*, which correctly scales the velocity profile closest to the wall. In fact, this is not a conclusion but a necessity. Since u_* is defined from the wall shear stress, which is turn proportional to the mean velocity gradient at the wall, the only way scaling the mean velocity profile there with u_* and ν would NOT work is if the experiments or computations were incorrect.[2]

So if scaling by u_* and ν is not the right question, what is? The real problem is how to relate these parameters to the remaining parameters of the flow! In other words, what is the friction law in terms of the boundary conditions and parameters of the problem? Clearly the friction law proposed by George & Capp (1) is incorrect, at least based on the data, and in fact in principle as well. This can be demonstrated using the momentum integral equation 7 as shown below.

First split the temperature integral in the buoyant sublayer in inner variables at $y/\eta_{It} = A$ so it can be written in two parts as:

$$u_*^2 - \left(\nu\frac{\partial U}{\partial y}\Big|_{y=h} - \overline{uv}\Big|_{y=h}\right) = g\beta T_{It}\eta_{It}\int_0^A \frac{T-T_w}{T_{It}} d\frac{y}{\eta_{It}} + g\beta T_o h\int_{A\eta/h}^1 g\beta\frac{T-T_{CL}}{T_o} d\frac{y}{h} \tag{20}$$

In the limit as $H_* \to \infty$, the two integrals are simply numbers, say I and II. Moreover, from the definitions above it follows that $T_{It}\eta_{It} = (g\beta F_o\alpha)^{1/2} = u_I^2$ and $T_o h = (g\beta F_o h)^{2/3} = u_o^2$, which are in fact just the squares of the inner and outer velocity scales proposed by George & Capp (1). Thus the integral of equation 20 can be rewritten as:

$$u_*^2 - \left(\nu\frac{\partial U}{\partial y}\Big|_{y=h} - \overline{uv}\Big|_{y=h}\right) = I\ u_{IGC}^2 + II\ u_{oGC}^2 \tag{21}$$

Dividing by u_{IGC}^2 and using the definitions yields the friction law as:

$$\frac{u_*^2}{u_{IGC}^2} - \frac{1}{u_{IGC}^2}\left(\nu\frac{\partial U}{\partial y}\Big|_{y=h} - \overline{uv}\Big|_{y=h}\right) = \left\{I + II\ H_*^{1/6}\right\} \tag{22}$$

But the right-hand side blows up in the limit as $H_* \to \infty$. Thus regardless of the problems presented by the second term on the right-hand-side, the George & Capp (1) choice of an inner scaling velocity is clearly not correct. In particular the dominant

[2]The same observation can be made for the mean temperature profile near the wall as well: it must by definition collapse when normalized using ΔT_w if the length scale is defined as $\alpha\Delta T_w/F_o$. So collapse in these variables only implies that the experimental measurements or computations are correct in this region.

contribution to momentum integral is not near the wall, but far from the wall. Hence the velocity near the wall is not a consequence primarily of the local buoyancy there, but is in fact largely a flow driven from the outside by the effects of buoyancy there. This is actually somewhat counterintuitive, because almost all of the temperature profile occurs near the wall; but it is the residual outer temperature profile that dominates the integral because of the much greater distance over which it must be integrated.[3]

The Friction Law

In fact it appears that the proper choice for an inner scaling velocity parameter should have been the outer scaling velocity, at least in the limit as $H_* \to \infty$ and including the extra terms from the centerline. To see this, divide equation 21 by $u_o^2 = (g\beta F_o h)^{2/3}$ to obtain:

$$\frac{u_*^2}{u_o^2} - \frac{1}{u_o^2}\left(\nu\frac{\partial U}{\partial y}\bigg|_{y=h} - \overline{uv}\bigg|_{y=h}\right) = \left\{I\ H_*^{-1/6} + II\right\} \tag{23}$$

It is easy to see that this indeed slowly approaches a constant, II, in the limit as $H_* \to \infty$. Thus, as noted above the friction is primarily determined by the outer velocity scaling, and the inner velocity would be properly chosen proportional to it.

Implications for The Velocity Profile In The Buoyant Sublayer

Thus it appears that at least in the limit as $H_* \to \infty$, the inner and outer velocity scales are the same; i.e., $u_* \propto (g\beta F_o h)^{1/3}$. It is well-known (c.f., George and Castillo (9)) that if the inner and outer velocity scales are the same, an immediate consequence is that the mean velocity profile in the overlap region must be logarithmic. Thus an immediate (and somewhat surprising) conclusion of the above is that the mean velocity profile in the overlap region should vary logarithmically. Thus in inner variables:

$$\frac{U}{u_*} = K_1\ ln\frac{yu_*}{\nu} + B(Pr) \tag{24}$$

where $B(Pr)$ may vary from fluid to fluid. In outer variables, the corresponding mean velocity profile would be:

$$\frac{U}{(g\beta F_o h)^{1/3}} = \frac{u_*}{(g\beta F_o h)^{1/3}}\left\{K_1\ ln\frac{y}{h} + B_1\right\} \tag{25}$$

[3]Interestingly, the George & Capp (1) mistake is similar to that often made for forced boundary layers where it is argued that most of the energy dissipation is near the wall, when in fact the opposite is true. Even though indeed the peak in the dissipation is very near the wall, the integral of the dissipation is dominated by the overlap region and outer flow.

where B_1 should be universal only if the coherent structures of the core region are the same. Clearly this should be a matter for investigation.

Note that it might be tempting to associate K_1 with the usual von Karman parameter, $1/\kappa$. As noted by George (10), there is really no reason to believe this parameter to be the same for different flows, and most certainly not for this one for which the constant stress layer does not exist.

SUMMARY AND CONCLUSION

A new theory has been proposed for the velocity profiles of a turbulent natural convection boundary layer in a differentially heated vertical channel. The mean velocity and temperature scaling parameters for inner and outer region of the flow have been derived using local similarity solutions at the limit of infinite local H-number. The inner and outer scaling parameters for the mean temperature profile is the same as suggested by George & Capp (1), as is the outer scale for the mean velocity. The inner velocity scale and friction law, however, are quite different. In particular both the inner and outer velocity scales are the same in the limit; and given in the limit of infinite H_* by $u_* \propto (g\beta F_o h)^{1/3}$. An immediate consequence is that the mean velocity profile in the buoyant sublayer is logarithmic. The shear stress is primarily determined by the buoyancy integral and the value of the total stress at the centerline, neither of which can be assumed to be zero. The latter represents a significant difference between channel flow and the natural convection boundary layer next to a vertical surface, and opens the possibility for the dependence of the flow on the nature of the turbulence in the core region; i.e., coherent structures versus incoherent motions.

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