ENERGY BALANCE MEASUREMENTS IN AN AXISYMMETRIC TURBULENT BUOYANT PLUME IN A NEUTRAL ENVIRONMENT

Aamir Shabbir and William K. George

Turbulence Research Laboratory
Department of Mechanical and Aerospace Engineering
University at Buffalo/SUNY
Buffalo, NY 14260.

ABSTRACT

Velocity and temperature fields were measured in an axisymmetric turbulent buoyant plume using a three wire probe. The ambient temperature was monitored using thermocouples placed at different heights to make sure that it was not stratified. Source conditions were also monitored so as to ascertain the rate at which buoyancy was added to the flow. The axisymmetry of the flow was checked using a planer array of sixteen thermocouples. The measurements satisfy both the integral and differential forms of the mean energy and momentum equations to within experimental error. The rates of thermal and mechanical energy dissipation were obtained as the closing terms in balances of the turbulence kinetic energy and temperature variance equations.

HISTORICAL REVIEW

Due to the complex nature of turbulence, an experimental approach is to think of model experiments which isolate a particular physical aspect of the phenomenon. Besides lending insight into the physics, such experiments also augument the theoretical development of the subject by verifying various theories and mathematicl models. The laboratory plume is one such flow which allows us to study the effect of buoyancy on turbulence.

The plume has been the subject of study since the similarity analysis of Zel'dovich (1937). Schmidt (1941) used mixing length type hypotheses to obtain expressions for the velocity and temperature profiles for both plane and axisymmetric plumes and compared the results with his own measurements.

Batchelor (1954) proposed similarity solutions for turbulent plumes for both the plane and the axisymmetric geometries in neutral and stratified environments. For an axisymmetric plume in a neutral environment, the mean vertical velocity and buoyancy field was shown to be given by

$$U = F^{1/3} z^{-1/3} f_1(r/z) \qquad (1)$$

$$g\beta \Delta T = F^{-2/3} z^{5/3} t_1(r/z) \qquad (2)$$

where F is the rate at which buoyancy is added at the source, z is the height, and where the functions f and t are to be determined from experiments. Batchelor discarded the solutions for stably stratified environments, arguing that these were physically unrealistic. His conclusions were disputed by George and Beuther (1982) on physical grounds and they showed measurements of plumes in stably stratified environments which collapsed in similarity variables to substantiate their claim.

Rouse, Yih and Humpherys (1952) also obtained the similarity relation for the two dimensional and axisymmetric plumes and experimentaly verified their fuctional forms. They generated plumes by using gas burners arranged in the appropriate formation, and then used thermocouples and wind-vane anemometers to measure the mean buoyancy and vertical velocity. More recently Rao and Brzustowski (1969) generated a plume by a burning wick heat source, and measured the velocity field by using hot-wires and quasi-linearization.

The first attempt to measure the mean and turbulence quantities using hot-wire anemometry with modern digital techniques was by George, Alpert and Tamanini (1977). A two wire probe was used to measure the temperature and velocities, and the corresponding similarity profiles were given approximately by

$$f(\eta) = 3.4 \exp(-55 r^2/z^2) \qquad (3)$$

$$t(\eta) = 9.1 \exp(-65 r^2/z^2) \qquad (4)$$

These are substantially different from those measured by Rouse et al (1952), the differences being attributable to the techniques used and a possible stratification of the Rouse facility (v. Beuther 1980). The rms values of temperature and velocities were found to be 40% and 28% respectively. The correlation coefficient between the temperature and vertical velocity was found to be 0.6 - 0.7, a result believed to be reasonable in plumes where the velocity and temperature field strongly influence each other. It was also found that about 15% of the total vertical heat transport was contributed by the fluctuations. Nakagone and Hirata (1975) also made similar measurements in an independent effort, but obtained lower values of the rms values of temperature and velocity (33% and 26% respectively) and also obtained a lower value for the vertical velocity temperature correlation coefficient of 0.45.

Beuther (1980) (see also Beuther and George 1980) used a three wire probe identical to that one used here to measure up to fourth moments for an axisymmetric plume. His measurements were taken in a stably stratified environment in which temperature increased with height, and therefore can not be compared with those cited above. For the sake of completeness his results are summarized below.

$$f(\eta) = 3.8 \exp(-69 \eta^2) \qquad (5)$$

$$g(\eta) = 10.4 \exp(-85 \eta^2) \qquad (6)$$

The temperature and velocity intensities were 30% and 26% at the centerline. The various balances were also computed to establish the accuracy of the experiment.

established in all experiments, there is still some disagreement about the centerline values of the mean profiles and the spreading rates. Chen and Rodi (1980) reviewed the existing experimental data on plumes and recommended profiles which were very close to those of George et al. (1977).

List (1982) in his review suggests that all the measurements in the axisymmetric geometry have not been taken in the fully developed region. He also points out that none of these studies equated the integrated buoyancy flux with the source buoyancy flux. The buoyancy flux F_0 which directly appears in the similarity relations was obtained in these studies by integrating the measured temperature and velocty profiles. This could be in error since the probes used, hot wires, do not have the ability to resolve flow reversals believed to be present at the outer edges of the flow. The LDA measurements of Capp (1983), however, show that these errors have a negligible effect on integral relations.

In order to resolve the various differences in the experiments discussed above Shabbir and George (1985) presented single wire measurements. In that experiment source conditions were carefully monitored to calculate the buoyancy parameter F which was used to normalize all the profiles. The experiment conserved buoyancy within 5-10% Based on these measurements they recommended the following buoyancy and velocity profiles for an axisymmetric turbulent buoyant plume in a neutral environment.

$$t = 9.4 \exp(-69\eta^2) \quad (7)$$

$$f = 3.2 \exp(-57\eta^2) \quad (8)$$

These are about the same as recommended by George et al. (1977) and by Baker et al. (1980).

OBJECTIVE OF THE PRESENT WORK.

The objective of this work is to measure the various moments up to fourth order so that the turbulence kinetic energy and temperature variance balances can be carried out. The mechanical and thermal dissipations are obtained as the closing entries in these balances. These balances show how the energy is budgeted and which terms or physical phenomena dominate the different regions of the flow. They also provide a data base for evaluating various turbulence models and closure assumptions. Obviously various hot wire errors will affect the measurements of the moments, but it is expected that in the central core of the flow where the turbulence intensities are small, these errors will be small.

INSTRUMENTATION.

The probe used was the same as used by Beuther (1980) which was a combination of a cross-wire and a temperature wire. The temperature wire was 1 micron in diameter and was heated with a current of 0.15 milliamperes. At such low current the wire was virtually insensitive to velocity. The x-wires were made of 5 micron gold-plated wire and had an ℓ/d of 125. An overheat ratio of 0.4 was used for the optimum response to both the temperature and velocity. All the velocity sensors were operated in the constant temperature mode using DISA 55M anemometer systems. For the temperature wire the 55M20 bridge was used with the 55M01 system.

To check the axisymmetry of the flow and to find the center of the plume, an array of sixteen thermocouples was used which were arranged in a 4 X 4 grid. The thermocouple scanner used for acquiring data from these thermocouples was made using four millivolt conditioners manufactured by Analog Devices (Model 2B54A). These conditioners were specially designed for thermocouple applications. A gain of 1000 was employed was converted into volts. The scanner was driven by TTL logic and was interfaced with a PDP 11/34. The output from the scanner was sampled and digitized using a 16 bit A/D converter. The scanner permitted taking as many as 400 samples per second, however, the fastest rate used in the experiments was 32 samples per second. All the thermocouples used in the experiment were Copper-Constantan and were ice referenced.

The outputs from all the anemometers and signal conditioners were digitized by an 16 bit A/D converter interfaced with the PDP 11/34. The voltages were recorded on a magnetic disk for later data conversion.

CALIBRATION SCHEMES.

The calibration techniques were similar to those used by Beuther (1980). The constant current wire gave a linear response to temperature change. Since for the velocity wire the output voltage is a function both of velocity and temperature it was handled by expressing the wire Nusselt number as a function of the Reynolds number (see Beuther 1980, George et al 1987). In particular,

$$Re = \sum_{n=0}^{4} A_n N_u^{n/2} \quad (9)$$

The coefficients A_n are temperature independent, the temperature dependence entering through the Nusselt number. For angle response of the x-wire a Champagne-type relation was used which incoporated a velocity-dependent k-factor, i.e.

$$U_{eff}/U_o = [\cos^2\phi + k^2(U_o)\sin^2\phi]^{1/2} \quad (10)$$

Note that k was found to be dependent on the velocity vector U at the low velocities encountered in this experiment (see Beuther et. al. 1987).

ERRORS IN THE MEASUREMENTS

There are three primary sources of error in a x-wire signal at low velocities rectification, cross-flow and lack of directional sensitivity. Tutu and Chevray (1975) have pointed out that rectification errors are more subtle and serious for x-wires than for single wires.

One manifestation of the rectification phenomenon is the occurrence of voltage pairs which could not be resolved into velocity pairs from the angle calibration. In other words, the instantaneous voltage pairs obtained can not be inverted by equation (10). For such data, the word "dropout" is probably a more accurate description than "rectification". Dropout is usually caused by a high intensity in the u or v component and is especially troublesome when the mean velocity is low (see Beuther et.al 1987). The dropout is small at the center of flows such as plumes but has been observed to be as big as 40% at the outer edges (see Beuther 1980, Beuther et. al. 1987).

Since the dropped data points are associated with the low velocities, its net effect would bias the velocities toward the higher end. Whether this results in the overestimation of higher moments or not is a difficult question to answer. It should also be noted that like dropout, the flow reversals on the wire are also associated with low mean velocities and by dropping such data points one <u>might</u> be reducing the rectification errors.

From the above discussion it is clear that it is very difficult to quantify the various hot wires errors. As a word of caution it is recommended that all the higher moments must be interpreted carefully beyond $\eta = 0.1$ where turbulence intensities begin to increase significantly.

SOURCE CONDITIONS

Source conditions for the experiment are given in Table I. Since both the momentum and buoyancy are added at the source, the flow near the source is more like a buoyant jet and not a plume. However as the air moves up vertically and the flow develops, the buoyancy overwhelms the momentum added at the source. After a certain distance from the source the flow is governed by the buoyancy alone. How far away this happens is determined by the source conditions.

Kotsovinos(1977) and Baker (1980) have defined a buoyancy length scale L based on the source buoyancy and momentum to characterize this distance

$$L = M_o^{3/4}/F_o^{1/2}. \qquad (11)$$

Baker (1980) has shown that the buoyant jet will reach an asymptotic plume like condition for $\zeta > 5$ where $\zeta = z/L$. For the source conditions of this study, this criteria is met (ζ ranges from 8 to 12).

An accurate knowledge of the source conditions is also essential to check whether or not the experiment conserves buoyancy. Also the rate at which buoyancy is added at the source appears in all the similarity relations for velocity. In view of the fact that plumes in stratified environments also satisfy similarity relationships using the local buoyancy integral, it is more accurate to use the source buoyancy which must be conserved for the neutral environment than one obtained by integrating the hot wire measured velocity and temperature profiles.

AMBIENT CONDITIONS.

Beuther and George (1982) and Beuther (1980) have shown that a small stratification of the ambient can cause a significant loss or gain of buoyancy. This can appreciably change the shape of profiles. In the experiment of Beuther (1980) the ambient was stably stratified and the data collapsed in the similarity variables when the local value of the buoyancy flux was used.

Almost none of the studies reported in the literature have monitored or documented changes in the ambient temperature. In the present experiment the ambient was continuously monitored using seven copper-constantan thermocouples placed at different heights. Figure (3) shows the change in ambient

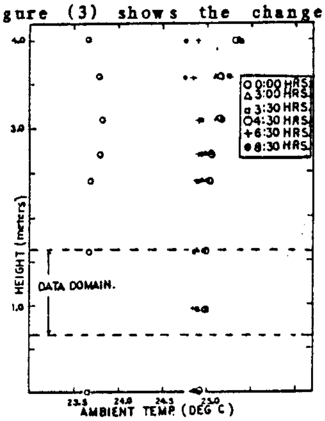

Figure 1. Variation of Ambient Temperature with height at different times.

temperature both as a function of height and time. It is seen that during the first two hours of operation a significant change in the temperature occurs. However, after this it changes very little, especially over the heights where the measurements were taken ($z < 1.8$ m.). For the measurements reported here, the temperature change from source to maximum height was less than 0.15 deg. C and can be shown to be consistent with a neutral environment, a fact confirmed by the relative constancy of the integrated buoyancy fluxes.

RESULTS.

The mean buoyancy profile shown in fig. (4) has a centerline value of 9.4 and is the same as that measured with a two wire parallel probe. The Gaussian curve shown is not the fit to the present data but is the one suggested by George et. al. (1977) and is reproduced here for comparison. The data collapse is very reasonable, and gives confidence that the plume is fully developed at the locations where the measurements are taken.

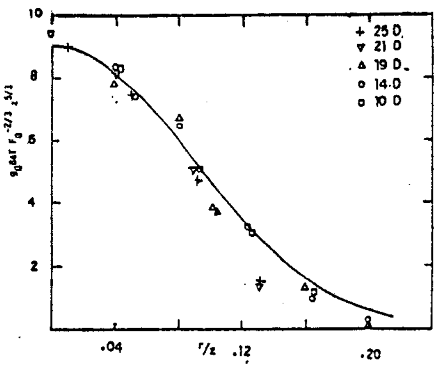

Figure 2. Mean Buoyancy profile.

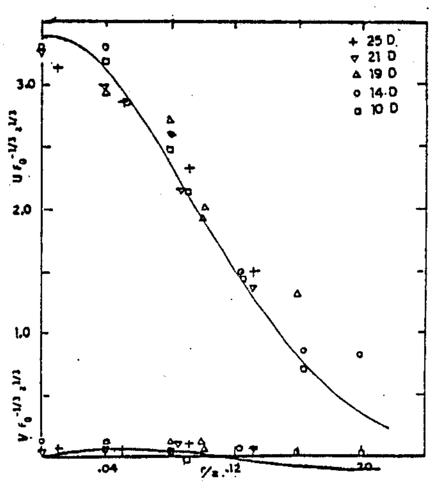

Figure 3. Mean Velocities.

The mean velocity profile shown in figure (5) has a centerline value of 3.4, and is again about the same as measured with a single wire (Shabbir and George 1985) except at outer edges where it is somewhat higher. The differences are due to the poor directional response of x-wires at the lower velocities, the differing cross-flow and rectification errors between single and x-wire probes.

The measured radial component of the velocity profile, also plotted in figure (5) is very small. This can also be obtained from the continuity equation by integrating the measured vertical velocity i.e.

$$V = \frac{-1}{r}\int_o^r \frac{\partial U}{\partial z}\, r dr \qquad (12)$$

The profile calculated from the streamwise mean velocity becomes negative (directed radially inwards) after $\eta = 0.1$ whereas the measured one does not. The difference can be attributed to the small values of V (which is of order δ/L) and the relatively large hot

The various second moments are shown in figures (4) through (9). The mean square values of temperature are about the same as measured with a single wire. The profile of $\overline{u^2}$ is about the same as measured with a single wire except at around η =0.2 where the x-wire measurement is slightly higher, the reason probably being the poor directional response of the x-wire at low velocities and the dropout phenonmena to be discussed latter in this chapter. The centerline value of the $\overline{v^2}$ is about 70% of u. The curves shown are the best fits to the data, the equations for which are tabulated in Table I along with those for the higher moments.

The centerline value of the $\overline{ut}$ correlation coefficient is is .68, and is slightly lower than the one obtained from a single wire. The correlation does show an off axis peak as observed in the single wire measurements. The radial heat flux $\overline{vt}$, which is very important to carry out the mean energy balance peaks at a value of 0.8 at about = .09 and is slightly higher than what Beuther(1980) measured under stratified conditions. Its shape is close to that of the derivative of the mean temperature. The shear stress $\overline{uv}$ has a maximum value of about 0.32 at η = 0.07 and is also slightly higher than what Beuther (1980) measured. It should be noted that the shear stress and the radial heat flux have finite values near the center rather than being zero. This is again due to the finite values of the various hot-wire error terms for these quantities.

The measured profiles were integrated across the flow to get the local buoyancy flux i.e

$$F_{local} = 2\pi \int_0^\infty g\beta(U\Delta T + \overline{ut})r dr \qquad (13)$$

For a neutral environment this must remain constant with height. The ratio of this local buoyancy flux to that added at the source is shown in fig. (12) and it is seen that measurements conserve this within 5-10%, consistent with the experimental errors. The constancy of this integral at the value measured at the source together with the neutral environment, and the overall self-preserving nature of the profiles lends considerable confidence to the experiment and would appearto overcome the objections of List (1982) cited earlier.

Some of the third moments are shown in figure (11). These will be used to balance the transport equations for temperature variance $\overline{t^2}$ and turbulence kinetic energy k. The moments $\overline{t^3}$ $\overline{u^3}$, $\overline{ut^2}$

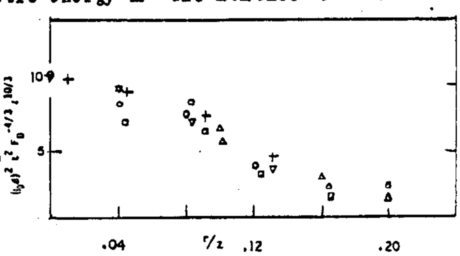

Figure 4. Temperature fluctuations.

and $\overline{u^2t}$ all start from a finite value at the center and then show a slight off-axis peak before rolling off. The moment $\overline{v^3}$ which represents the radial transport of the radial velocity fluctuations peaks at about η=.07. Again there is a small centerline finite value rather than being zero because of the cross-flow errors.

One disappointing feature of the measured third moments is their wide scatter. The record lengths would have to be substantially longer to achieve better statistical convergence, a very difficult problem in view of the large integral time scale and the difficulties in maintaining a stable environment for longer than 12 hours.

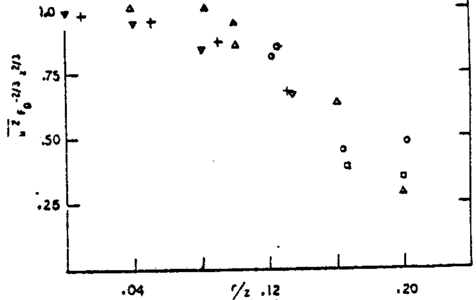

Figure 5. Vertical velocity fluctuations.

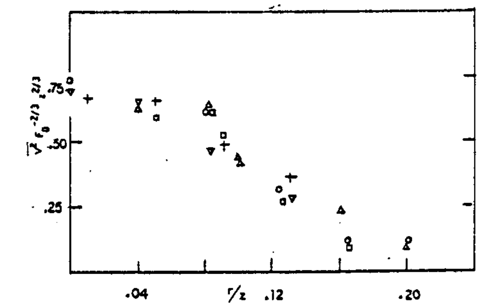

Figure 6. Radial velocity fluctuations.

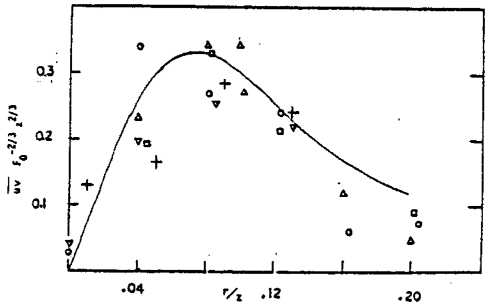

Figure 7. Shear stress

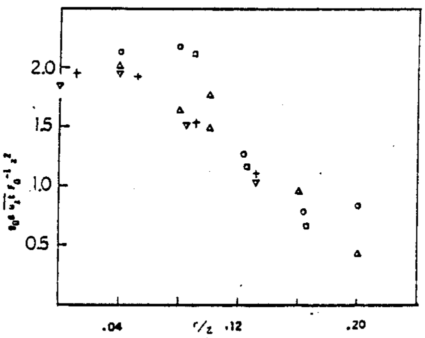

Figure 8. Vertical heat flux.

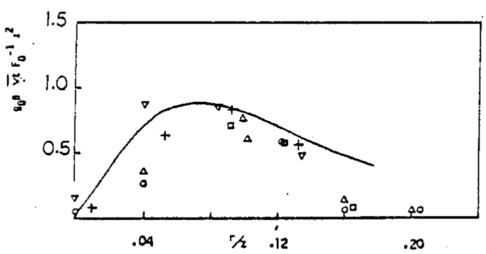

Figure 9. Radial heat flux.

BALANCES FOR THE MEAN ENERGY AND MOMENTUM EQUATIONS.

The measured profiles were substituted in the mean energy and mean momentum equations. The resulting balances are shown in figures (12) and (13) . The error shown is the amount with which one side is smaller or greater than the other. It is seen that

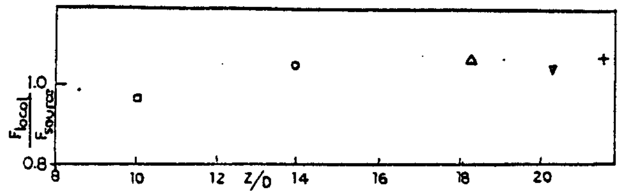

Figure 10. Buoyancy flux at different heights.

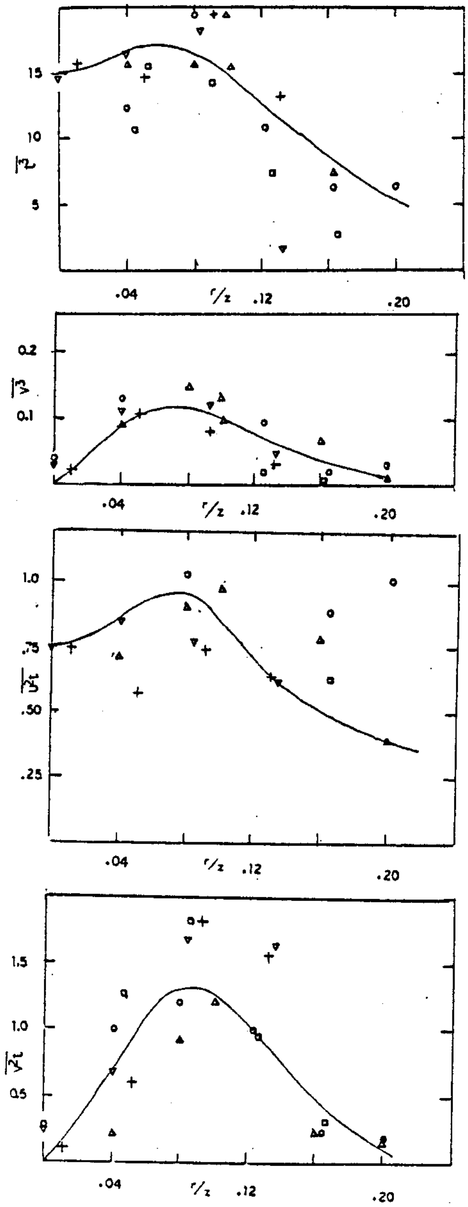

Figure 11. Various third moments.

these errors are within 10% to 15% of the advection term which are typical for such measurements (c.f. Beuther 1980). It should also be noted that the axial transport and advection terms are almost negligible relative to their radial counterparts. The buoyancy production is almost twice the advection of momentum in the vertical direction which is be expected in flows where buoyancy force is solely responsible for setting up the fluid motion.

It should be noted that these balances help to establish the consistency and accuracy of the experimental data since many of the previous free shear flow experiment are found not to satisfy the constraints imposed by the equation of motion.

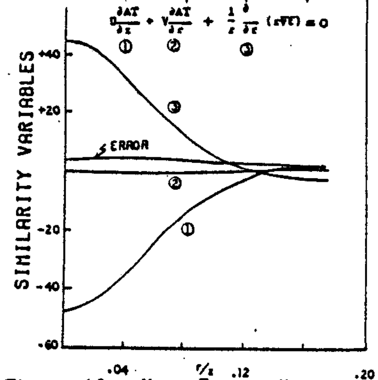

Figure 12. Mean Energy Balance.

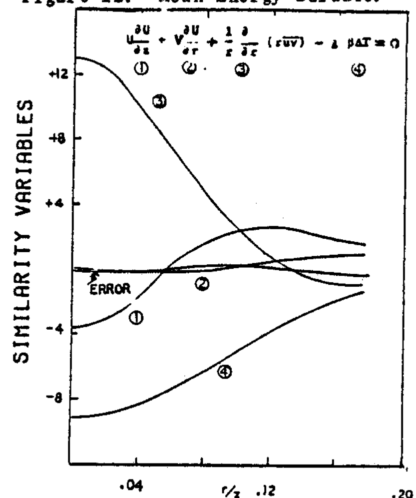

Figure 13. Mean Momentum Balance.

BALANCES FOR THE $\overline{t^2}$ AND k EQAUTIONS.

Since the rates of mechanical and thermal dissipation were not measured, the balances equations for the temperature variance $\overline{t^2}$ and turbulence kinetic energy k were evaluated to obtain their values. These balances also help to compare the magnitude of various other terms appearing in these equations. The transport equations for t and k are given by

$$U\frac{\partial \overline{t^2}}{\partial z} + V\frac{\partial \overline{t^2}}{\partial r} = -2(\overline{ut}\frac{\partial T}{\partial z} - \overline{vt}\frac{\partial T}{\partial r}) - (\frac{\partial}{\partial z}\overline{ut^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\overline{vt^2})) - \epsilon_t \quad (16)$$

$$U\frac{\partial \overline{q^2}}{\partial z} + V\frac{\partial \overline{q^2}}{\partial r} = \frac{\partial}{\partial z}(\frac{\overline{up}}{\rho} + \overline{q^2u}) + \frac{1}{r}\frac{\partial}{\partial r}[r(\frac{\overline{up}}{\rho} + \overline{q^2v})$$

$$- \overline{u^2}\frac{\partial U}{\partial z}\,\overline{uv}\frac{\partial U}{\partial r} - \overline{uv}\frac{\partial V}{\partial z} - \overline{v^2}\frac{\partial V}{\partial r} + g\beta\overline{ut} - \epsilon \quad (17)$$

Since at present there is no method of measuring the pressure diffusion terms it was decided to make an estimate of these. For this purpose we used the second order closure scheme of Lumley (1978) which gives

$$\overline{pu_i}/\rho = -\overline{q^2u_i}/5 \quad (18)$$

i.e the pressure transport in a given direction is about 20% of the turbulent transport in that direction. It is realized that there is no experimental verification of the above relation, but at the same time is believed that it is better to use this estimate rather than just setting the pressure diffusion equal to zero. This also means that the term labelled "disspation" in following graphs is really the sum of the dissipation, the errors due to the above assumption and the measurement errors present in

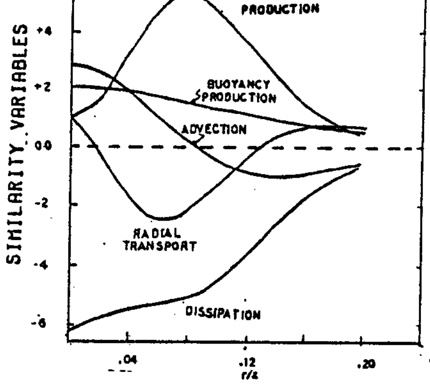

Figure 14. Turbulence Kinetic Energy Balance.

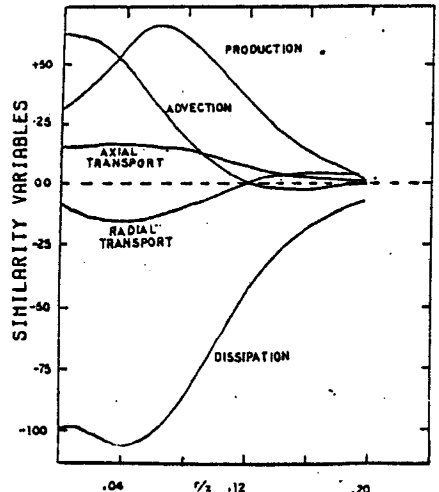

Figure 15. Turbulent Thermal Energy Balance.

the other terms. If the correction for the pressure diffusion term were not included, the "dissipation" terms would be smaller by approximately 8%.

Figures (14) and (15) show the balances for these equations. At the centerline the buoyancy production of turbulence energy is about 60% of the advection terms. After about η = 0.08 the advection term becomes negative, implying that at the outer edges the kinetic energy is being advected downwards. Also the axial transport of kinetic energy by the turbulence is very small as compared to the radial.

It should be mentioned that little confidence can be placed in these balances beyond $\eta \sim .1$ or so due to the various measurement errors associated with flow reversals and cross-flow on hot wires.

SUMMARY

The temperature and velocity measurements were carried out in a neutral environment axisymmetric plume using hot wire probes. In addition to the mean and rms values, various third and fourth order moments were computed from the instantaneous temperature and velocity. The measurements conserve buoyancy to within 5- 10% and satisfy the mean energy and momentum equations to within experimental error. The ratesof thermal amd mechanical dissipation were obtained as the closing terms in the balances for the temperature variance and the turbulence kinetic energy. It is cautioned that these measurements and balances should be interpreted carefully beyond $\eta \gtrsim 0.1$ where the local turbulence intensities begin to increase significantly, with a corresponding increase in the hot-wire errors.

ACKNOWLEDGEMENTS

This work was supported by the National Science

The authors are grateful to Professor Dale Taulbee for many helpful discussions about this work.

REFERENCES

Baker, C.B. (1980) An Analysis of the Turbulent Buoyant Jet. PhD Dissertation, SUNY at Buffalo, Buffalo, NY.

Batchelor, G.K. (1954) Heat Convection and Buoyancy Effects in Fluids. Quart. J. Royal Met. Soc. V.80, p. 339.

Beuther, P.D. (1980) Ph.D. Dissertation, SUNY at Buffalo, Buffalo, NY.

Beuther, P.D. George, W.K. and Shabbir, A. (1987) X-wire Response in Turbulent Flows of High Intensity Turbulence and Low Mean Velocities. Symp. on Thermal Anemometry, Cincinnati, OH.

Beuther, P.D. and George W.K. (1982). Proceedings of the International Heat Transfer Conference, Munich.

Capp, S. C. (1983). Ph.D. dissertation, SUNY at Buffalo.

Chen, C.J. and Rodi, W. (1980), Vertical Buoyant Jets: A Review of Experimental Data. Pergamon, NY.

George, W.K., Jr., Alpert, R.L., Tamanini, F. (1977) Turbulence Measurements in an Axisymmetric Buoyant Plume. Int. J. Heat Mass Trans. V.20, 1145-1154.

George, W.K. and Beuther, P.B., (1982) The Turbulent Buoyant Plume in a Stratified Environment. Proc. of Nat. Cong. Theor. and Appl. Mech., Cornell Univ., Ithaca, NY.

George, W.K., Beuther, P.D., and Shabbir, A. (1987) Polynomial Calibrations for Hot Wires in Thermally-Varying Flows. Symp. on Thermal Anemometry, Cincinnati, OH.

List, E. J. (1982). Turbulent Buoyant Jets and Plumes, ed. W. Rodi, Pergamon, NY.

Nakagome, H. and Hirata, M. (1977) The Structure of Turbulent Diffusion in an Axi-symmetrical Thermal Plume. Heat Trans. β Turbulent Buoyant Convection, (Spalding and Afgan, eds.) McGraw-Hill, NY 367-372.

Rao, V.K. and Brzustowski, T.A. (1969) Combust. Sci. Technol.,1, 171.

Rouse, J., Yih, C.S. and Humphrey, H.W. (1952) Gravitational Convection from a Boundary Source. Tellus, 4, p. 201-10.

Schmidt, W. (1941) Turbulent Propagation of a Stream of Air. ZAMM, V21 6.
Axisymmetric Turbulent Buoyant Plume in a Neutral Envronment. (1985) APS Bulletin

Shabbir, A. and George, W.K. (1985). APS Bulletin, Vol. 30, No. 10, Abstract DD6.

Shabbir, A. (1987) Ph.D. dissertation, SUNY at Buffalo.

Tamanini, F. (1979) The Effect of Buoyancy on the Turbulence Structure of Vertical Round Jets. ASME J. of Heat Transfer., V 100, 4, p. 659-664.

Tutu, N.K. and Chevray, R. (1975) Cross-wire Anemometry in High Intensity Turbulence, J. Fluid Mech., 71, p. 785.

Zel'dovich, Ya B. (1937) Limiting Laws for Turbulent Flows in Free Convection, Zh. EKSP. Teoret. Fiz. 7(12), 1463.